Pearson Physics Level 30
Unit V Momentum and Impulse: Unit V Review Solutions

Student Book pages 503–507

Vocabulary

1. **momentum:** vector quantity defined as the product of the mass of an object and its velocity. Momentum has units of kilogram-metres per second.
   impulse: vector quantity present when two or more objects interact. It is defined as the product of the net force on an object during the interaction and the interaction time. Impulse is also equal to the change in momentum of the object. Impulse has units of newton-seconds or kilogram-metres per second.
   one-dimensional collisions: interactions between two or more objects in which the motion of the objects before and after interaction occurs in one dimension.
   conservation of momentum: universal law that states that for an isolated system consisting of two or more objects, the momentum of the system remains constant.
   conservation of energy: universal law that states that for an isolated system, the total energy of the system remains constant.
   elastic collisions: interactions involving a system of two or more objects in which the total kinetic energy of the system is conserved. So the total initial kinetic energy is equal to the total final kinetic energy.
   inelastic collisions: interactions involving a system of two or more objects in which the total kinetic energy of the system is *not* conserved.
   two-dimensional collisions: interactions between two or more objects in which the motion of the objects before and/or after interaction occurs in two dimensions, e.g., the *x* and *y* directions.
   centre of mass: point on an object where all of its mass can be thought to be concentrated.

Knowledge

Chapter 9

2. Momentum and impulse are both vector quantities. An object has momentum whether or not it interacts with another object. However, an impulse can only be provided when two or more objects interact. Impulse is equivalent to the *change* in momentum of an object, so impulse depends on time while momentum is time independent.
3. The units of momentum are kg•m/s. The units of impulse are N•s, which are equivalent to kg•m/s from the relationship 1 N = 1 kg•m/s^2.
4. Since the answer involves a direction and the units kg•m/s, the student could have calculated either momentum or impulse.
5. In terms of momentum, Newton's second law states that a net force acting on an object is equal to how fast the momentum of the object is changing.
6. A vector quantity means that direction is just as important as magnitude. When two or more objects collide, the vector nature of momentum is important to consider to correctly interpret the motion of the objects immediately after the interaction.

7. An average net force means that the net force acting on an object is not constant over time. When two objects collide, the magnitude of the net force acting on each object increases rapidly to some maximum value, and then decreases as the objects begin to separate. So $\vec{F}_{\text{net}_{\text{ave}}} = \dfrac{\vec{F}_{\text{net}_{\text{max}}} - \vec{F}_{\text{net}_{\text{min}}}}{\Delta t}$, where $\vec{F}_{\text{net}_{\text{max}}}$ and $\vec{F}_{\text{net}_{\text{min}}}$ are the maximum and minimum net forces during the interaction, and Δt is the interaction time.

8. A more massive car (M) has greater momentum than a less massive car (m) travelling at the same velocity ($\vec{v}$). So to avoid an accident, the more massive car would require a greater force to change its velocity than the less massive car. This accounts for the increased number of accidents for more massive cars.

When an accident does occur, the impulse provided to the more massive car would be $\vec{F}_{\text{net}_M} \Delta t = M\Delta\vec{v}$, while that provided to the less massive car would be $\vec{F}_{\text{net}_m} \Delta t = m\Delta\vec{v}$. Each car would experience the same Δt, $\Delta\vec{v}$, and impulse ($\vec{F}_{\text{net}_M} \Delta t = \vec{F}_{\text{net}_m} \Delta t$). Since $M > m$, $\vec{F}_{\text{net}_M} < \vec{F}_{\text{net}_m}$ for the same impulse. So the net force exerted on the motorists in the less massive car would be greater than that exerted on the motorists in the more massive car. This accounts for the reduced survival rate for motorists of less massive cars.

9. The shocks on a mountain bike increase the interaction time during which the net force on the rider acts. So for the same impulse, this increased interaction time decreases the average net force acting on the rider's joints, which minimizes strain injuries.

10. To determine the momentum of an object, you would need to measure the mass of the object in kilograms and the velocity of the object in metres per second.

11. For an isolated system of two or more objects interacting in one or two dimensions, the mass of the system, the momentum of the system, and the total energy are all conserved quantities.

An example of a one-dimensional collision is two ball bearings colliding on a straight track.

An example of a two-dimensional collision is two billiard balls colliding off-centre and deflecting away from each other after the interaction.

12. If a system is isolated, the sum of the internal forces of the system is zero, and yields the law of conservation of momentum.

If the system is not isolated, the system experiences an impulse because a net force acts on it. In an non-isolated system, the conservation of momentum does not apply.

13. To avoid injury when landing on a hard surface, a gymnast should always bend her knees. For the same impulse, the bent knees increase the time it takes to stop her motion, resulting in a decreased net force on the leg joints, hips, and even the neck and spine.

14. If the net force on an object is zero, the momentum of the object will not change.

If a non-zero net force acts on the object, the magnitude of its momentum could decrease, increase, or remain constant.

***p* decreasing and increasing**: If the net force opposes the motion of the object, the object will slow down and its direction of motion may or may not change. For example, when a baseball bat strikes a baseball, the bat exerts a net force on the ball. So for part of the interaction, the magnitude of the momentum of the ball decreases from its initial value (p_i) to zero. At the instant the ball begins to be redirected away from the bat, the magnitude of the momentum of the ball increases from zero to a new value, possibly greater than p_i.

***p* increasing or constant**: If the net force acts in the same direction as the motion of the object, the speed of the object will either remain the same or increase. For example, when a student starts pushing on a skateboard from a stopped position, the net force on the skateboard acts in the same direction as the motion of the skateboard. The speed of the skateboard increases and the magnitude of its momentum also increases. If a small child pushes on the student, the push might not be great enough to change the speed of the skateboard. So the push will not change the momentum of the skateboard.

15. When $\Delta \vec{p}$ is divided by mass, you get $\Delta \vec{v}$, which is the change in velocity.

16. (a) For a given impulse, increasing the time interval decreases the net force during interaction.

(b) For a given impulse, decreasing the net force during interaction increases the time interval.

17.
- To catch a water balloon without breaking it, you need to simultaneously maximize the interaction time and minimize the net force on the balloon. Begin the catch with your hands as far up and forward as possible. Then decrease the momentum of the balloon as gradually and evenly as possible by extending your arms and hands as far down and back as appropriate to stop the motion of the balloon. Alternatively, you could use a blanket held at the corners to catch the balloon, provided that the interaction time is maximized by releasing the tautness of the blanket, as appropriate, during the catch.
- The hiking boot must have maximum cushioning in the sole. This increases the interaction time when each foot collides with the rough ground. The net force exerted by the ground on the foot would have to be minimized as well. With each step, the first contact point with the ground is usually the heel, so this part would need extra shock-absorbing material. The upper of the shoe should also protect the wearer from collisions with branches and sharp rocks.
- To shoot an arrow with maximum velocity using a bow, the bow string must be pulled back as far as possible to enable it to exert the greatest possible forward force on the arrow. The arrow must be released in such a way that the friction of the string with the archer's fingers or arm is minimized.
- To throw a javelin the farthest possible distance, you would extend your arm back as far as possible. When the javelin is released, the net force on the javelin will be exerted over the greatest possible time interval. You would also run forward, putting as much of your bodyweight into the throw as possible. You would twist your torso and throw your body forward; both of these movements increase the net force acting on the javelin and the interaction time.
- For a car to accelerate, the tires must exert a backward force on the ground. From Newton's third law, the ground in turn will exert a forward force of equal magnitude on the car. On an icy road, the force of friction exerted by the tires on the ice must be maximized. Winter tires will increase the force of friction. In order to have the maximum possible acceleration, the accelerator must be pressed slowly and

evenly. This will maximize the interaction time during which the frictional force acts. If the tires spin too quickly, the force of friction causes the ice to melt, producing a thin film of water. This watery layer on the ice reduces the force of friction acting on the tires, so the tires will slide instead of roll and the car will not be able to accelerate.

18. A hunter always presses the butt of a shotgun tight against the shoulder before firing to control the recoil of the gun after firing. From the conservation of momentum, the forward change in momentum of the bullet results in a backward change in momentum of the gun.

If the shotgun is not pressed against the hunter's shoulder, the backward motion of the shotgun will hit the hunter's shoulder causing pain and possible injury. If the shotgun is pressed against the shoulder, the gun and hunter form one combined mass, which is much greater than the mass of the shotgun alone. So the backward motion of this greater mass will be far less than that of the shotgun alone.

19. Draw the vector on a coordinate plane, such as *xy* or north-east, with the tail of the vector at the origin. From the tip of the vector, draw a perpendicular line to one of the axes. One of the components of the vector lies along one of the axes, and the other component lies along the perpendicular from the axis to the tip of the vector. When both components are added together, you get the original vector. To find the magnitude of each component, either measure the lengths using a scale vector diagram, or use the trigonometric functions sine or cosine.

20. The conservation of momentum and the conservation of energy are universal laws because they apply throughout the universe and have no known exceptions.

21. The law of conservation of momentum requires an isolated system because any external net force acting on the system will provide an impulse. That impulse will change the momentum of some or all of the objects in the system, and the momentum of the system will no longer be conserved.

22. **(a)** First calculate the initial and final momenta of each object where both the mass and velocities are known. From the law of conservation of momentum, the vector sum of the initial momentum of the system must equal the vector sum of the final momentum of the system. Decide on an appropriate scale and draw the known momentum vectors, going from tip to tail and making sure the lengths and angles are accurately drawn. The vector needed to return to the start will be the unknown quantity. Measure the length and angle associated with that vector. Then use the equation $p = mv$ to determine the unknown speed.

(b) Resolve all known velocities into components, such as x and y, or north and east. Apply the law of conservation of momentum to each direction to find the components of the unknown velocity. Then use the Pythagorean theorem and the tangent function to find the magnitude and direction of the unknown velocity.

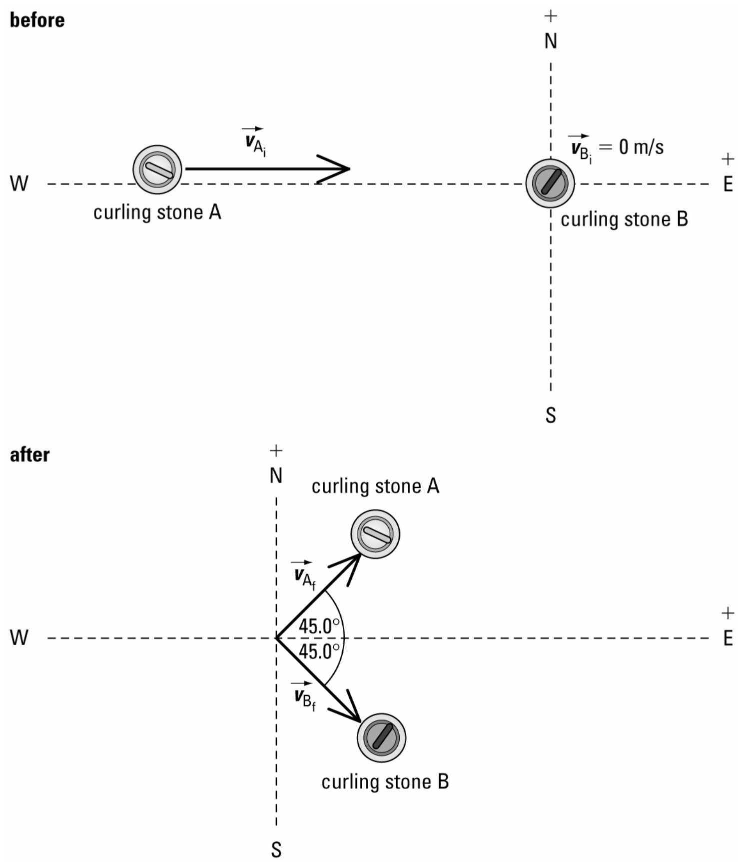
23. When the engines of a rocket burn fuel, the escaping exhaust gas has mass and considerable speed. The backward change in momentum of the gas provides a forward impulse on the rocket. From the law of conservation of momentum, the backward change in momentum of the gas is equal to the forward change in momentum of the rocket. The rocket moves forward because the exhaust gas exerts a net force on the rocket due to its changing momentum, not because it is pushing against anything.

24. The less massive fragment will have the greater speed because its inertia is less than the other fragment. When two objects of unequal mass experience the same net force, the less massive object will have the greater acceleration since its motion is easier to change.

25. It is important to find the velocities of all objects in the system immediately after collision to minimize the effect of external frictional forces so that the conservation of momentum can be applied. External frictional forces would change the final velocity of each object after collision and the conservation of momentum would not apply.

26. The mass of the system, the momentum of the system, and the total energy are all conserved in a collision.

27.



28. Since the wall is fixed to Earth, the ball-wall-Earth system is isolated. So the law of conservation of momentum applies and the momentum of the system is constant. The change in momentum of the ball is equal to the change in momentum of the wall-Earth combination. You cannot detect the change in momentum of the wall-Earth combination because Earth's mass is much greater than the mass of the ball.

29. The neutron and the neutrino are two subatomic particles that were discovered using the law of conservation of momentum and the conservation of energy.

30. In an inelastic collision, some of the kinetic energy of the system is converted to other forms of energy, such as heat, sound, and light. However, the *total* energy of the system remains constant.

31. When no external net force acts on a system, the conservation of momentum applies. So the momentum of the system remains constant, $\vec{p}_{sys_i} = \vec{p}_{sys_f}$.

If there is only one object in the system, the conservation of momentum of the system would be $m_A \vec{v}_{A_i} = m_A \vec{v}_{A_f}$. If you divide both sides by m_A, you get $\vec{v}_{A_i} = \vec{v}_{A_f}$.

This equation means that if no external net force acts on an object, the velocity of the object remains constant. This is equivalent to Newton's first law, which states that an object will continue being at rest or moving at constant velocity unless acted upon by an external non-zero net force.

32. The collision between the bullet and armour plating is inelastic because the total kinetic energy of the system is not conserved. Energy is required to break the bonds of the molecules in the ceramic plate. So most of the kinetic energy of the bullet will be converted to potential energy, heat energy, and sound.

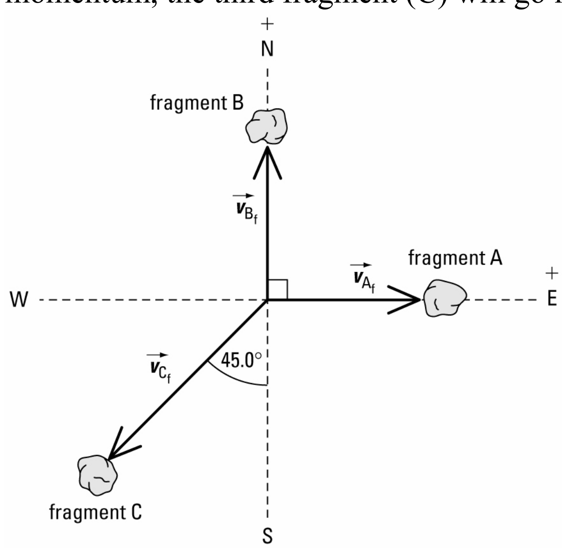
33. The compact car will experience the greater change in its direction of motion just after impact. Since each vehicle will be provided with the same impulse during impact and the compact car has less mass than the van, the change in velocity of the car will be greater for the same impulse.

34. It is possible for the conservation of momentum to be valid even if two objects move faster just before, than just after, collision. For example, suppose a bullet is fired at a ballistic pendulum that is swinging toward the bullet at some initial speed v_{p_i}.

The initial speed of the bullet v_{b_i} is considerable. At the instant the bullet becomes embedded in the pendulum, the bullet provides an impulse to the pendulum. This impulse will cause the pendulum to slow down and maybe swing in the opposite direction but at a much slower speed than v_{b_i} or v_{p_i}.

35. When a jet fighter shoots ammunition, the forward change in momentum of the bullets results in a backward change in momentum of the aircraft, from the conservation of momentum. Since the change in momentum of the bullet is large, the reduced velocity of the aircraft would be noticeable.

36. If one fragment (A) goes east, another (B) goes north, then from the conservation of momentum, the third fragment (C) will go in approximately the SW direction.

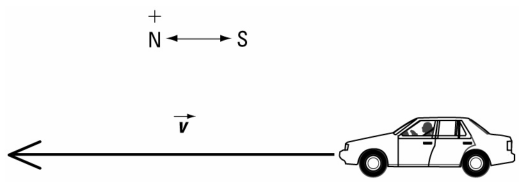


Applications

37. ***Given***

m = 1600 kg

$\vec{v}$ = 8.5 m/s [N]



Required

momentum of car ($\vec{p}$)

Analysis and Solution

The momentum of the car is in the direction of its velocity. So use the scalar form of $\vec{p} = m\vec{v}$ to find the magnitude of the momentum.

$$
\begin{aligned}
p &= mv \\
&= (1600 \text{ kg})(8.5 \text{ m/s}) \\
&= 1.4 \times 10^4 \text{ kg•m/s}
\end{aligned}
$$

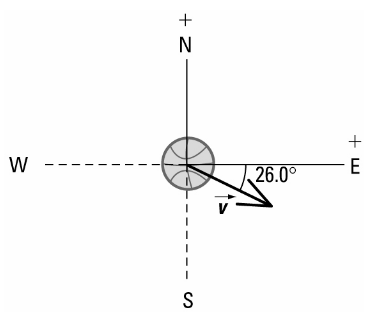
Paraphrase

The momentum of the car is 1.4×10^4 kg•m/s [N].

38. ***Given***

m = 575 g

$\vec{v}$ = 12.4 m/s [26.0º S of E]



Required

momentum vector diagram ($\vec{p}$)

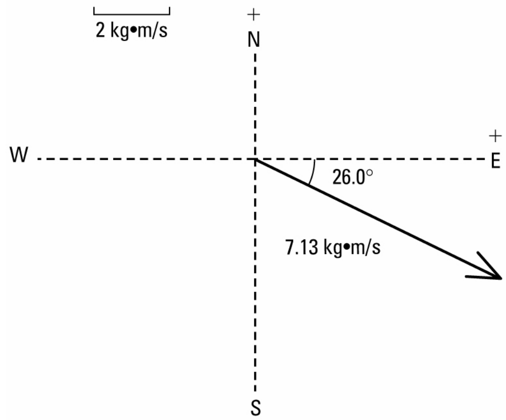
Analysis and Solution

The momentum of the basketball is in the direction of its velocity. So use the scalar form of $\vec{p} = m\vec{v}$ to find the magnitude of the momentum.

$$
\begin{aligned}
p &= mv \\
&= (575 \not{\text{g}})\left(\frac{1 \text{ kg}}{1000 \not{\text{g}}}\right)(12.4 \text{ m/s}) \\
&= 7.13 \text{ kg•m/s}
\end{aligned}
$$

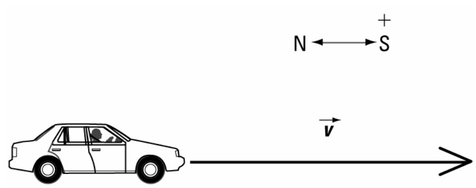
Paraphrase

The momentum of the basketball is 7.13 kg•m/s [26.0º S of E]. The momentum vector diagram is shown below.



39. ***Given***

m = 1250 kg
$\vec{v}$ = 14.8 m/s [S]



Required

momentum of car ($\vec{p}$)

Analysis and Solution

The momentum of the car is in the direction of its velocity. So use the scalar form of $\vec{p} = m\vec{v}$ to find the magnitude of the momentum.

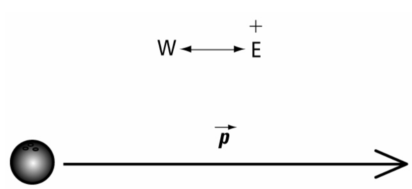
$$\begin{aligned} p &= mv \\ &= (1250 \text{ kg})(14.8 \text{ m/s}) \\ &= 1.85 \times 10^4 \text{ kg•m/s} \end{aligned}$$

Paraphrase

The momentum of the car is 1.85×10^4 kg•m/s [S].

40. ***Given***

$\vec{p}$ = 28 kg•m/s [E]
v = 4.5 m/s



Required
mass of bowling ball (m)

Analysis and Solution
The momentum of the bowling ball is in the direction of its velocity. So use the scalar form of $\vec{p} = m\vec{v}$ to find the mass.

$$p = mv$$

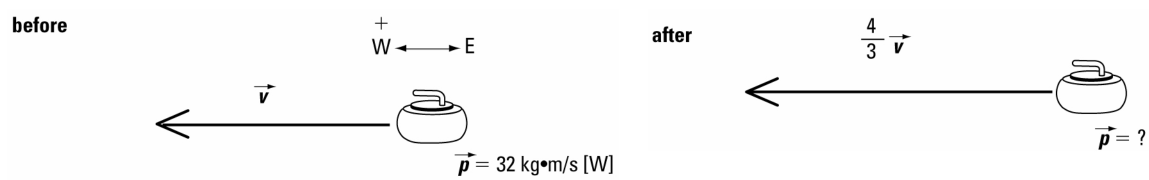
$$m = \frac{p}{v}$$

$$= \frac{28\ \text{kg}\bullet\frac{\cancel{\text{m}}}{\cancel{\text{s}}}}{4.5\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}}$$

$$= 6.2\ \text{kg}$$

Paraphrase
The mass of the bowling ball is 6.2 kg.

41. ***Analysis and Solution***
From the equation $\vec{p} = m\vec{v}$, $p \propto m$ and $p \propto v$.
The figure below represents the situation of the problem.



$p \propto \frac{7}{8}m$ and $p \propto \frac{4}{3}v$

Calculate the factor change of p.

$$\frac{7}{8} \times \frac{4}{3} = \frac{7}{6}$$

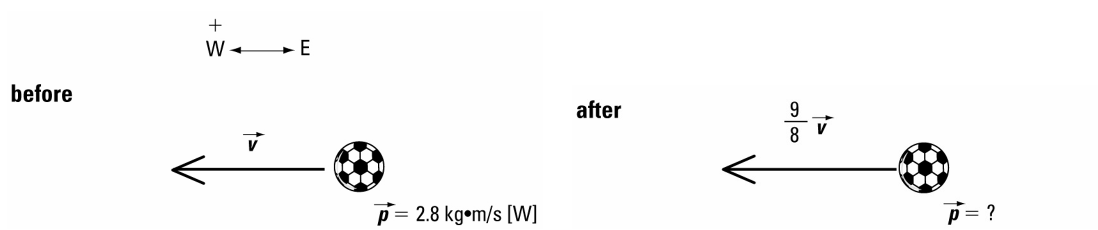
Calculate p.

$$\frac{7}{6}p = \frac{7}{6} \times (32\ \text{kg}\bullet\text{m/s})$$

$$= 37\ \text{kg}\bullet\text{m/s}$$

The new momentum will be 37 kg•m/s [W].

42. ***Analysis and Solution***
From the equation $\vec{p} = m\vec{v}$, $p \propto m$ and $p \propto v$.
The figure below represents the situation of the problem.



$p \propto \frac{3}{4} m$ and $p \propto \frac{9}{8} v$

Calculate the factor change of p.

$$\frac{3}{4} \times \frac{9}{8} = \frac{27}{32}$$

Calculate p.

$$\frac{27}{32} p = \frac{27}{32} \times (2.8 \text{ kg•m/s})$$
$$= 2.4 \text{ kg•m/s}$$

The new momentum will be 2.4 kg•m/s [W].

43. (a) ***Given***

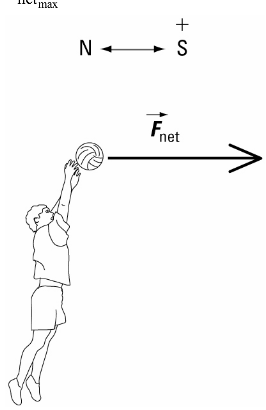
$t_i = 1.0$ ms

$t_f = 4.0$ ms

$F_{net_i} = 0$ N

$F_{net_f} = 0$ N

$F_{net_{max}} = 5000$ N



Required

impulse provided to volleyball

Analysis and Solution

The impulse and velocity after impact are in the south direction.

The magnitude of the impulse is equal to the area under the net force-time graph.

Calculate the time interval.

$$\Delta t = t_f - t_i$$
$$= 4.0 \text{ ms} - 1.0 \text{ ms}$$
$$= 3.0 \text{ ms or } 3.0 \times 10^{-3} \text{ s}$$

$$\text{magnitude of impulse} = \frac{1}{2}(\Delta t)(F_{\text{net}_{\max}})$$
$$= \frac{1}{2}(3.0 \times 10^{-3}\ \text{s})(5000\ \text{N})$$
$$= 7.5\ \text{N•s}$$
$$\text{impulse} = 7.5\ \text{N•s [S]}$$

Paraphrase

The impulse provided to the volleyball is 7.5 N•s [S].

(b) ***Given***

impulse = 7.5 N•s [S] from part (a)

$\vec{v}_i$ = 18 m/s [N]

$\vec{v}_f$ = 11 m/s [S]

Required

mass of volleyball (m)

Analysis and Solution

Impulse is numerically equal to $m\Delta\vec{v}$ or $m(\vec{v}_f - \vec{v}_i)$.

$$+7.5\ \text{N•s} = m(\vec{v}_f - \vec{v}_i)$$
$$= m\{+11\ \text{m/s} - (-18\ \text{m/s})\}$$
$$7.5\ \text{N•s} = m(29\ \text{m/s})$$
$$m = \frac{7.5\ \text{N•s}}{29\ \text{m/s}}$$
$$= \frac{7.5\left(\text{kg•}\frac{\text{m}}{\text{s}^2}\right)(\text{s})}{29\ \frac{\text{m}}{\text{s}}}$$
$$= 0.26\ \text{kg}$$

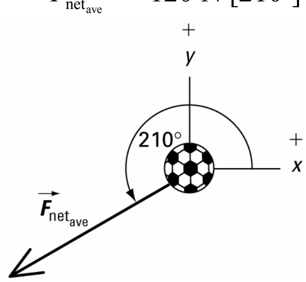
Paraphrase

The mass of the volleyball is 0.26 kg.

44. (a) ***Given***

Δt = 0.0252 s

$\vec{F}_{\text{net}_{\text{ave}}}$ = 120 N [210º]



Required

impulse provided to soccer ball

Analysis and Solution

The impulse and the average net force are in the same direction.

$$\begin{aligned}\text{magnitude of impulse} &= (F_{\text{net}_{\text{ave}}})(\Delta t)\\ &= (120\text{ N})(0.0252\text{ s})\\ &= 3.02\text{ N•s}\\ \text{impulse} &= 3.02\text{ N•s [210º]}\end{aligned}$$

Paraphrase

The impulse provided to the soccer ball is 3.02 N•s [210º].

(b) ***Given***

impulse = 3.024 N•s [210º] from part (a)
m = 0.44 kg

Required

change in velocity of soccer ball ($\Delta \vec{v}$)

Analysis and Solution

Impulse is numerically equal to $m\Delta \vec{v}$.

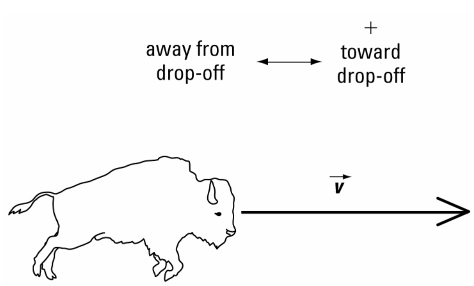
$$\begin{aligned}+3.024\text{ N•s} &= m\Delta \vec{v}\\ \Delta \vec{v} &= \frac{+3.024\text{ N•s}}{m}\\ &= \frac{+3.024\text{ N•s}}{0.44\text{ kg}}\\ &= \frac{+3.024\left(\cancel{\text{kg}}\bullet\frac{\text{m}}{\text{s}^{\cancel{2}}}\right)(\cancel{\text{s}})}{0.44\ \cancel{\text{kg}}}\\ &= +6.9\text{ m/s}\end{aligned}$$

Paraphrase

The change in velocity of the soccer ball is 6.9 m/s [210º].

45. ***Given***

m = 900 kg
$\vec{v}_{\text{i}}$ = 6.0 m/s [toward drop-off]
Δt = 2.0 s
$\vec{v}_{\text{f}}$ = 0 m/s



Required

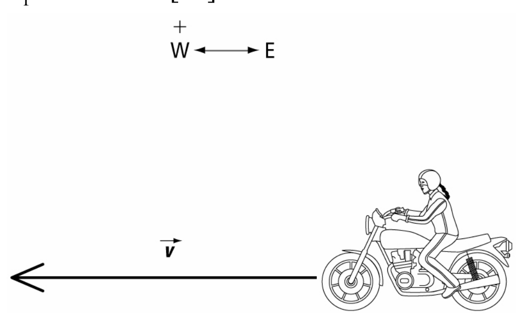
horizontal force bison must exert to stop ($\vec{F}_{\text{b on gr}}$)

Analysis and Solution
From Newton's third law, if the bison exerts a net force toward the drop-off, the ground will exert a force of equal magnitude but opposite direction on the bison. It is this reaction force acting for 2.0 s that provides the required impulse to stop the bison.
Use the equation of impulse to calculate the net force that the ground exerts on the bison.

$$\begin{aligned}
\vec{F}_{\text{net}}\Delta t &= m\Delta\vec{v} \\
\vec{F}_{\text{net}} &= \frac{m\Delta\vec{v}}{\Delta t} \\
&= \frac{m(\vec{v}_{\text{f}} - \vec{v}_{\text{i}})}{\Delta t} \\
&= \frac{(900\text{ kg})(0\ \frac{\text{m}}{\text{s}} - 6.0\ \frac{\text{m}}{\text{s}})}{2.0\text{ s}} \\
&= -2.7 \times 10^{3}\text{ kg•m/s}^{2} \\
\vec{F}_{\text{gr on b}} &= 2.7 \times 10^{3}\text{ N [away from drop-off]} \\
\vec{F}_{\text{b on gr}} &= 2.7 \times 10^{3}\text{ N [toward drop-off]}
\end{aligned}$$

Paraphrase
The bison must exert a horizontal force of 2.7×10^3 N [toward drop-off] to stop in time.

46. (a) ***Given***
$m = 275$ kg
$\vec{v}_{\text{i}} = 0$ m/s
$\vec{v}_{\text{f}} = 20.0$ m/s [W]



Required
minimum impulse on rider-motorcycle system

Analysis and Solution
Impulse is numerically equal to $m\Delta\vec{v}$ or $m(\vec{v}_{\text{f}} - \vec{v}_{\text{i}})$.

$$\begin{aligned}
\text{impulse} &= m\Delta\vec{v} \\
&= (275\text{ kg})(+20.0\text{ m/s} - 0\text{ m/s}) \\
&= +5.50 \times 10^{3}\text{ N•s}
\end{aligned}$$

$= 5.50 \times 10^3$ N•s [W]

Paraphrase

The minimum impulse on the rider-motorcycle system is 5.50×10^3 N•s [W].

(b) ***Given***

$\vec{F}_{net_{ave}} = 710$ N [E]

Required

minimum time interval (Δt)

Analysis and Solution

In order for the rider-motorcycle system to accelerate west, the system must be provided with an impulse directed west. So the average net force must also be directed west.

Use the equation of impulse to calculate the time interval.

$$\text{magnitude of impulse} = (F_{net_{ave}})(\Delta t)$$

$$5.50 \times 10^3 \text{ N•s} = (710 \text{ N})(\Delta t)$$

$$\Delta t = \frac{5.50 \times 10^3 \ \cancel{\text{N}}\text{•s}}{710 \ \cancel{\text{N}}}$$

$$= 7.75 \text{ s}$$

Paraphrase

The minimum time interval is 7.75 s.

(c) From Newton's third law, if the wheels exert a net force east, the ground will exert a force of equal magnitude but opposite direction on the wheels. It is this reaction force acting for a specific time interval that provides the required impulse to accelerate the motorcycle.

(d) It is necessary to specify a minimum impulse because if the impulse provided to the system has a magnitude less than the minimum, the system will not attain a final speed of 20.0 m/s. Similarly, a minimum time is necessary because if the time interval is too long, the system will not have the required acceleration for the given average net force.

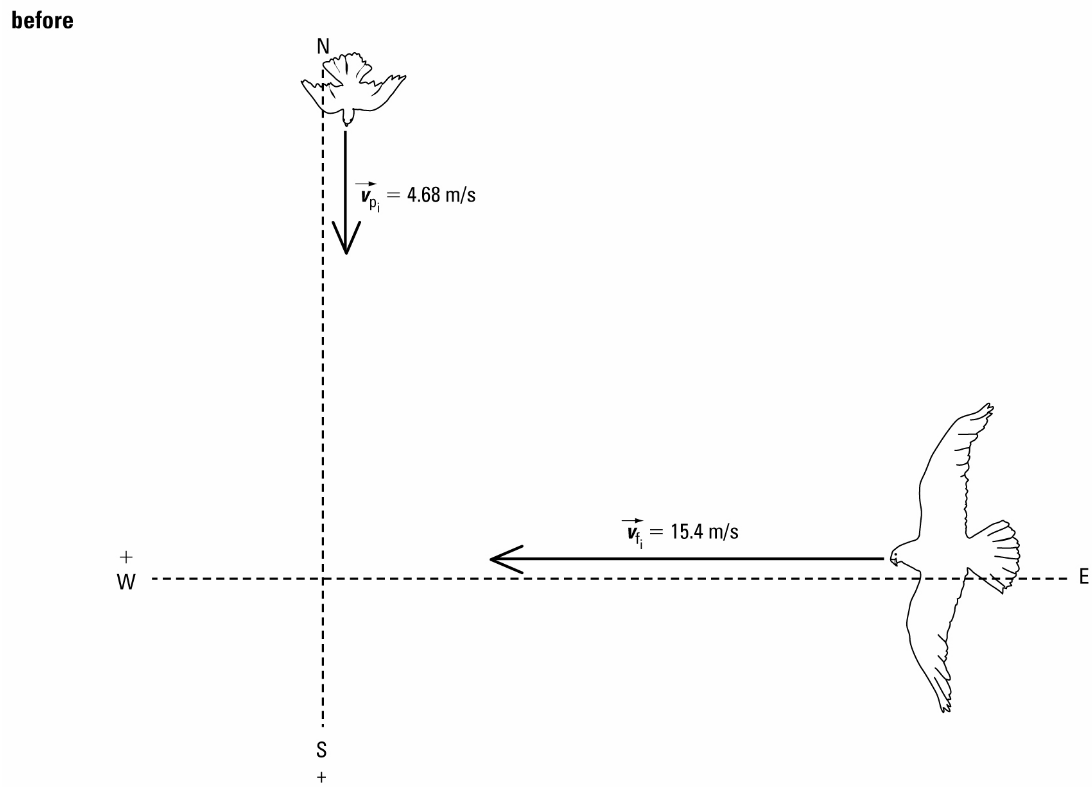
47. ***Given***

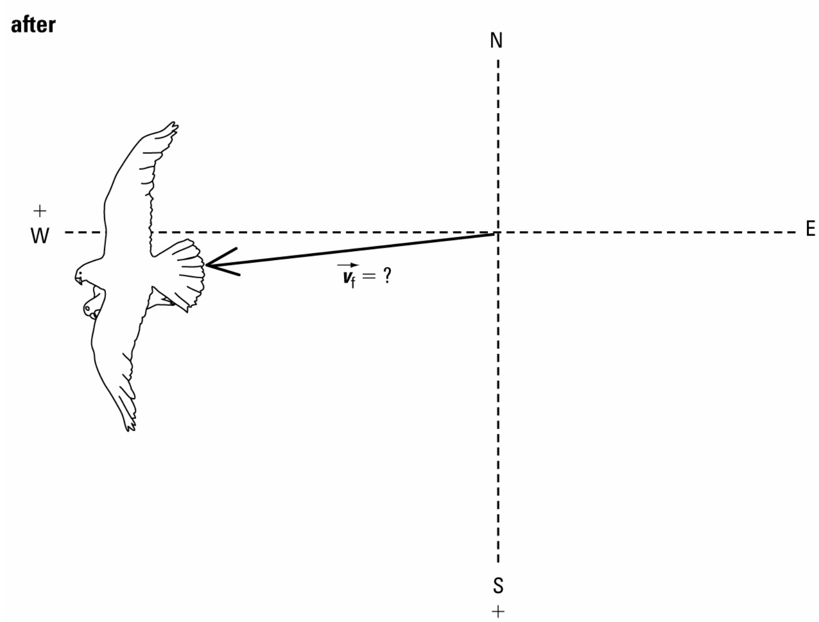
$m_f = 1.15$ kg

$m_p = 0.423$ kg

$\vec{v}_{f_i} = 15.4$ m/s [W]

$\vec{v}_{p_i} = 4.68$ m/s [S]





Required

final velocity of centre of mass ($\vec{v}_f$)

Analysis and Solution

Choose the falcon and the pigeon as an isolated system.

The falcon captured the pigeon. So both birds have the same final velocity.

Resolve all velocities into west and south components.

Vector	West component	South component
$\vec{v}_{f_i}$	15.4 m/s	0
$\vec{v}_{p_i}$	0	4.68 m/s

Apply the law of conservation of momentum to the system in the west and south directions.

W direction

$$p_{sys_{iW}} = p_{sys_{fW}}$$

$$p_{f_{iW}} + p_{p_{iW}} = p_{sys_{fW}}$$

$$m_f v_{f_{iW}} + 0 = (m_f + m_p)\, v_{fW}$$

$$\begin{aligned} v_{fW} &= \left(\frac{m_f}{m_f + m_p}\right) v_{f_{iW}} \\ &= \left(\frac{1.15\text{ kg}}{1.15\text{ kg} + 0.423\text{ kg}}\right)(15.4\text{ m/s}) \\ &= \left(\frac{1.15\ \cancel{\text{kg}}}{1.573\ \cancel{\text{kg}}}\right)(15.4\text{ m/s}) \\ &= 11.26\text{ m/s} \end{aligned}$$

S direction

$$p_{sys_{iS}} = p_{sys_{fS}}$$

$$p_{f_{iS}} + p_{p_{iS}} = p_{sys_{fS}}$$

$$0 + m_p v_{p_{iS}} = (m_f + m_p)\, v_{fS}$$

$$\begin{aligned} v_{fS} &= \left(\frac{m_p}{m_f + m_p}\right) v_{p_{iS}} \\ &= \left(\frac{0.423\text{ kg}}{1.15\text{ kg} + 0.423\text{ kg}}\right)(4.68\text{ m/s}) \\ &= \left(\frac{0.423\ \cancel{\text{kg}}}{1.573\ \cancel{\text{kg}}}\right)(4.68\text{ m/s}) \\ &= 1.259\text{ m/s} \end{aligned}$$

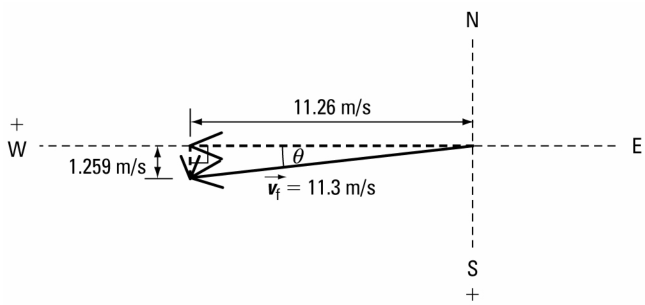
Use the Pythagorean theorem to find the magnitude of $\vec{v}_f$.

$$\begin{aligned} v_f &= \sqrt{(v_{fW})^2 + (v_{fS})^2} \\ &= \sqrt{(11.26\text{ m/s})^2 + (1.259\text{ m/s})^2} \\ &= 11.3\text{ m/s} \end{aligned}$$

Use the tangent function to find the direction of $\vec{v}_f$.

$$\begin{aligned} \tan\theta &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{1.259\ \cancel{\frac{\text{m}}{\text{s}}}}{11.26\ \cancel{\frac{\text{m}}{\text{s}}}} \\ &= 0.1118 \\ \theta &= \tan^{-1}(0.1118) \\ &= 6.4° \end{aligned}$$

From the figure below, this angle is between $\vec{v}_f$ and the west direction.



$$\vec{v}_f = 11.3 \text{ m/s [6.4° S of W]}$$

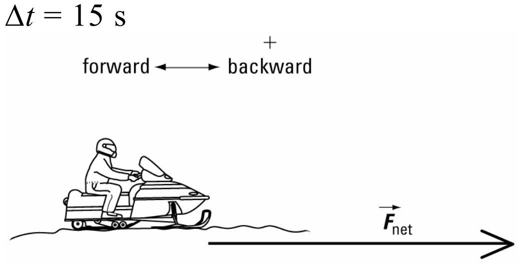
Paraphrase

The velocity of the centre of mass of both birds will be 11.3 m/s [6.4° S of W] immediately after the interaction.

48. (a) ***Given***

$F_{net} = 508$ N

$\Delta t = 15$ s



Required

impulse provided to snowmobile-driver system

Analysis and Solution

From Newton's third law, if the snowmobile exerts a net force backward on the snow, the snow will exert a force of equal magnitude but opposite direction on the snowmobile. It is this reaction force acting for 15 s that provides the required impulse to accelerate the snowmobile.

$$\begin{aligned} \text{magnitude of impulse} &= (F_{net_{ave}})(\Delta t) \\ &= (508 \text{ N})(15 \text{ s}) \\ &= 7.6 \times 10^3 \text{ N•s} \\ \text{impulse} &= 7.6 \times 10^3 \text{ N•s [forward]} \end{aligned}$$

Paraphrase

The snow will provide an impulse of 7.6×10^3 N•s [forward] on the snowmobile.

(b) ***Given***

impulse = 7.62×10^3 N•s [forward] from part (a)

$m_s = 275$ kg

$m_d = 75$ kg

Required
change in velocity of snowmobile ($\Delta \vec{v}$)

Analysis and Solution
The snowmobile and driver move together as a unit. So calculate the total mass.

$$\begin{aligned} m_T &= m_s + m_d \\ &= 275 \text{ kg} + 75 \text{ kg} \\ &= 350 \text{ kg} \end{aligned}$$

Impulse is numerically equal to $m\Delta \vec{v}$.

$$\begin{aligned} -7.62 \times 10^3 \text{ N•s} &= m_T \Delta \vec{v} \\ \Delta \vec{v} &= \frac{-7.62 \times 10^3 \text{ N•s}}{m_T} \\ &= \frac{-7.62 \times 10^3 \text{ N•s}}{350 \text{ kg}} \\ &= \frac{-7.62 \times 10^3 \left(\cancel{\text{kg}} \bullet \frac{\text{m}}{\text{s}^{\cancel{2}}}\right)(\cancel{\text{s}})}{350 \cancel{\text{kg}}} \\ &= -22 \text{ m/s} \end{aligned}$$

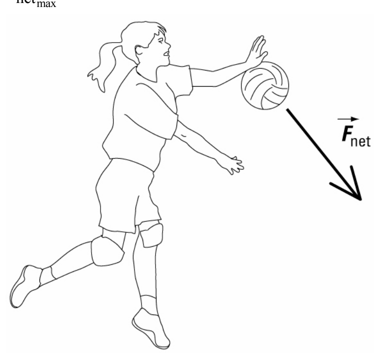
Paraphrase
The change in velocity of the snowmobile is 22 m/s [forward].

49. (a) ***Given***

$t_i = 1.0$ ms $t_f = 5.0$ ms

$F_{net_i} = 0$ N $F_{net_f} = 0$ N

$F_{net_{max}} = 1500$ N



Required
magnitude of impulse provided to volleyball

Analysis and Solution
The magnitude of the impulse is equal to the area under the net force-time graph.
Calculate the time interval.

$$\begin{aligned} \Delta t &= t_f - t_i \\ &= 5.0 \text{ ms} - 1.0 \text{ ms} \end{aligned}$$

$= 4.0 \text{ ms or } 4.0 \times 10^{-3} \text{ s}$

$$\text{magnitude of impulse} = \frac{1}{2}(\Delta t)(F_{\text{net}_{\text{max}}})$$

$$= \frac{1}{2}(4.0 \times 10^{-3} \text{ s})(1500 \text{ N})$$

$$= 3.0 \text{ N•s}$$

Paraphrase

The magnitude of the impulse provided to the volleyball is 3.0 N•s.

(b) ***Given***

magnitude of impulse = 3.00 N•s from part (a)

$\vec{v}_i = 0$ m/s

$m = 275$ g

Required

final speed of volleyball (v_f)

Analysis and Solution

The impulse and velocity after impact are in the same direction.

Impulse is numerically equal to $m\Delta\vec{v}$ or $m(\vec{v}_f - \vec{v}_i)$.

$$+3.00 \text{ N•s} = m(\vec{v}_f - \vec{v}_i)$$

$$= m(\vec{v}_f - 0)$$

$$= m\vec{v}_f$$

$$\vec{v}_f = \frac{+3.00 \text{ N•s}}{m}$$

$$= \frac{+3.00\left(\cancel{\text{kg}}\bullet\frac{\text{m}}{\text{s}^{\cancel{2}}}\right)(\cancel{\text{s}})}{(275\ \cancel{\text{g}})\left(\frac{1\ \cancel{\text{kg}}}{1000\ \cancel{\text{g}}}\right)}$$

$$= +11 \text{ m/s}$$

$$= 11 \text{ m/s}$$

Paraphrase

The final speed of the volleyball is 11 m/s when it leaves the player's hand.

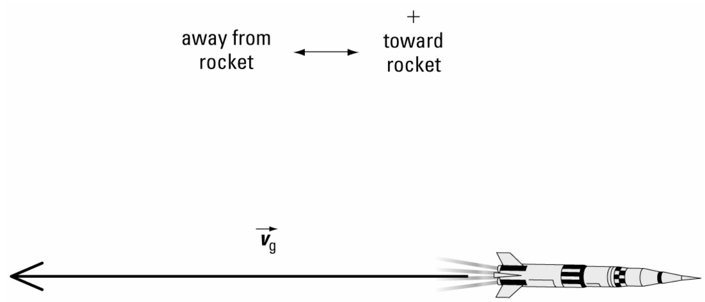
50. ***Given***

$m = 520$ kg

$\Delta t = 0.40$ s

$\vec{v}_i = 0$ m/s

$\vec{v}_f = 5.0 \times 10^4$ m/s [away from rocket]



Required

net force on rocket ($\vec{F}_{\text{gas on r}}$)

Analysis and Solution

From Newton's third law, if the rocket exerts a net force on the exhaust gas to expel it, the gas will exert a force of equal magnitude but opposite direction on the rocket. It is this reaction force acting for 0.40 s that provides the required impulse to accelerate the rocket.
Use the equation of impulse to calculate the net force that the rocket exerts on the gas.

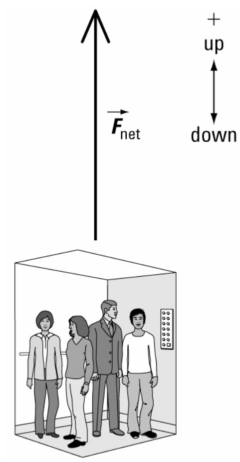
$$\begin{aligned}
\vec{F}_{\text{net}}\,\Delta t &= m\Delta\vec{v} \\
\vec{F}_{\text{net}} &= \frac{m\Delta\vec{v}}{\Delta t} \\
&= \frac{m(\vec{v}_{\text{f}} - \vec{v}_{\text{i}})}{\Delta t} \\
&= \frac{(520\ \text{kg})\left(-5.0\times10^{4}\ \frac{\text{m}}{\text{s}} - 0\ \frac{\text{m}}{\text{s}}\right)}{0.40\ \text{s}} \\
&= -6.5 \times 10^{7}\ \text{kg}\bullet\text{m/s}^2 \\
\vec{F}_{\text{r on gas}} &= 6.5 \times 10^{7}\ \text{N [away from rocket]} \\
\vec{F}_{\text{gas on r}} &= 6.5 \times 10^{7}\ \text{N [toward rocket]}
\end{aligned}$$

Paraphrase

The exhaust gas will exert a force of 6.5×10^7 N [toward rocket] on the rocket.

51. ***Given***

$m = 1700$ kg
$\Delta t = 8.8$ s
$\vec{v}_{\text{i}} = 0$ m/s
$\vec{v}_{\text{f}} = 4.5$ m/s [up]



Required
net force on cable ($\vec{F}_{\text{e on c}}$)

Analysis and Solution
From Newton's third law, if the elevator exerts a net force on the cable, the cable will exert a force of equal magnitude but opposite direction on the elevator. It is this reaction force acting for 8.8 s that provides the required impulse to accelerate the elevator.
Use the equation of impulse to calculate the net force that the cable exerts on the elevator.

$$\vec{F}_{net}\Delta t = m\Delta\vec{v}$$

$$\begin{aligned}\vec{F}_{net} &= \frac{m\Delta\vec{v}}{\Delta t}\\ &= \frac{m(\vec{v}_f - \vec{v}_i)}{\Delta t}\\ &= \frac{(1700\text{ kg})(+4.5\ \frac{\text{m}}{\text{s}} - 0\ \frac{\text{m}}{\text{s}})}{8.8\text{ s}}\\ &= +8.7 \times 10^2\text{ kg•m/s}^2\end{aligned}$$

$$\vec{F}_{\text{c on e}} = 8.7 \times 10^2\text{ N [up]}$$

$$\vec{F}_{\text{e on c}} = 8.7 \times 10^2\text{ N [down]}$$

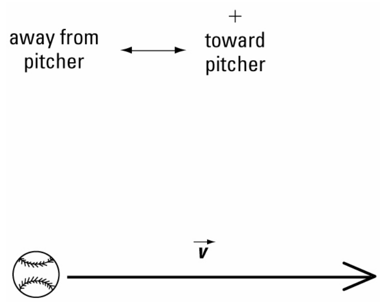
Paraphrase
The elevator must exert a force of 8.7×10^2 N [down] on the cable.

52. (a) ***Given***

$m = 0.146$ kg

$\vec{v}_i = 40$ m/s [away from pitcher]

$\vec{v}_f = 45$ m/s [toward pitcher]



Required

impulse provided to baseball

Analysis and Solution

The impulse and velocity after impact are in the same direction.
Impulse is numerically equal to $m\Delta\vec{v}$ or $m(\vec{v}_f - \vec{v}_i)$.

$$\begin{aligned}\text{impulse} &= m(\vec{v}_f - \vec{v}_i) \\ &= (0.146\text{ kg})\{+45\text{ m/s} - (-40\text{ m/s})\} \\ &= +12\text{ kg•m/s} \\ &= 12\text{ kg•m/s [toward pitcher]}\end{aligned}$$

Paraphrase

The impulse provided to the baseball is 12 kg•m/s [toward pitcher].

(b) ***Given***

impulse = 12.4 kg•m/s [toward pitcher] from part (a)

Δt = 8.0 ms

Required

average net force on ball ($\vec{F}_{\text{net}_{\text{ave}}}$)

Analysis and Solution

Use the equation of impulse to calculate the average net force that the bat exerts on the ball.

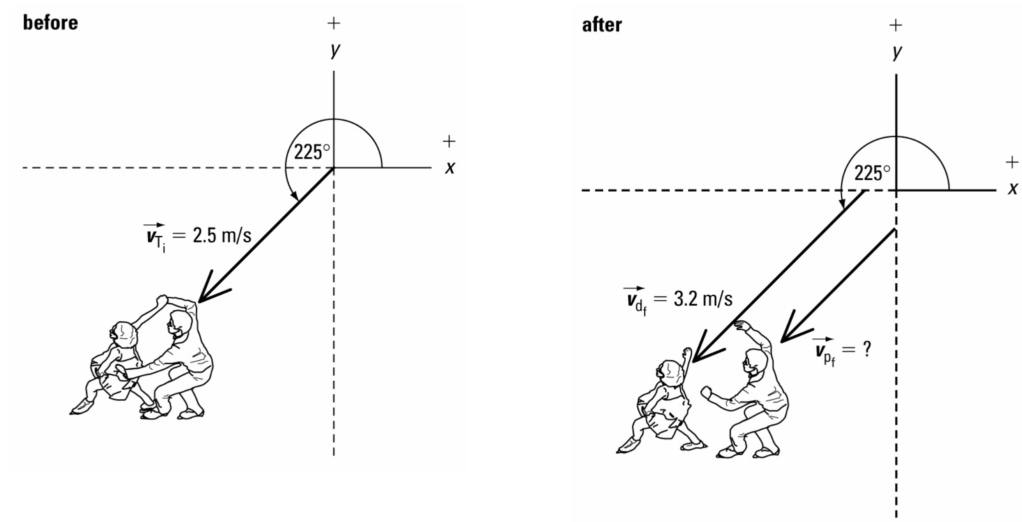
$$\begin{aligned}+12.4\text{ kg•m/s} &= \vec{F}_{\text{net}_{\text{ave}}}\Delta t \\ \vec{F}_{\text{net}_{\text{ave}}} &= \frac{+12.4\text{ kg•m/s}}{\Delta t} \\ &= \frac{+12.4\text{ kg•}\frac{\text{m}}{\text{s}}}{(8.0\text{ }\cancel{\text{ms}})\left(\frac{1\text{ s}}{1000\text{ }\cancel{\text{ms}}}\right)} \\ &= +1.6 \times 10^3\text{ kg•m/s}^2 \\ \vec{F}_{\text{net}_{\text{ave}}} &= 1.6 \times 10^3\text{ N [toward pitcher]}\end{aligned}$$

Paraphrase
The average net force that the bat exerts on the ball is 1.6×10^3 N [toward pitcher].

53. ***Given***

$m_p = 80$ kg $m_d = 45$ kg

$\vec{v}_{T_i} = 2.5$ m/s [225°] $\vec{v}_{d_f} = 3.2$ m/s [225°]



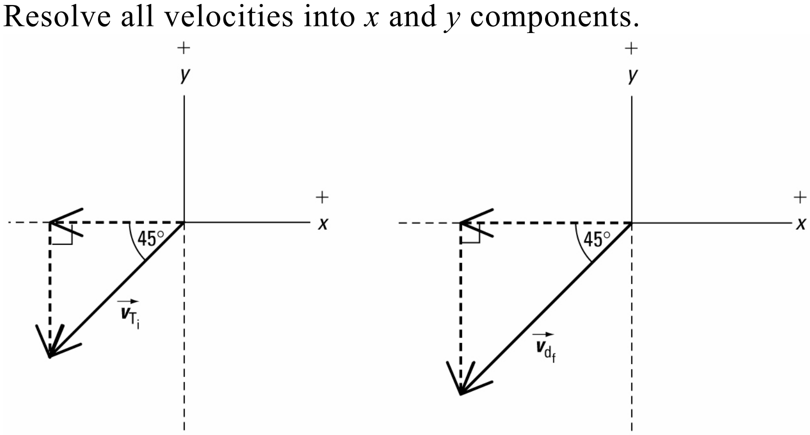
Required

final velocity of partner ($\vec{v}_{p_f}$)

Analysis and Solution
Choose the ice dancer and her partner as an isolated system.
Both skaters move together initially. So find the total mass.

$$\begin{aligned} m_T &= m_p + m_d \\ &= 80 \text{ kg} + 45 \text{ kg} \\ &= 125 \text{ kg} \end{aligned}$$

Resolve all velocities into x and y components.



Vector	*x* component	*y* component
$\vec{v}_{T_i}$	–(2.5 m/s)(cos 45°)	–(2.5 m/s)(sin 45°)
$\vec{v}_{d_f}$	–(3.2 m/s)(cos 45°)	–(3.2 m/s)(sin 45°)

Apply the law of conservation of momentum to the system in the *x* and *y* directions.

x direction

$$p_{sys_{ix}} = p_{sys_{fx}}$$

$$p_{sys_{ix}} = p_{p_{fx}} + p_{d_{fx}}$$

$$m_T v_{T_{ix}} = m_p v_{p_{fx}} + m_d v_{d_{fx}}$$

$$\left(\frac{m_T}{m_p}\right) v_{T_{ix}} = v_{p_{fx}} + \left(\frac{m_d}{m_p}\right) v_{d_{fx}}$$

$$v_{p_{fx}} = \left(\frac{m_T}{m_p}\right) v_{T_{ix}} - \left(\frac{m_d}{m_p}\right) v_{d_{fx}}$$

$$= \left(\frac{125\ \cancel{kg}}{80\ \cancel{kg}}\right)\{-(2.5\text{ m/s})(\cos 45°)\} - \left(\frac{45\ \cancel{kg}}{80\ \cancel{kg}}\right)\{-(3.2\text{ m/s})(\cos 45°)\}$$

$$= -1.49\text{ m/s}$$

y direction

$$p_{sys_{iy}} = p_{sys_{fy}}$$

$$p_{sys_{iy}} = p_{p_{fy}} + p_{d_{fy}}$$

$$m_T v_{T_{iy}} = m_p v_{p_{fy}} + m_d v_{d_{fy}}$$

$$\left(\frac{m_T}{m_p}\right) v_{T_{iy}} = v_{p_{fy}} + \left(\frac{m_d}{m_p}\right) v_{d_{fy}}$$

$$v_{p_{fy}} = \left(\frac{m_T}{m_p}\right) v_{T_{iy}} - \left(\frac{m_d}{m_p}\right) v_{d_{fy}}$$

$$= \left(\frac{125\ \cancel{kg}}{80\ \cancel{kg}}\right)\{-(2.5\text{ m/s})(\sin 45°)\} - \left(\frac{45\ \cancel{kg}}{80\ \cancel{kg}}\right)\{-(3.2\text{ m/s})(\sin 45°)\}$$

$$= -1.49\text{ m/s}$$

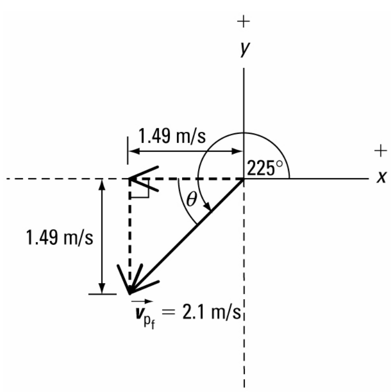
Use the Pythagorean theorem to find the magnitude of $\vec{v}_{p_f}$.

$$\begin{aligned} v_{p_f} &= \sqrt{\left(v_{p_{fx}}\right)^2 + \left(v_{p_{fy}}\right)^2} \\ &= \sqrt{(-1.49 \text{ m/s})^2 + (-1.49 \text{ m/s})^2} \\ &= 2.1 \text{ m/s} \end{aligned}$$

Use the tangent function to find the direction of $\vec{v}_{p_f}$.

$$\begin{aligned} \tan\theta &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{1.49 \cancel{\frac{\text{m}}{\text{s}}}}{1.49 \cancel{\frac{\text{m}}{\text{s}}}} \\ &= 1.000 \\ \theta &= \tan^{-1}(1.000) \\ &= 45° \end{aligned}$$

From the figure below, this angle is between $\vec{v}_{p_f}$ and the negative x-axis. So the direction of $\vec{v}_{p_f}$ measured *counterclockwise* from the positive x-axis is 180° + 45° = 225°.



$$\vec{v}_{p_f} = 2.1 \text{ m/s } [225°]$$

Paraphrase

The velocity of the partner will be 2.1 m/s [225°] immediately after the push.

54. (a) ***Given***

$m_A = 87.0$ kg $\quad m_B = 73.9$ kg

$\vec{v}_{A_i} = 1.21$ m/s [N] $\quad \vec{v}_{B_i} = 1.51$ m/s [S]

$\vec{v}_{B_f} = 1.43$ m/s [N]

Required

final velocity of student A if collision is elastic ($\vec{v}_{A_f}$)

Analysis and Solution

Choose both students as an isolated system.
If the collision is elastic, the total kinetic energy of the system is conserved.

$$E_{k_i} = E_{k_f}$$

$$\frac{1}{2}m_A\left(v_{A_i}\right)^2 + \frac{1}{2}m_B\left(v_{B_i}\right)^2 = \frac{1}{2}m_A\left(v_{A_f}\right)^2 + \frac{1}{2}m_B\left(v_{B_f}\right)^2$$

$$\left(v_{A_i}\right)^2 + \left(\frac{m_B}{m_A}\right)\left(v_{B_i}\right)^2 = \left(v_{A_f}\right)^2 + \left(\frac{m_B}{m_A}\right)\left(v_{B_f}\right)^2$$

$$\left(v_{A_f}\right)^2 = \left(v_{A_i}\right)^2 + \left(\frac{m_B}{m_A}\right)\left\{\left(v_{B_i}\right)^2 - \left(v_{B_f}\right)^2\right\}$$

$$= (1.21 \text{ m/s})^2 + \left(\frac{73.9 \text{ kg}}{87.0 \text{ kg}}\right)\left\{(1.51 \text{ m/s})^2 - (1.43 \text{ m/s})^2\right\}$$

$$= 1.66 \text{ m}^2/\text{s}^2$$

$$v_{A_f} = 1.29 \text{ m/s}$$

Since both students run toward each other and bounce apart after collision, the final velocity of student A will be directed south.

$$\vec{v}_{A_f} = 1.29 \text{ m/s [S]}$$

Paraphrase

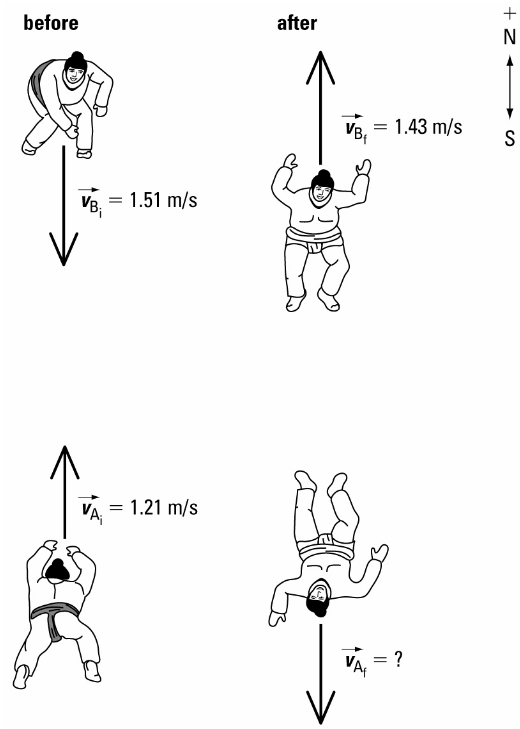
If the collision is elastic, the velocity of student A will be 1.29 m/s [S] immediately after the interaction.

(b) ***Given***

$m_A = 87.0$ kg
$m_B = 73.9$ kg

$\vec{v}_{A_i} = 1.21$ m/s [N]
$\vec{v}_{B_i} = 1.51$ m/s [S]

$\vec{v}_{B_f} = 1.43$ m/s [N]



Required

final velocity of student A ($\vec{v}_{A_f}$)

Analysis and Solution

Choose both students as an isolated system.
Apply the law of conservation of momentum to the system.

$$\vec{p}_{sys_i} = \vec{p}_{sys_f}$$

$$\vec{p}_{A_i} + \vec{p}_{B_i} = \vec{p}_{A_f} + \vec{p}_{B_f}$$

$$m_A\vec{v}_{A_i} + m_B\vec{v}_{B_i} = m_A\vec{v}_{A_f} + m_B\vec{v}_{B_f}$$

$$\vec{v}_{A_i} + \left(\frac{m_B}{m_A}\right)\vec{v}_{B_i} = \vec{v}_{A_f} + \left(\frac{m_B}{m_A}\right)\vec{v}_{B_f}$$

$$\vec{v}_{A_f} = \vec{v}_{A_i} + \left(\frac{m_B}{m_A}\right)(\vec{v}_{B_i} - \vec{v}_{B_f})$$

$$= +1.21 \text{ m/s} + \left(\frac{73.9 \text{ kg}}{87.0 \text{ kg}}\right)(-1.51 \text{ m/s} - 1.43 \text{ m/s})$$

$$= -1.29 \text{ m/s}$$

$$\vec{v}_{A_f} = 1.29 \text{ m/s [S]}$$

Paraphrase

Using the conservation of momentum, the velocity of student A will be 1.29 m/s [S] immediately after the interaction.

(c) Since the final velocity using the conservation of momentum is the same as that using the conservation of kinetic energy, the collision is elastic. If the collision were inelastic, the final velocity in parts (a) and (b) would not be the same.

55. ***Given***

$m_c = 1380$ kg $\qquad m_p = 5.45$ kg

$\vec{v}_{c_i} = 0$ m/s $\qquad \vec{v}_{p_i} = 0$ m/s

$\vec{v}_{p_f} = 190$ m/s [forward]

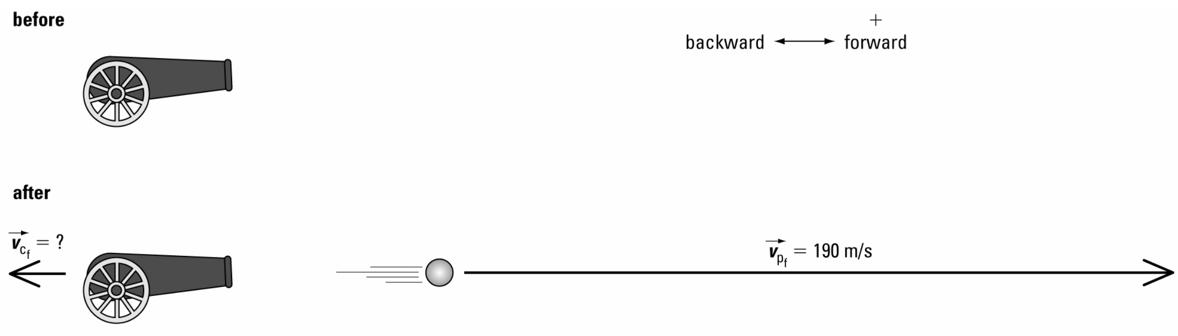


Diagram is not to scale.

Required

final velocity of cannon ($\vec{v}_{c_f}$)

Analysis and Solution

Choose the cannon and projectile as an isolated system.
The cannon and projectile each have an initial velocity of zero. So the system has an initial momentum of zero.

$$\vec{p}_{sys_i} = 0$$

Apply the law of conservation of momentum to the system.

$$\vec{p}_{sys_i} = \vec{p}_{sys_f}$$

$$\vec{p}_{sys_i} = \vec{p}_{c_f} + \vec{p}_{p_f}$$

$$0 = m_c\vec{v}_{c_f} + m_p\vec{v}_{p_f}$$

$$\vec{v}_{c_f} = -\left(\frac{m_p}{m_c}\right)\vec{v}_{p_f}$$

$$= -\left(\frac{5.45 \cancel{\text{kg}}}{1380 \cancel{\text{kg}}}\right)(+190 \text{ m/s})$$

$$= -0.750 \text{ m/s}$$

$$\vec{v}_{c_f} = 0.750 \text{ m/s [backward]}$$

Paraphrase

The velocity of the cannon will be 0.750 m/s [backward] immediately after firing the projectile.

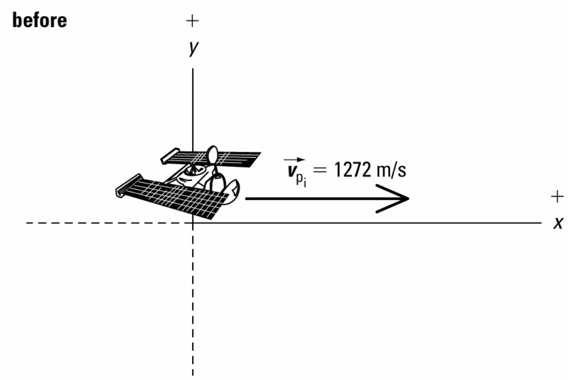
56. ***Given***

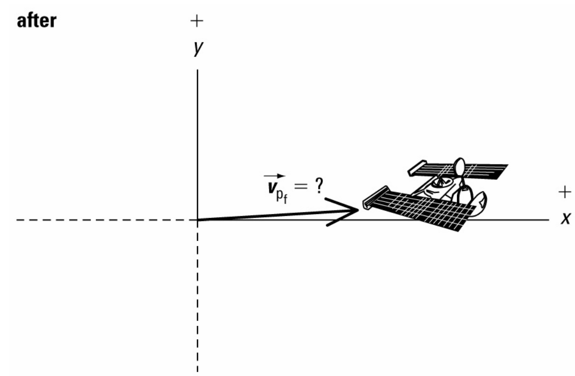
$m_p = 3650$ kg

$\vec{v}_{p_i} = 1272$ m/s [0°]

$\vec{F}_{net} = 1.80 \times 10^4$ N [90.0°]

$\Delta t = 15.6$ s





Required

final velocity of space probe ($\vec{v}_{p_f}$)

Analysis and Solution

Since the impulse is perpendicular to the initial velocity of the space probe, resolve $\vec{F}_{net}$ and $\vec{v}_{p_i}$ into x and y components to determine the impulse along each axis.

Vector	*x* component	*y* component
$\vec{F}_{net}$	0	1.80×10^4 N
$\vec{v}_{p_i}$	1272 m/s	0

Impulse is numerically equal to $m\Delta\vec{v}$ or $m(\vec{v}_f - \vec{v}_i)$.

Use the equation of impulse to calculate the final velocity of the space probe in the x and y directions.

$$\vec{F}_{net}\Delta t = m\Delta\vec{v}$$

x direction

$$F_{net_x}\Delta t = m(v_{p_{fx}} - v_{p_{ix}})$$

$$0 = v_{p_{fx}} - v_{p_{ix}}$$

y direction

$$F_{net_y}\Delta t = m(v_{p_{fy}} - v_{p_{iy}})$$

$$F_{net_y}\Delta t = m(v_{p_{fy}} - 0)$$

$$v_{p_{fx}} = v_{p_{ix}} = 1272 \text{ m/s}$$

$$v_{p_{fy}} = \frac{F_{net_y} \Delta t}{m} = \frac{\left(1.80 \times 10^4 \text{ kg} \cdot \frac{\text{m}}{\text{s}^2}\right)(15.6 \text{ s})}{3650 \text{ kg}} = 76.93 \text{ m/s}$$

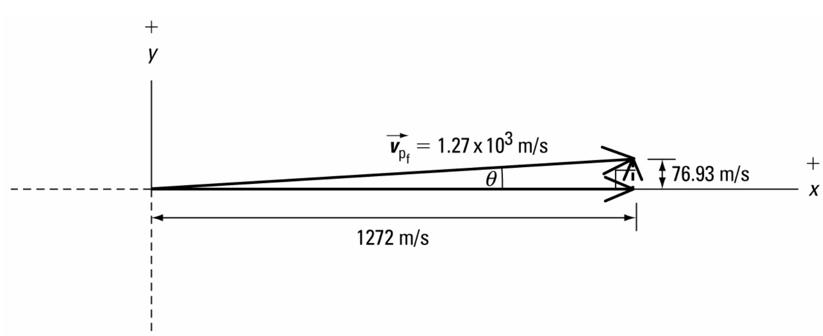
Use the Pythagorean theorem to find the magnitude of $\vec{v}_{p_f}$.

$$\begin{aligned} v_{p_f} &= \sqrt{\left(v_{p_{fx}}\right)^2 + \left(v_{p_{fy}}\right)^2} \\ &= \sqrt{(1272 \text{ m/s})^2 + (76.93 \text{ m/s})^2} \\ &= 1.27 \times 10^3 \text{ m/s} \end{aligned}$$

Use the tangent function to find the direction of $\vec{v}_{p_f}$.

$$\begin{aligned} \tan\theta &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{76.93 \frac{\text{m}}{\text{s}}}{1272 \frac{\text{m}}{\text{s}}} \\ &= 0.0605 \\ \theta &= \tan^{-1}(0.0605) \\ &= 3.5° \end{aligned}$$

From the figure below, this angle is between $\vec{v}_{p_f}$ and the positive x-axis.



$$\vec{v}_{p_f} = 1.27 \times 10^3 \text{ m/s [3.5°]}$$

Paraphrase

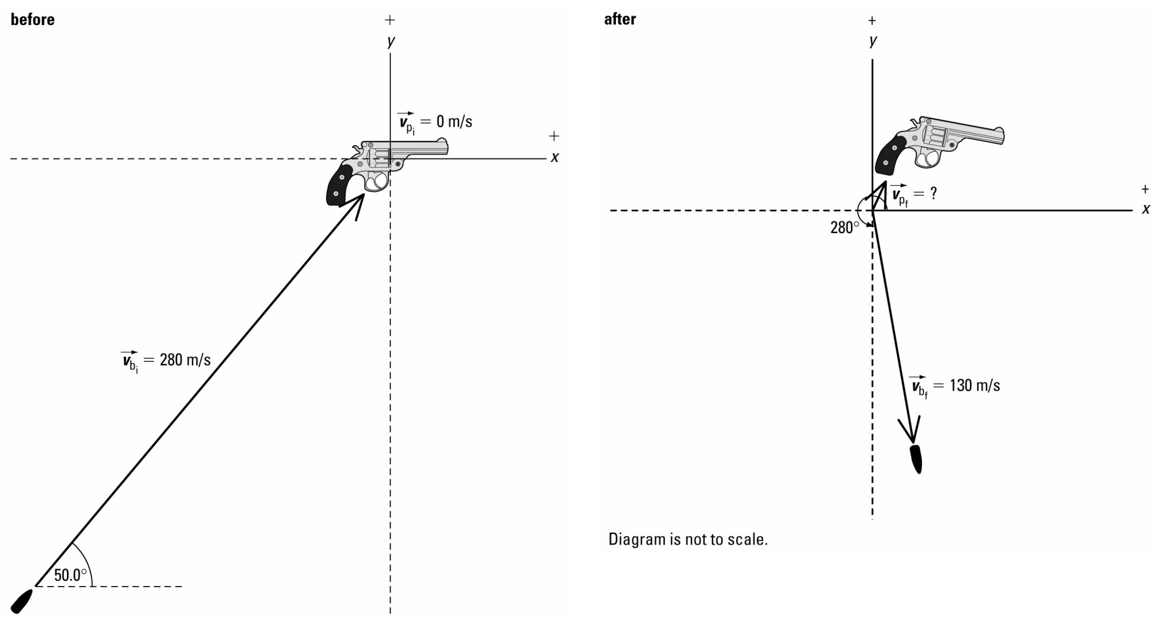
The newly adjusted velocity of the space probe will be 1.27×10^3 m/s [3.5°].

57. ***Given***

$m_p = 1.60$ kg $m_b = 15$ g

$\vec{v}_{p_i} = 0$ m/s $\vec{v}_{b_i} = 280$ m/s [50.0°]

$\vec{v}_{b_f} = 130$ m/s [280°]



Required

final velocity of pistol ($\vec{v}_{p_f}$)

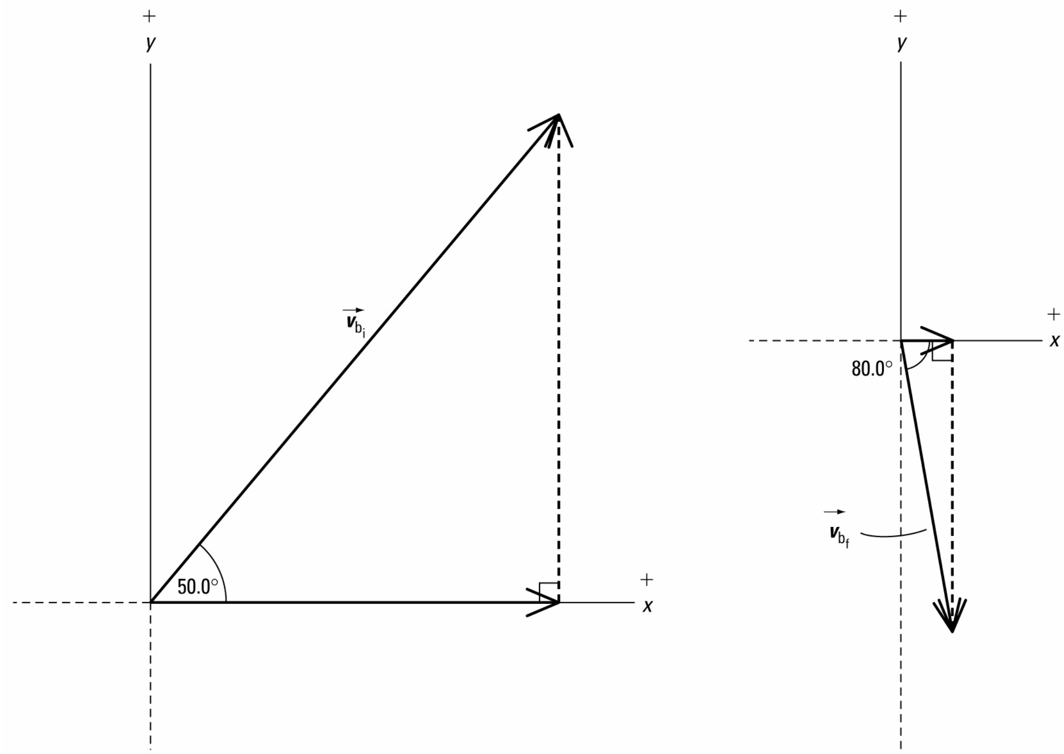
Analysis and Solution

Convert the mass of the bullet to kilograms.

$$m_b = 15 \text{ g} \times \frac{1 \text{ kg}}{1000 \text{ g}}$$

$$= 0.015 \text{ kg}$$

Choose the pistol and bullet as an isolated system.
Resolve all velocities into x and y components.



Vector	*x* component	*y* component
$\vec{v}_{b_i}$	(280 m/s)(cos 50.0°)	(280 m/s)(sin 50.0°)
$\vec{v}_{b_f}$	(130 m/s)(cos 80.0°)	–(130 m/s)(sin 80.0°)

The pistol has an initial velocity of zero. So its initial momentum is zero.

$$\vec{p}_{p_i} = 0$$

Apply the law of conservation of momentum to the system in the *x* and *y* directions.

x direction

$$\begin{aligned}
p_{sys_{ix}} &= p_{sys_{fx}} \\
p_{p_{ix}} + p_{b_{ix}} &= p_{p_{fx}} + p_{b_{fx}} \\
0 + m_b v_{b_{ix}} &= m_p v_{p_{fx}} + m_b v_{b_{fx}} \\
\left(\frac{m_b}{m_p}\right) v_{b_{ix}} &= v_{p_{fx}} + \left(\frac{m_b}{m_p}\right) v_{b_{fx}} \\
v_{p_{fx}} &= \left(\frac{m_b}{m_p}\right)(v_{b_{ix}} - v_{b_{fx}}) \\
&= \left(\frac{0.015\ \cancel{\text{kg}}}{1.60\ \cancel{\text{kg}}}\right)\{(280\text{ m/s})(\cos 50.0°) - (130\text{ m/s})(\cos 80.0°)\} \\
&= 1.48\text{ m/s}
\end{aligned}$$

y direction

$$p_{\text{sys}_{\text{i}y}} = p_{\text{sys}_{\text{f}y}}$$

$$p_{\text{p}_{\text{i}y}} + p_{\text{b}_{\text{i}y}} = p_{\text{p}_{\text{f}y}} + p_{\text{b}_{\text{f}y}}$$

$$0 + m_{\text{b}} v_{\text{b}_{\text{i}y}} = m_{\text{p}} v_{\text{p}_{\text{f}y}} + m_{\text{b}} v_{\text{b}_{\text{f}y}}$$

$$\left(\frac{m_{\text{b}}}{m_{\text{p}}}\right) v_{\text{b}_{\text{i}y}} = v_{\text{p}_{\text{f}y}} + \left(\frac{m_{\text{b}}}{m_{\text{p}}}\right) v_{\text{b}_{\text{f}y}}$$

$$\begin{aligned} v_{\text{p}_{\text{f}y}} &= \left(\frac{m_{\text{b}}}{m_{\text{p}}}\right)(v_{\text{b}_{\text{i}y}} - v_{\text{b}_{\text{f}y}}) \\ &= \left(\frac{0.015\ \cancel{\text{kg}}}{1.60\ \cancel{\text{kg}}}\right)\{(280\text{ m/s})(\sin 50.0°) - [-(130\text{ m/s})(\sin 80.0°)]\} \\ &= 3.21\text{ m/s} \end{aligned}$$

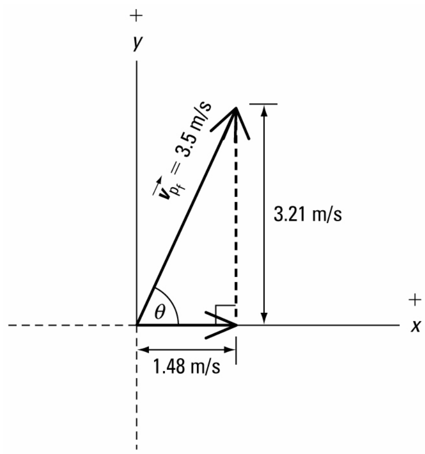
Use the Pythagorean theorem to find the magnitude of $\vec{v}_{\text{p}_{\text{f}}}$.

$$\begin{aligned} v_{\text{p}_{\text{f}}} &= \sqrt{\left(v_{\text{p}_{\text{f}x}}\right)^2 + \left(v_{\text{p}_{\text{f}y}}\right)^2} \\ &= \sqrt{(1.48\text{ m/s})^2 + (3.21\text{ m/s})^2} \\ &= 3.5\text{ m/s} \end{aligned}$$

Use the tangent function to find the direction of $\vec{v}_{\text{p}_{\text{f}}}$.

$$\begin{aligned} \tan\theta &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{3.21\ \cancel{\frac{\text{m}}{\text{s}}}}{1.48\ \cancel{\frac{\text{m}}{\text{s}}}} \\ &= 2.176 \\ \theta &= \tan^{-1}(2.176) \\ &= 65.3° \end{aligned}$$

From the figure below, this angle is between $\vec{v}_{p_f}$ and the positive x-axis.



$\vec{v}_{p_f} = 3.5$ m/s [65.3°]

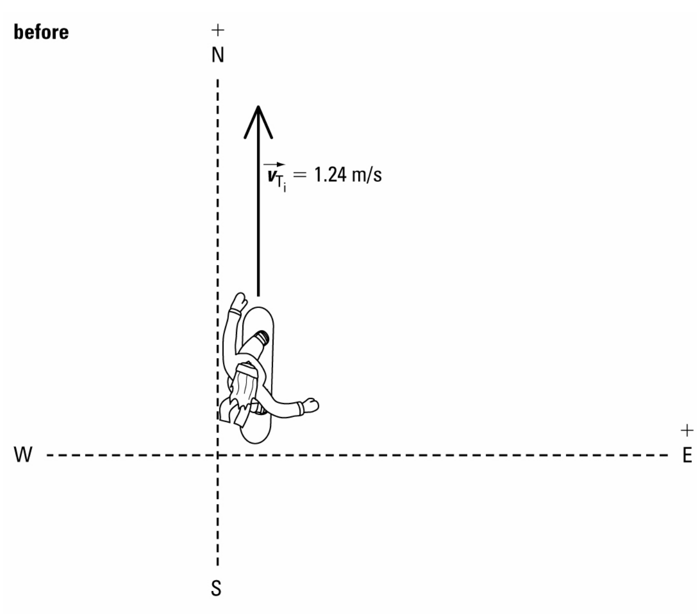
Paraphrase

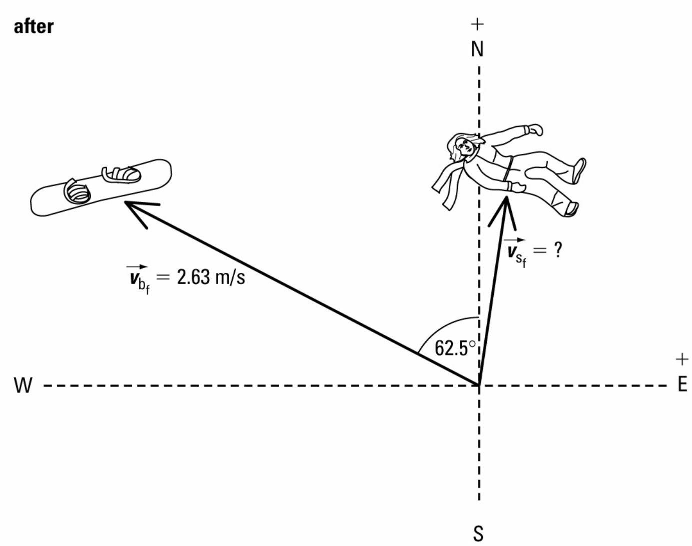
The velocity of the pistol will be 3.5 m/s [65.3°] immediately after the interaction.

58. ***Given***

$m_s = 52.5$ kg $\quad$ $m_b = 4.06$ kg

$\vec{v}_{T_i} = 1.24$ m/s [N] $\quad$ $\vec{v}_{b_f} = 2.63$ m/s [62.5° W of N]





Required

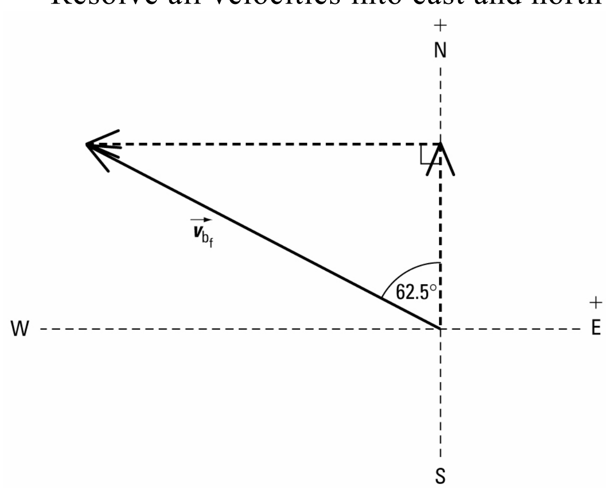
final velocity of snowboarder ($\vec{v}_{s_f}$)

Analysis and Solution

Choose the snowboarder and snowboard as an isolated system.
The snowboarder and snowboarder have the same initial velocity.
So, calculate the total mass.

$$\begin{aligned} m_T &= m_s + m_b \\ &= 52.5 \text{ kg} + 4.06 \text{ kg} \\ &= 56.56 \text{ kg} \end{aligned}$$

Resolve all velocities into east and north components.



Vector	East component	North component
$\vec{v}_{T_i}$	0	1.24 m/s
$\vec{v}_{b_f}$	–(2.63 m/s)(sin 62.5°)	(2.63 m/s)(cos 62.5°)

Apply the law of conservation of momentum to the system in the east and north directions.

E direction

$$p_{sys_{iE}} = p_{sys_{fE}}$$

$$p_{sys_{iE}} = p_{s_{fE}} + p_{b_{fE}}$$

$$m_T v_{T_{iE}} = m_s v_{s_{fE}} + m_b v_{b_{fE}}$$

$$0 = m_s v_{s_{fE}} + m_b v_{b_{fE}}$$

$$v_{s_{fE}} = -\left(\frac{m_b}{m_s}\right) v_{b_{fE}}$$

$$= -\left(\frac{4.06 \ \cancel{kg}}{52.5 \ \cancel{kg}}\right)\{-(2.63 \text{ m/s})(\sin 62.5°)\}$$

$$= 0.1804 \text{ m/s}$$

N direction

$$p_{sys_{iN}} = p_{sys_{fN}}$$

$$p_{sys_{iN}} = p_{s_{fN}} + p_{b_{fN}}$$

$$m_T v_{T_{iN}} = m_s v_{s_{fN}} + m_b v_{b_{fN}}$$

$$\left(\frac{m_T}{m_s}\right) v_{T_{iN}} = v_{s_{fN}} + \left(\frac{m_b}{m_s}\right) v_{b_{fN}}$$

$$v_{s_{fN}} = \left(\frac{m_T}{m_s}\right) v_{T_{iN}} - \left(\frac{m_b}{m_s}\right) v_{b_{fN}}$$

$$= \left(\frac{56.56 \ \cancel{kg}}{52.5 \ \cancel{kg}}\right)(1.24 \text{ m/s}) - \left(\frac{4.06 \ \cancel{kg}}{52.5 \ \cancel{kg}}\right)(2.63 \text{ m/s})(\cos 62.5°)$$

$$= 1.242 \text{ m/s}$$

Use the Pythagorean theorem to find the magnitude of $\vec{v}_{s_f}$.

$$v_{s_f} = \sqrt{\left(v_{s_{fE}}\right)^2 + \left(v_{s_{fN}}\right)^2}$$

$$= \sqrt{(0.1804 \text{ m/s})^2 + (1.242 \text{ m/s})^2}$$

$$= 1.26 \text{ m/s}$$

Use the tangent function to find the direction of $\vec{v}_{s_f}$.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

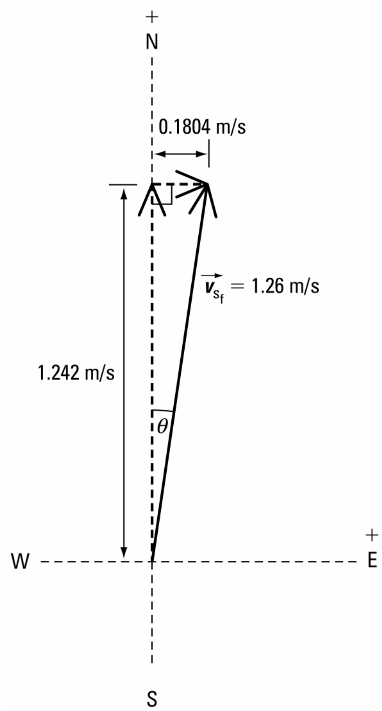
$$= \frac{1.242\ \frac{\text{m}}{\text{s}}}{0.1804\ \frac{\text{m}}{\text{s}}}$$

$$= 6.884$$

$$\theta = \tan^{-1}(6.884)$$

$$= 81.7°$$

From the figure below, this angle is between $\vec{v}_{s_f}$ and the east direction. So the direction of $\vec{v}_{s_f}$ measured from north is 90.0° – 81.7° = 8.3°.



$$\vec{v}_{s_f} = 1.26 \text{ m/s [8.3° E of N]}$$

Paraphrase

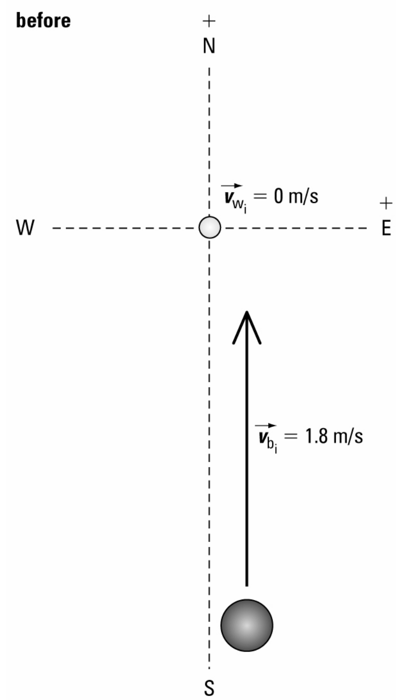
The velocity of the snowboarder will be 1.26 m/s [8.3° E of N] immediately after the kick.

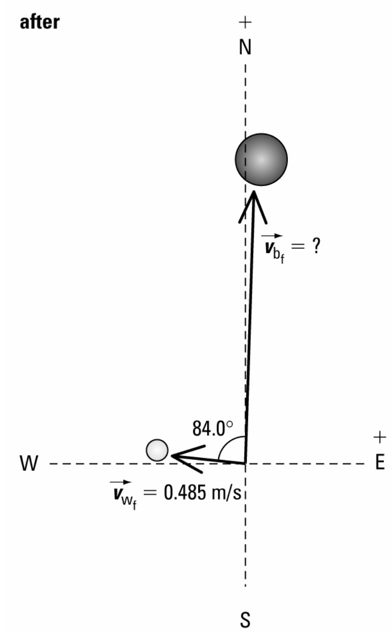
59. (a) ***Given***

m_b = 1.26 kg m_w = 0.145 kg

$\vec{v}_{b_i}$ = 1.8 m/s [N] $\vec{v}_{w_i}$ = 0 m/s

$\vec{v}_{w_f}$ = 0.485 m/s [84.0° W of N]



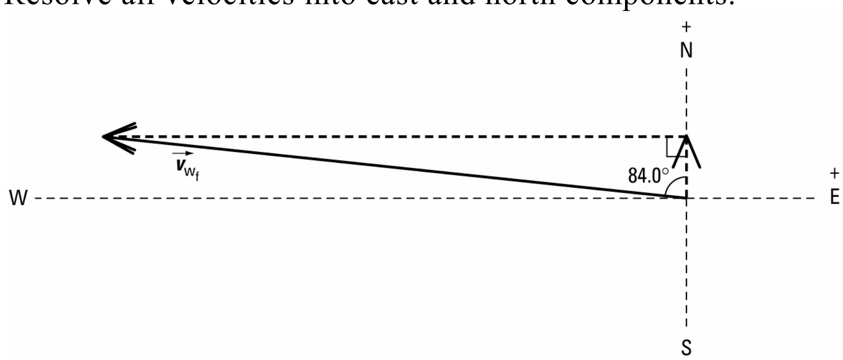


Required

final velocity of brown bocce ball ($\vec{v}_{b_f}$)

Analysis and Solution

Choose the brown and white bocce balls as an isolated system.
Resolve all velocities into east and north components.



Vector	East component	North component
$\vec{v}_{b_i}$	0	1.8 m/s
$\vec{v}_{w_f}$	–(0.485 m/s)(sin 84.0°)	(0.485 m/s)(cos 84.0°)

The white ball has an initial velocity of zero. So its initial momentum is zero.

$$\vec{p}_{w_i} = 0$$

Apply the law of conservation of momentum to the system in the east and north directions.

E direction

$$p_{sys_{iE}} = p_{sys_{fE}}$$

$$p_{b_{iE}} + p_{w_{iE}} = p_{b_{fE}} + p_{w_{fE}}$$

$$m_b v_{b_{iE}} + 0 = m_b v_{b_{fE}} + m_w v_{w_{fE}}$$

$$0 + 0 = m_b v_{b_{fE}} + m_w v_{w_{fE}}$$

$$v_{b_{fE}} = -\left(\frac{m_w}{m_b}\right) v_{w_{fE}}$$

$$= -\left(\frac{0.145\ \cancel{kg}}{1.26\ \cancel{kg}}\right)\{-(0.485\text{ m/s})(\sin 84.0°)\}$$

$$= 0.0555\text{ m/s}$$

N direction

$$p_{sys_{iN}} = p_{sys_{fN}}$$

$$p_{b_{iN}} + p_{w_{iN}} = p_{b_{fN}} + p_{w_{fN}}$$

$$m_b v_{b_{iN}} + 0 = m_b v_{b_{fN}} + m_w v_{w_{fN}}$$

$$v_{b_{iN}} = v_{b_{fN}} + \left(\frac{m_w}{m_b}\right) v_{w_{fN}}$$

$$v_{b_{fN}} = v_{b_{iN}} - \left(\frac{m_w}{m_b}\right) v_{w_{fN}}$$

$$= 1.8\text{ m/s} - \left(\frac{0.145\ \cancel{kg}}{1.26\ \cancel{kg}}\right)(0.485\text{ m/s})(\cos 84.0°)$$

$$= 1.79\text{ m/s}$$

Use the Pythagorean theorem to find the magnitude of $\vec{v}_{b_f}$.

$$v_{b_f} = \sqrt{\left(v_{b_{fE}}\right)^2 + \left(v_{b_{fN}}\right)^2}$$

$$= \sqrt{(0.0555\text{ m/s})^2 + (1.79\text{ m/s})^2}$$

$$= 1.8\text{ m/s}$$

Use the tangent function to find the direction of $\vec{v}_{b_f}$.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

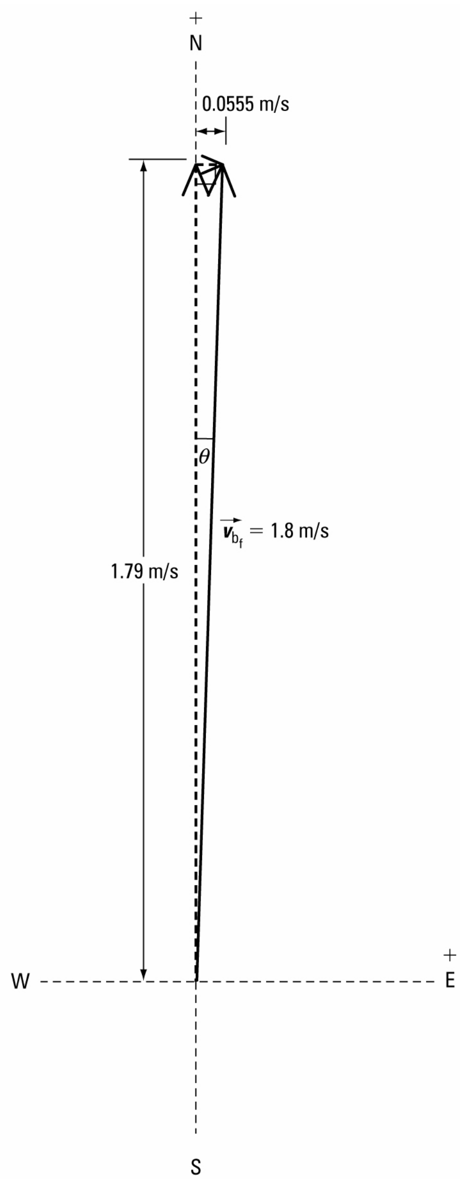
$$= \frac{1.79\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}}{0.0555\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}}$$

$$= 32.25$$

$$\theta = \tan^{-1}(32.25)$$

$$= 88.2°$$

From the figure below, this angle is between $\vec{v}_{b_f}$ and the east direction. So the direction of $\vec{v}_{b_f}$ measured from north is 90.0° – 88.2° = 1.8°.



$\vec{v}_{b_f}$ = 1.8 m/s [1.8° E of N]

Paraphrase

The velocity of the brown bocce ball will be 1.8 m/s [1.8° E of N] immediately after the collision.

(b) ***Given***

m_b = 1.26 kg

m_w = 0.145 kg

$\vec{v}_{b_i}$ = 1.8 m/s [N]

$\vec{v}_{w_i}$ = 0 m/s

$\vec{v}_{b_f}$ = 1.79 m/s [1.8° E of N] from part (a)

$\vec{v}_{w_f}$ = 0.485 m/s [84.0° W of N]

Required

determine if the collision is elastic

Analysis and Solution

Choose both bocce balls as an isolated system.
Calculate the total initial kinetic energy and the total final kinetic energy of the system.

$$
\begin{aligned}
E_{k_i} &= \frac{1}{2} m_b \left(v_{b_i}\right)^2 + \frac{1}{2} m_w \left(v_{w_i}\right)^2 \\
&= \frac{1}{2} (1.26 \text{ kg})(1.8 \text{ m/s})^2 + 0 \\
&= 2.0 \text{ kg•m}^2/\text{s}^2 \\
&= 2.0 \text{ J}
\end{aligned}
$$

$$
\begin{aligned}
E_{k_f} &= \frac{1}{2} m_b \left(v_{b_f}\right)^2 + \frac{1}{2} m_w \left(v_{w_f}\right)^2 \\
&= \frac{1}{2} (1.26 \text{ kg})(1.79 \text{ m/s})^2 + \frac{1}{2} (0.145 \text{ kg})(0.485 \text{ m/s})^2 \\
&= 2.0 \text{ kg•m}^2/\text{s}^2 \\
&= 2.0 \text{ J}
\end{aligned}
$$

Since $E_{k_i} = E_{k_f}$, the collision is elastic.

Paraphrase

The collision between the brown and white bocce balls is elastic.

60. ***Given***

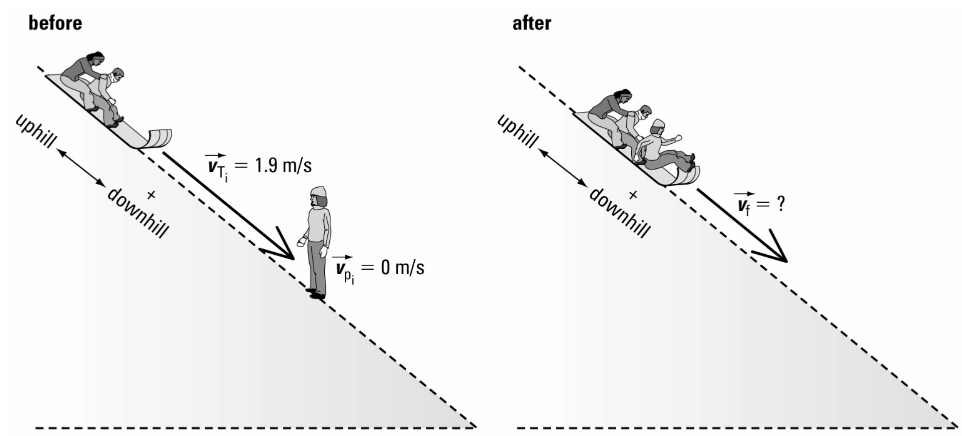
m_{tw} = 128 kg

m_t = 2.0 kg

m_p = 60 kg

$\vec{v}_{T_i}$ = 1.9 m/s [downhill]

$\vec{v}_{p_i}$ = 0 m/s



Required

final velocity of three people-toboggan system ($\vec{v}_f$)

Analysis and Solution

Choose all three people and the toboggan as an isolated system.
Two people and the toboggan have the same initial velocity. So calculate their total mass.

$$\begin{aligned} m_T &= m_{tw} + m_t \\ &= 128 \text{ kg} + 2.0 \text{ kg} \\ &= 130.0 \text{ kg} \end{aligned}$$

The third person has an initial velocity of zero. So that person's initial momentum is zero.

$$\vec{p}_{p_i} = 0$$

Apply the law of conservation of momentum to the system.

$$\begin{aligned} \vec{p}_{sys_i} &= \vec{p}_{sys_f} \\ \vec{p}_{T_i} + \vec{p}_{p_i} &= \vec{p}_{sys_f} \\ m_T \vec{v}_{T_i} + 0 &= (m_T + m_p)\vec{v}_f \\ \vec{v}_f &= \left(\frac{m_T}{m_T + m_p}\right)\vec{v}_{T_i} \\ &= \left(\frac{130.0 \text{ kg}}{130.0 \text{ kg} + 60 \text{ kg}}\right)(+1.9 \text{ m/s}) \\ &= \left(\frac{130.0 \not{\text{kg}}}{190.0 \not{\text{kg}}}\right)(1.9 \text{ m/s}) \\ &= +1.3 \text{ m/s} \\ \vec{v}_f &= 1.3 \text{ m/s [downhill]} \end{aligned}$$

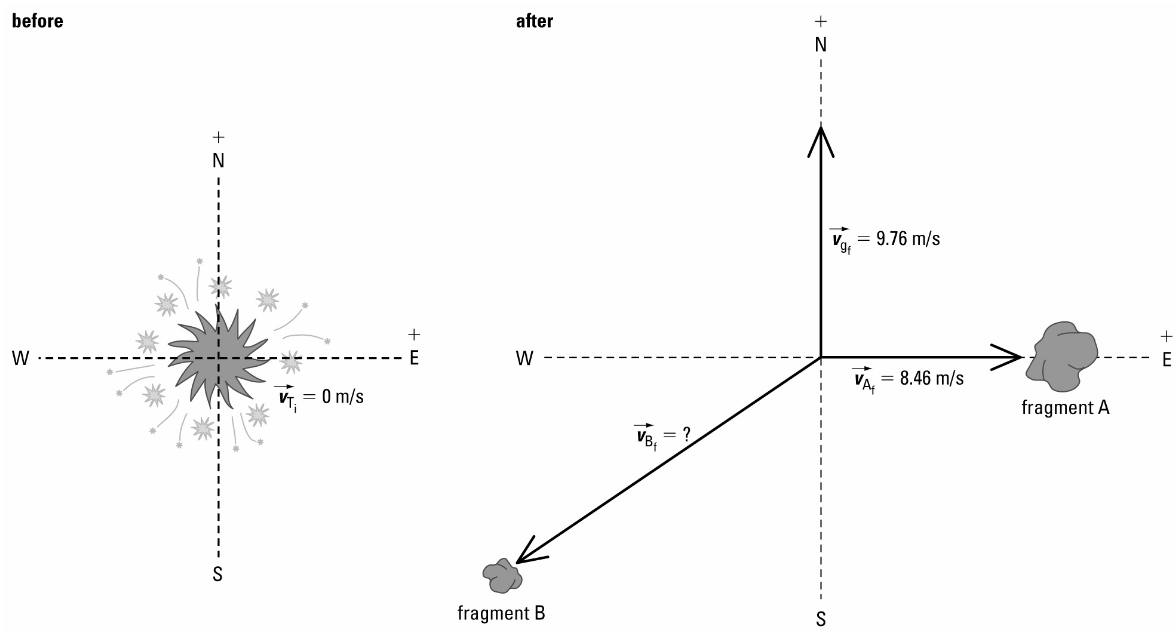
Paraphrase

The velocity of the three people-toboggan system will be 1.3 m/s [downhill] immediately after the interaction.

61. ***Given***

$m_A = 0.0958$ kg $\quad$ $m_B = 0.0627$ kg $\quad$ $m_g = 0.0562$ kg

$\vec{v}_{A_f} = 8.46$ m/s [E] $\quad$ $\vec{v}_{g_f} = 9.76$ m/s [N]



Required

final velocity of fragment B ($\vec{v}_{B_f}$)

Analysis and Solution

Choose the original paint can, both fragments, and the gas as an isolated system. The paint can has an initial velocity of zero. So the system has an initial momentum of zero.

$$\vec{p}_{sys_i} = 0$$

The momentum of each fragment and the gas is in the same direction as its velocity.

Calculate the momentum of fragment A and the gas.

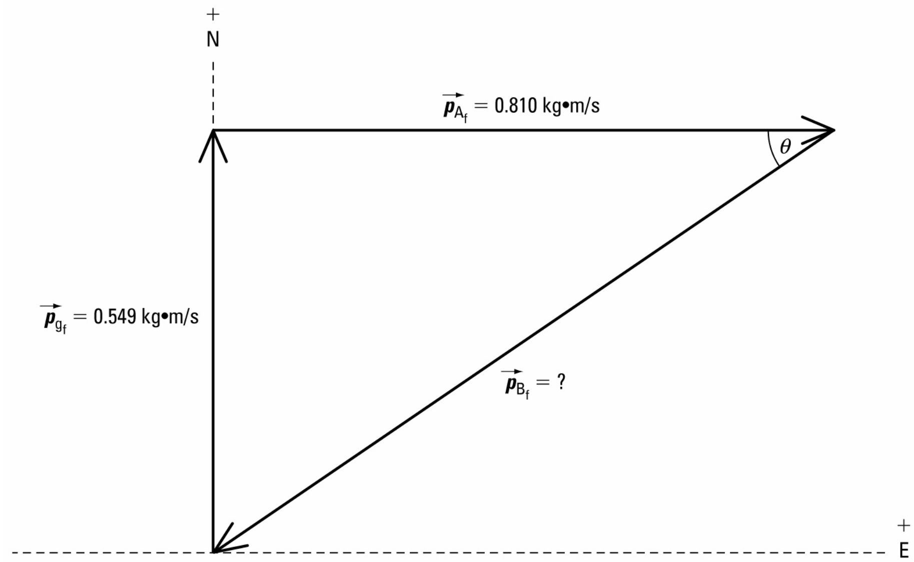
$$\begin{aligned} p_{A_f} &= m_A v_{A_f} \\ &= (0.0958 \text{ kg})(8.46 \text{ m/s}) \\ &= 0.810 \text{ kg•m/s} \\ \vec{p}_{A_f} &= 0.810 \text{ kg•m/s [E]} \end{aligned}$$

$$\begin{aligned} p_{g_f} &= m_g v_{g_f} \\ &= (0.0562 \text{ kg})(9.76 \text{ m/s}) \\ &= 0.549 \text{ kg•m/s} \\ \vec{p}_{g_f} &= 0.549 \text{ kg•m/s [N]} \end{aligned}$$

Apply the law of conservation of momentum to the system.

$$\begin{aligned} \vec{p}_{sys_i} &= \vec{p}_{sys_f} \\ 0 &= \vec{p}_{A_f} + \vec{p}_{B_f} + \vec{p}_{g_f} \end{aligned}$$

Use a vector addition diagram to determine the momentum of fragment B.



From the figure above, careful measurements give p_{B_f} = 0.979 kg•m/s and θ = 34.1° S of W.

Divide the momentum of fragment B by its mass to find the velocity.

$$p_{B_f} = m_B v_{B_f}$$

$$v_{B_f} = \frac{p_{B_f}}{m_B}$$

$$= \frac{0.979\ \cancel{\text{kg}} \bullet \frac{\text{m}}{\text{s}}}{0.0627\ \cancel{\text{kg}}}$$

$$= 15.6 \text{ m/s}$$

$$\vec{v}_{B_f} = 15.6 \text{ m/s [34.1° S of W]}$$

Paraphrase

The velocity of fragment B will be 15.6 m/s [34.1° S of W] immediately after the explosion.

62. (a) ***Given***

m_c = 0.185 kg m_b = 0.046 kg

$\vec{v}_{c_i}$ = 28.5 m/s [forward] $\vec{v}_{b_i}$ = 0 m/s

$\vec{v}_{b_f}$ = 45.7 m/s [forward]

Required

final speed of golf club if collision is elastic ($\vec{v}_{c_f}$)

Analysis and Solution

Choose the golf club and golf ball as an isolated system.

If the collision is elastic, the total kinetic energy of the system is conserved.

$$E_{k_i} = E_{k_f}$$

$$\frac{1}{2}m_c\left(v_{c_i}\right)^2 + \frac{1}{2}m_b\left(v_{b_i}\right)^2 = \frac{1}{2}m_c\left(v_{c_f}\right)^2 + \frac{1}{2}m_b\left(v_{b_f}\right)^2$$

$$\left(v_{c_i}\right)^2 + 0 = \left(v_{c_f}\right)^2 + \left(\frac{m_b}{m_c}\right)\left(v_{b_f}\right)^2$$

$$\left(v_{c_f}\right)^2 = \left(v_{c_i}\right)^2 - \left(\frac{m_b}{m_c}\right)\left(v_{b_f}\right)^2$$

$$= (28.5\text{ m/s})^2 - \left(\frac{0.046\ \cancel{\text{kg}}}{0.185\ \cancel{\text{kg}}}\right)(45.7\text{ m/s})^2$$

$$= 2.9 \times 10^2\ \text{m}^2/\text{s}^2$$

$$v_{c_f} = 17\text{ m/s}$$

Paraphrase

If the collision between the golf club and golf ball is elastic, the speed of the golf club will be 17 m/s immediately after the interaction.

(b) ***Given***

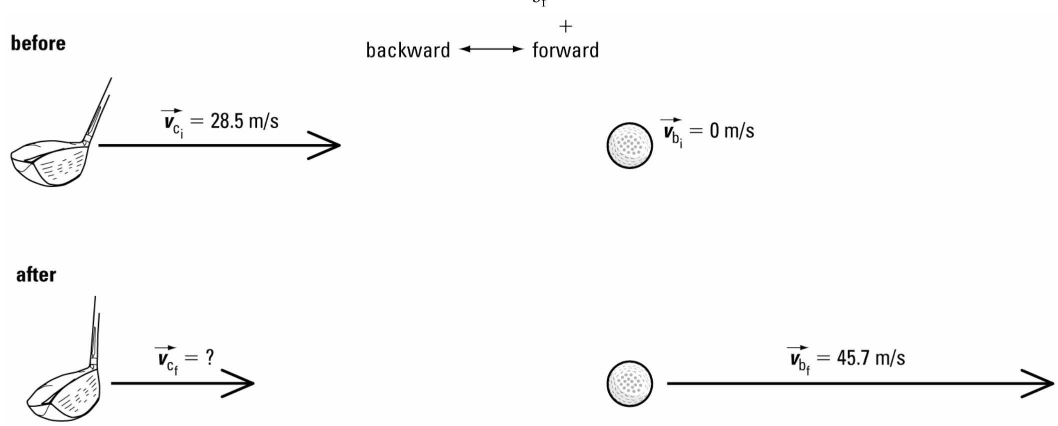
$m_c = 0.185$ kg

$\vec{v}_{c_i} = 28.5$ m/s [forward]

$m_b = 0.046$ kg

$\vec{v}_{b_i} = 0$ m/s

$\vec{v}_{b_f} = 45.7$ m/s [forward]



Required

final velocity of golf club ($\vec{v}_{A_f}$)

Analysis and Solution

Choose the golf club and golf ball as an isolated system.
The golf ball has an initial velocity of zero. So its initial momentum is zero.

$$\vec{p}_{b_i} = 0$$

Apply the law of conservation of momentum to the system.

$$\vec{p}_{sys_i} = \vec{p}_{sys_f}$$

$$\vec{p}_{c_i} + \vec{p}_{b_i} = \vec{p}_{c_f} + \vec{p}_{b_f}$$

$$m_c \vec{v}_{c_i} + 0 = m_c \vec{v}_{c_f} + m_b \vec{v}_{b_f}$$

$$\vec{v}_{c_i} = \vec{v}_{c_f} + \left(\frac{m_b}{m_c}\right) \vec{v}_{b_f}$$

$$\vec{v}_{c_f} = \vec{v}_{c_i} - \left(\frac{m_b}{m_c}\right) \vec{v}_{b_f}$$

$$= +28.5 \text{ m/s} - \left(\frac{0.046 \cancel{\text{ kg}}}{0.185 \cancel{\text{ kg}}}\right)(+45.7 \text{ m/s})$$

$$= +17 \text{ m/s}$$

$$\vec{v}_{c_f} = 17 \text{ m/s [forward]}$$

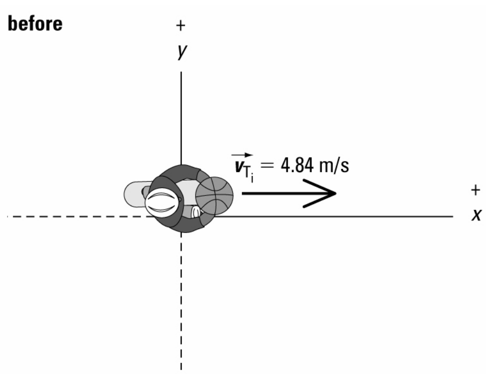
Paraphrase

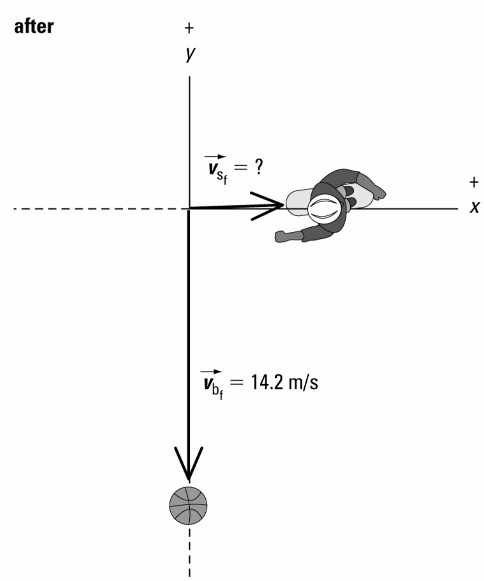
Using the conservation of momentum, the velocity of the golf club will be 17 m/s [forward] immediately after the interaction.

63. ***Given***

$m_s = 50.2$ kg $\quad m_b = 0.600$ kg

$\vec{v}_{T_i} = 4.84$ m/s [0°] $\quad \vec{v}_{b_f} = 14.2$ m/s [270°]





Required

final velocity of student-skateboard combination ($\vec{v}_{s_f}$)

Analysis and Solution

Choose the student, skateboard, and basketball as an isolated system.
The student, skateboard, and basketball have the same initial velocity.
So calculate the total mass.

$$m_T = m_s + m_b$$
$$= 50.2 \text{ kg} + 0.600 \text{ kg}$$
$$= 50.800 \text{ kg}$$

Resolve all velocities into x and y components.

Vector	*x* component	*y* component
$\vec{v}_{T_i}$	4.84 m/s	0
$\vec{v}_{b_f}$	0	–(14.2 m/s)

Apply the law of conservation of momentum to the system in the x and y directions.

x direction

$$p_{sys_{ix}} = p_{sys_{fx}}$$

$$p_{sys_{ix}} = p_{p_{fx}} + p_{b_{fx}}$$

$$m_T v_{T_{ix}} = m_s v_{s_{fx}} + m_b v_{b_{fx}}$$

$$m_T v_{T_{ix}} = m_s v_{s_{fx}} + 0$$

$$v_{s_{fx}} = \left(\frac{m_T}{m_s}\right) v_{T_{ix}}$$

$$= \left(\frac{50.800\ \cancel{kg}}{50.2\ \cancel{kg}}\right)(4.84\text{ m/s})$$

$$= 4.898\text{ m/s}$$

y direction

$$p_{sys_{iy}} = p_{sys_{fy}}$$

$$p_{sys_{iy}} = p_{p_{fy}} + p_{b_{fy}}$$

$$m_T v_{T_{iy}} = m_s v_{s_{fy}} + m_b v_{b_{fy}}$$

$$0 = m_s v_{s_{fy}} + m_b v_{b_{fy}}$$

$$v_{s_{fy}} = -\left(\frac{m_b}{m_s}\right) v_{b_{fy}}$$

$$= -\left(\frac{0.600\ \cancel{kg}}{50.2\ \cancel{kg}}\right)\{-(14.2\text{ m/s})\}$$

$$= 0.1697\text{ m/s}$$

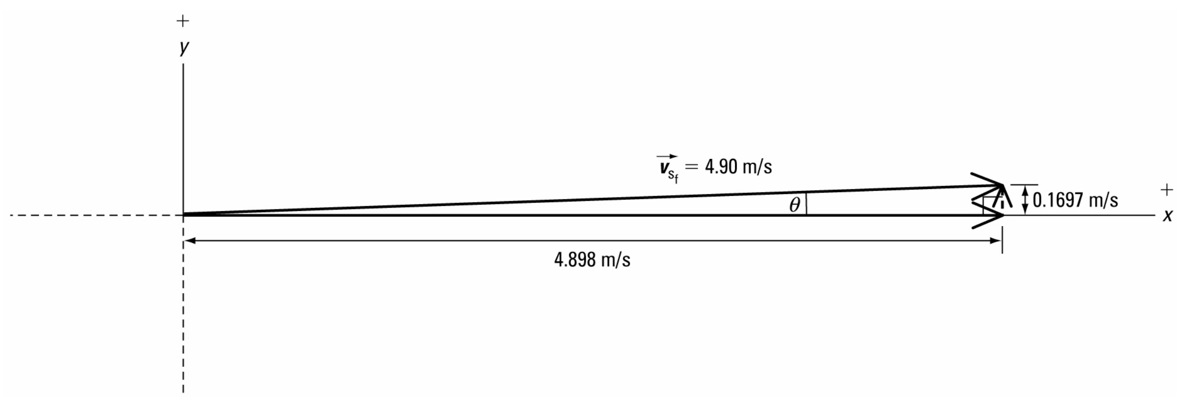
Use the Pythagorean theorem to find the magnitude of $\vec{v}_{s_f}$.

$$v_{s_f} = \sqrt{\left(v_{s_{fx}}\right)^2 + \left(v_{s_{fy}}\right)^2}$$
$$= \sqrt{(4.898\ \text{m/s})^2 + (0.1697\ \text{m/s})^2}$$
$$= 4.90\ \text{m/s}$$

Use the tangent function to find the direction of $\vec{v}_{s_f}$.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{0.1697\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}}{4.898\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}}$$
$$= 0.03465$$
$$\theta = \tan^{-1}(0.03465)$$
$$= 2.0°$$

From the figure below, this angle is between $\vec{v}_{s_f}$ and the positive x-axis.



$$\vec{v}_{s_f} = 4.90\ \text{m/s [2.0°]}$$

Paraphrase

The velocity of the student-skateboard combination will be 4.90 m/s [2.0°] immediately after the throw.

64. (a) ***Given***

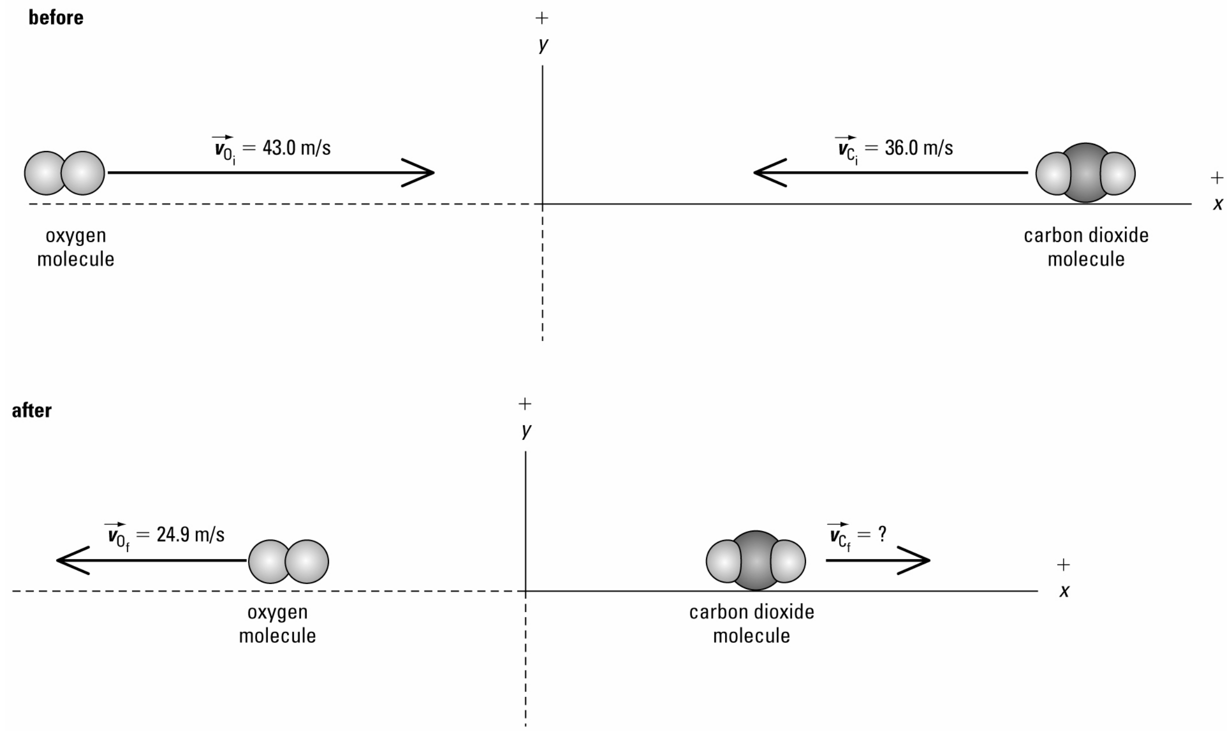
$m_O = 5.316 \times 10^{-23}$ kg

$m_C = 7.311 \times 10^{-23}$ kg

$\vec{v}_{O_i} = 43.0$ m/s [0°]

$\vec{v}_{C_i} = 36.0$ m/s [180°]

$\vec{v}_{O_f} = 24.9$ m/s [180°]



Required

final velocity of carbon dioxide molecule ($\vec{v}_{C_f}$)

Analysis and Solution

Choose the oxygen and carbon dioxide molecules as an isolated system.
Apply the law of conservation of momentum to the system.

$$\vec{p}_{sys_i} = \vec{p}_{sys_f}$$

$$\vec{p}_{O_i} + \vec{p}_{C_i} = \vec{p}_{O_f} + \vec{p}_{C_f}$$

$$m_O\vec{v}_{O_i} + m_C\vec{v}_{C_i} = m_O\vec{v}_{O_f} + m_C\vec{v}_{C_f}$$

$$\left(\frac{m_O}{m_C}\right)\vec{v}_{O_i} + \vec{v}_{C_i} = \left(\frac{m_O}{m_C}\right)\vec{v}_{O_f} + \vec{v}_{C_f}$$

$$\vec{v}_{C_f} = \vec{v}_{C_i} + \left(\frac{m_O}{m_C}\right)(\vec{v}_{O_i} - \vec{v}_{O_f})$$

$$= -36.0 \text{ m/s} + \left(\frac{5.316\times10^{-23} \cancel{\text{kg}}}{7.311\times10^{-23} \cancel{\text{kg}}}\right)\{+43.0 \text{ m/s} - [-(24.9 \text{ m/s})]\}$$

$$= +13.4 \text{ m/s}$$

$$\vec{v}_{C_f} = 13.4 \text{ m/s [0°]}$$

Paraphrase

The velocity of the carbon dioxide molecule is 13.4 m/s [0°] immediately after the collision.

(b) ***Given***

$m_O = 5.316 \times 10^{-23}$ kg $\qquad m_C = 7.311 \times 10^{-23}$ kg

$\vec{v}_{O_i} = 43.0$ m/s [0°] $\qquad \vec{v}_{C_i} = 36.0$ m/s [180°]

$\vec{v}_{O_f} = 24.9$ m/s [180°] $\qquad \vec{v}_{C_f} = 13.37$ m/s [0°] from part (a)

Required

determine if the collision is elastic

Analysis and Solution

Choose the oxygen and carbon dioxide molecules as an isolated system. Calculate the total initial kinetic energy and the total final kinetic energy of the system.

$$\begin{aligned} E_{k_i} &= \frac{1}{2} m_O \left(v_{O_i}\right)^2 + \frac{1}{2} m_C \left(v_{C_i}\right)^2 \\ &= \frac{1}{2}(5.316 \times 10^{-23}\text{ kg})(43.0\text{ m/s})^2 + \frac{1}{2}(7.311 \times 10^{-23}\text{ kg})(36.0\text{ m/s})^2 \\ &= 9.65 \times 10^{-20}\text{ kg•m}^2/\text{s}^2 \\ &= 9.65 \times 10^{-20}\text{ J} \end{aligned}$$

$$\begin{aligned} E_{k_f} &= \frac{1}{2} m_O \left(v_{O_f}\right)^2 + \frac{1}{2} m_C \left(v_{C_f}\right)^2 \\ &= \frac{1}{2}(5.316 \times 10^{-23}\text{ kg})(24.9\text{ m/s})^2 + \frac{1}{2}(7.311 \times 10^{-23}\text{ kg})(13.37\text{ m/s})^2 \\ &= 2.30 \times 10^{-20}\text{ kg•m}^2/\text{s}^2 \\ &= 2.30 \times 10^{-20}\text{ J} \end{aligned}$$

Since $E_{k_i} \neq E_{k_f}$, the collision is inelastic.

Paraphrase

The collision between the oxygen and carbon dioxide molecules is inelastic.

65. ***Given***

$m_e = 9.11 \times 10^{-31}$ kg $\qquad m_p = 1.67 \times 10^{-27}$ kg

$\vec{v}_{n_i} = 0$ m/s $\qquad \vec{v}_{e_f} = 4.35 \times 10^5$ m/s [E] $\qquad \vec{v}_{p_f} = 14.8$ m/s [E]

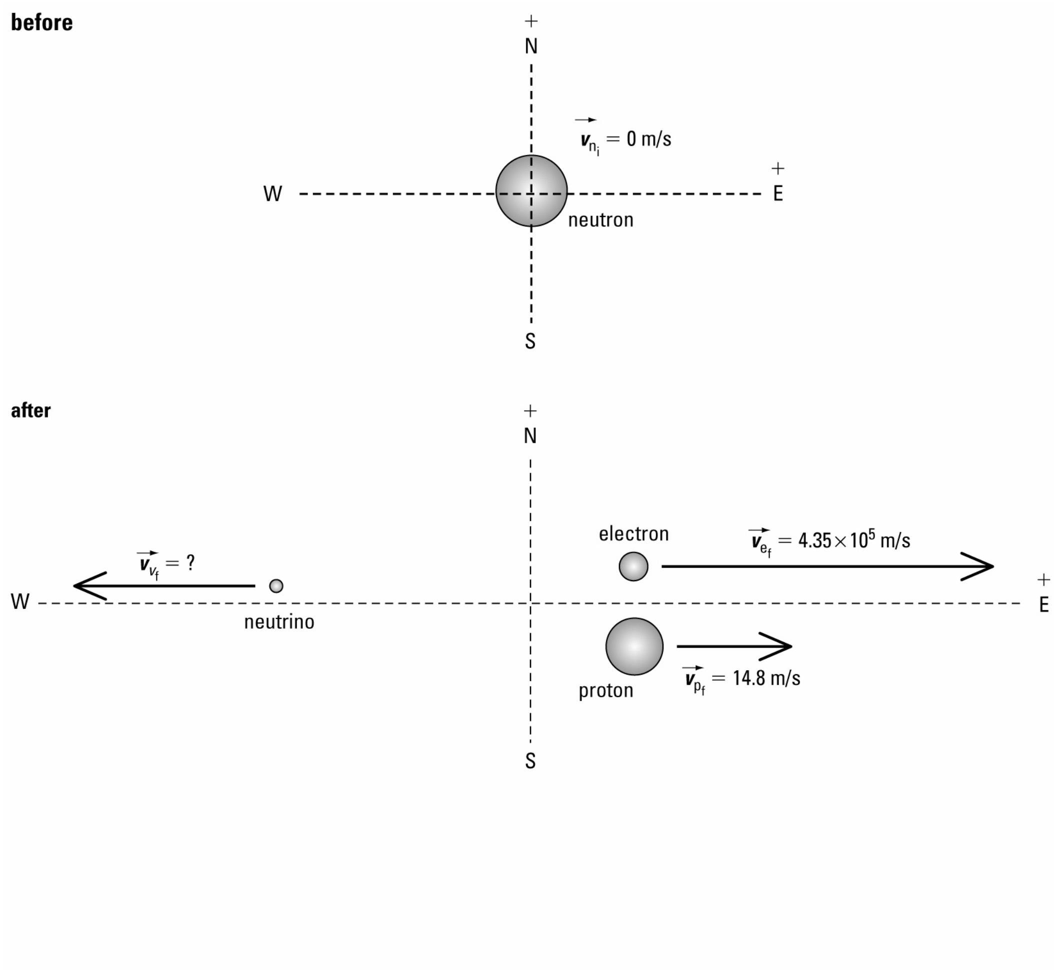


Diagram is not to scale.

Required

momentum of neutrino ($\vec{p}_{\nu_f}$)

Analysis and Solution

Choose the neutron, electron, proton, and neutrino as an isolated system.
The neutron is stationary. So the initial momentum of the system is zero.

$$\vec{p}_{sys_i} = 0$$

Apply the law of conservation of momentum to the system.

$$\vec{p}_{sys_i} = \vec{p}_{sys_f}$$

$$\vec{p}_{sys_i} = \vec{p}_{e_f} + \vec{p}_{p_f} + \vec{p}_{\nu_f}$$

$$0 = m_e \vec{v}_{e_f} + m_p \vec{v}_{p_f} + \vec{p}_{\nu_f}$$

$$\vec{p}_{\nu_f} = -m_e \vec{v}_{e_f} - m_p \vec{v}_{p_f}$$

$$\vec{p}_{\nu_f} = -(9.11 \times 10^{-31} \text{ kg})(+4.35 \times 10^5 \text{ m/s}) - (1.67 \times 10^{-27} \text{ kg})(+14.8 \text{ m/s})$$

$$= -4.21 \times 10^{-25} \text{ kg•m/s}$$

$\vec{p}_{\nu_f} = 4.21 \times 10^{-25}$ kg•m/s [W]

Paraphrase

The momentum of the neutrino that is released is 4.21×10^{-25} kg•m/s [W].

66. (a) ***Given***

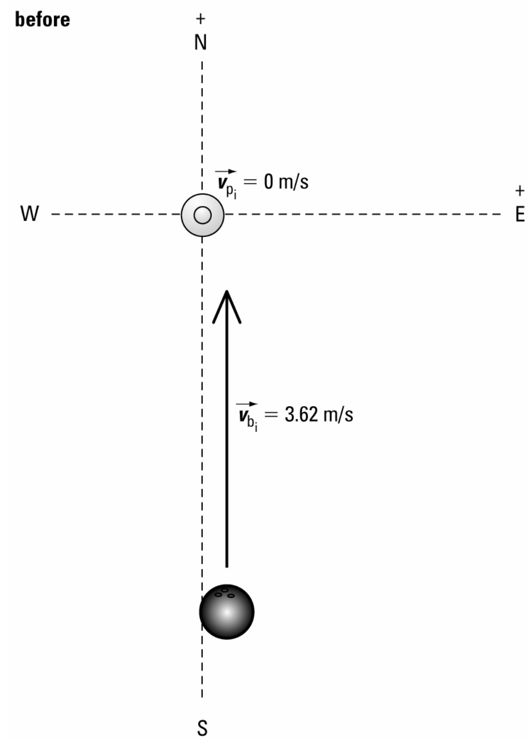
$m_b = 8.95$ kg

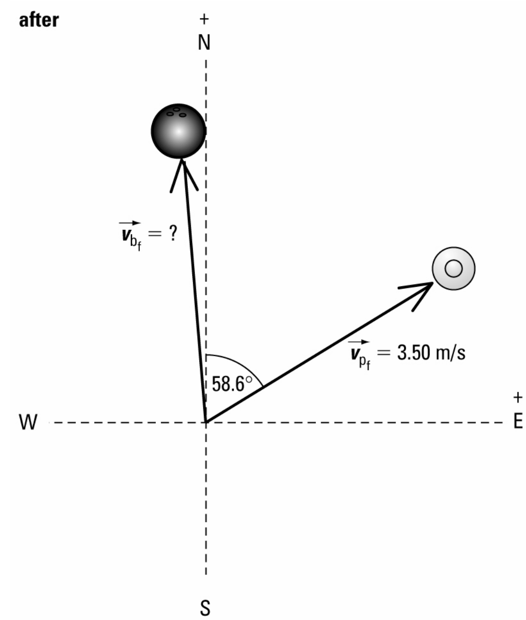
$\vec{v}_{b_i} = 3.62$ m/s [N]

$m_p = 0.856$ kg

$\vec{v}_{p_i} = 0$ m/s

$\vec{v}_{p_f} = 3.50$ m/s [58.6° E of N]



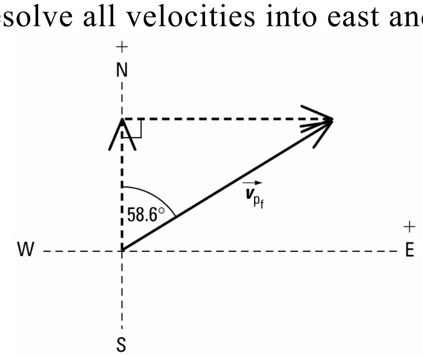


Required

final velocity of bowling ball ($\vec{v}_{b_f}$)

Analysis and Solution

Choose the bowling ball and bowling pin as an isolated system.
Resolve all velocities into east and north components.



Vector	East component	North component
$\vec{v}_{b_i}$	0	3.62 m/s
$\vec{v}_{p_f}$	(3.50 m/s)(sin 58.6°)	(3.50 m/s)(cos 58.6°)

The bowling pin has an initial velocity of zero. So its initial momentum is zero.

$$\vec{p}_{p_i} = 0$$

Apply the law of conservation of momentum to the system in the east and north directions.

E direction

$$\begin{aligned}
p_{sys_{iE}} &= p_{sys_{fE}} \\
p_{b_{iE}} + p_{p_{iE}} &= p_{b_{fE}} + p_{p_{fE}} \\
m_b v_{b_{iE}} + 0 &= m_b v_{b_{fE}} + m_p v_{p_{fE}} \\
0 + 0 &= m_b v_{b_{fE}} + m_p v_{p_{fE}} \\
v_{b_{fE}} &= -\left(\frac{m_p}{m_b}\right) v_{p_{fE}} \\
&= -\left(\frac{0.856\ \cancel{\text{kg}}}{8.95\ \cancel{\text{kg}}}\right)(3.50\text{ m/s})(\sin 58.6°) \\
&= -0.2857\text{ m/s}
\end{aligned}$$

N direction

$$\begin{aligned}
p_{sys_{iN}} &= p_{sys_{fN}} \\
p_{b_{iN}} + p_{p_{iN}} &= p_{b_{fN}} + p_{p_{fN}} \\
m_b v_{b_{iN}} + 0 &= m_b v_{b_{fN}} + m_p v_{p_{fN}} \\
v_{b_{iN}} &= v_{b_{fN}} + \left(\frac{m_p}{m_b}\right) v_{p_{fN}} \\
v_{b_{fN}} &= v_{b_{iN}} - \left(\frac{m_p}{m_b}\right) v_{p_{fN}} \\
&= 3.62\text{ m/s} - \left(\frac{0.856\ \cancel{\text{kg}}}{8.95\ \cancel{\text{kg}}}\right)(3.50\text{ m/s})(\cos 58.6°) \\
&= 3.446\text{ m/s}
\end{aligned}$$

Use the Pythagorean theorem to find the magnitude of $\vec{v}_{b_f}$.

$$v_{b_f} = \sqrt{\left(v_{b_{fE}}\right)^2 + \left(v_{b_{fN}}\right)^2}$$

$$= \sqrt{(-0.2857\text{ m/s})^2 + (3.446\text{ m/s})^2}$$

$$= 3.46\text{ m/s}$$

Use the tangent function to find the direction of $\vec{v}_{b_f}$.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

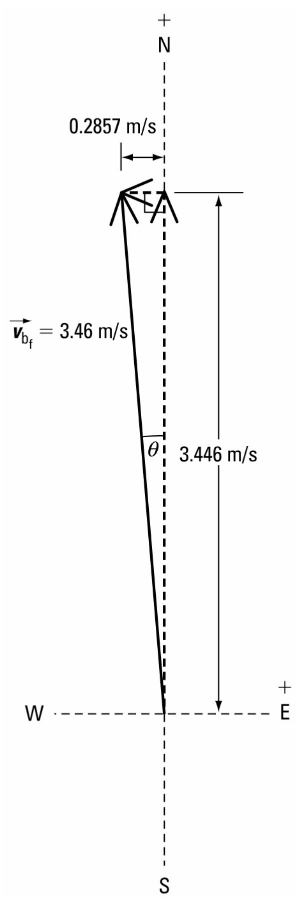
$$= \frac{3.446\ \cancel{\frac{\text{m}}{\text{s}}}}{0.2857\ \cancel{\frac{\text{m}}{\text{s}}}}$$

$$= 12.06$$

$$\theta = \tan^{-1}(12.06)$$

$$= 85.3°$$

From the figure below, this angle is between $\vec{v}_{b_f}$ and the west direction. So the direction of $\vec{v}_{b_f}$ measured from north is 90.0° – 85.3° = 4.7°.



$\vec{v}_{b_f}$ = 3.46 m/s [4.7° W of N]

Paraphrase

The velocity of the bowling ball will be 3.46 m/s [4.7° W of N] immediately after the collision.

(b) ***Given***

m_b = 8.95 kg

$\vec{v}_{b_i}$ = 3.62 m/s [N]

$\vec{v}_{b_f}$ = 3.457 m/s [4.7° W of N] from part (a)

m_p = 0.856 kg

$\vec{v}_{p_i}$ = 0 m/s

$\vec{v}_{p_f}$ = 3.50 m/s [58.6° E of N]

Required

determine if the collision is elastic

Analysis and Solution

Choose the bowling ball and bowling pin as an isolated system.
Calculate the total initial kinetic energy and the total final kinetic energy of the system.

$$\begin{aligned}
E_{k_i} &= \frac{1}{2} m_b \left(v_{b_i}\right)^2 + \frac{1}{2} m_p \left(v_{p_i}\right)^2 \\
&= \frac{1}{2}(8.95 \text{ kg})(3.62 \text{ m/s})^2 + 0 \\
&= 58.6 \text{ kg•m}^2/\text{s}^2 \\
&= 58.6 \text{ J}
\end{aligned}$$

$$\begin{aligned}
E_{k_f} &= \frac{1}{2} m_b \left(v_{b_f}\right)^2 + \frac{1}{2} m_p \left(v_{p_f}\right)^2 \\
&= \frac{1}{2}(8.95 \text{ kg})(3.457 \text{ m/s})^2 + \frac{1}{2}(0.856 \text{ kg})(3.50 \text{ m/s})^2 \\
&= 58.7 \text{ kg•m}^2/\text{s}^2 \\
&= 58.7 \text{ J}
\end{aligned}$$

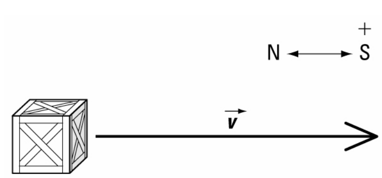
Since $E_{k_i} \neq E_{k_f}$, the collision is inelastic.

Paraphrase

The collision between the bowling ball and bowling pin is inelastic.

67. (a) ***Given***

p = 560 kg•m/s
$\vec{v}$ = 50.4 km/h [S]



Required

mass of crate (m)

Analysis and Solution

Convert the speed of the crate to metres per second.

$$v = \frac{50.4\ \text{km}}{1\ \text{h}} \times \frac{1000\ \text{m}}{1\ \text{km}} \times \frac{1\ \text{h}}{3600\ \text{s}}$$

$$= 14.0\ \text{m/s}$$

The momentum of the crate is in the direction of its velocity. So use the scalar form of $\vec{p} = m\vec{v}$ to find the mass.

$$p = mv$$

$$m = \frac{p}{v}$$

$$= \frac{560\ \text{kg}\bullet\frac{\text{m}}{\text{s}}}{14.0\ \frac{\text{m}}{\text{s}}}$$

$$= 40.0\ \text{kg}$$

Paraphrase

The mass of the crate is 40.0 kg.

(b) ***Given***

$\Delta t = 5.25$ s

Required

impulse provided to the road

Analysis and Solution

From Newton's third law, if the tires of the truck exert a net force south on the road, the road will exert a force of equal magnitude but opposite direction on the tires. It is this reaction force acting for 5.25 s that provides the required impulse to stop the truck.

Impulse is numerically equal to $m\Delta\vec{v}$ or $m(\vec{v}_f - \vec{v}_i)$. Calculate the impulse provided to the truck.

$$(\text{impulse})_t = m_T(\vec{v}_f - \vec{v}_i)$$

$$= m_T(0 - 14.0\ \text{m/s})$$

$$= -14.0 m_T\ \text{kg}\bullet\text{m/s}$$

$$= 14.0 m_T\ \text{kg}\bullet\text{m/s [N]}$$

$$(\text{impulse})_r = 14.0 m_T\ \text{kg}\bullet\text{m/s [S]}$$

Paraphrase

The driver would have to provide an impulse of $14.0m_T$ kg•m/s [S] on the road, where m_T is the total mass of the truck, driver, and crate.

(c) ***Given***

(impulse)$_r$ = 14.0m_T kg•m/s [S] from part (b)

Δt = 5.25 s

μ_s = 0.30

Required

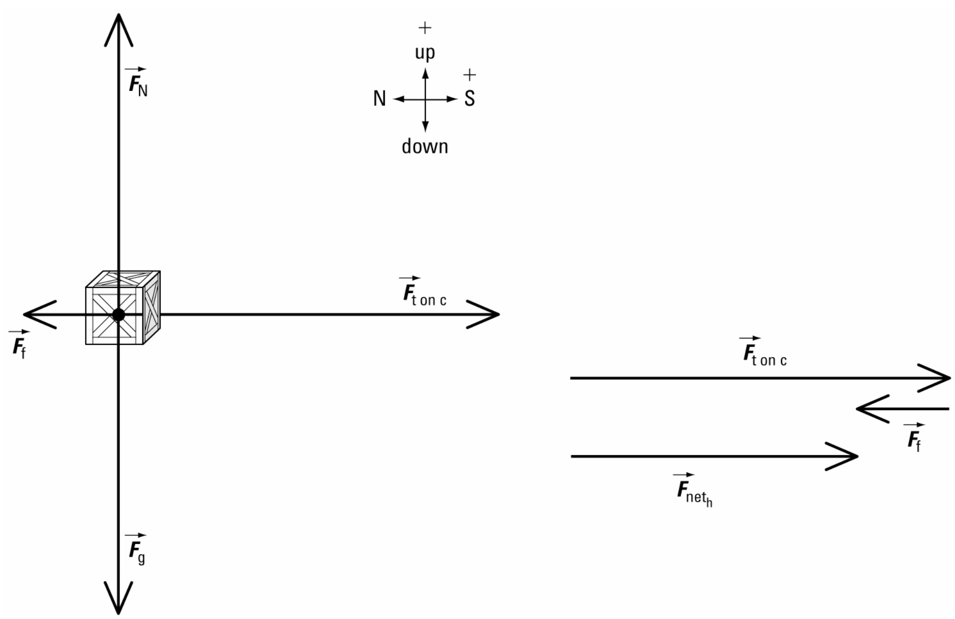
net force on crate ($\vec{F}_{net}$)

Analysis and Solution

Use the equation of impulse to calculate the force that the truck exerts on the crate.

$$
\begin{aligned}
+14.0m_T \text{ kg•m/s} &= \vec{F}_{\text{t on c}} \Delta t \\
\vec{F}_{\text{t on c}} &= \frac{+14.0m_T \text{ kg•m/s}}{\Delta t} \\
&= \frac{+14.0m_T \text{ kg•m/s}}{5.25 \text{ s}} \\
&= +2.67m_T \text{ kg•m/s}^2 \\
\vec{F}_{\text{t on c}} &= 2.67m_T \text{ N [S]}
\end{aligned}
$$

Draw a free-body diagram for the crate.



The crate is not accelerating up or down. So in the vertical direction, F_{net_v} = 0 N.

Write equations to find the net force on the crate in both the horizontal and vertical directions.

horizontal direction

$$\vec{F}_{\text{net}_\text{h}} = \vec{F}_{\text{t on c}} + \vec{F}_{\text{f}_\text{static}}$$

$$F_{\text{net}_\text{h}} = F_{\text{t on c}} + F_{\text{f}_\text{static}}$$

$$= F_{\text{t on c}} + (-\mu_\text{s} F_\text{N})$$

$$= F_{\text{t on c}} - \mu_\text{s} F_\text{N}$$

vertical direction

$$\vec{F}_{\text{net}_\text{v}} = \vec{F}_\text{N} + \vec{F}_\text{g}$$

$$F_{\text{net}_\text{v}} = F_\text{N} + F_\text{g}$$

$$0 = F_\text{N} + (-mg)$$

$$= F_\text{N} - mg$$

$$F_\text{N} = mg$$

Substitute $F_\text{N} = mg$ into the equation for F_{net_h}.

$$F_{\text{net}_\text{h}} = F_{\text{t on c}} - \mu_\text{s} mg$$

$$= +2.67 m_\text{T}\ \text{N} - (0.30)(40.0\ \text{kg})(9.81\ \text{m/s}^2)$$

$$= (2.67 m_\text{T} - 118)\ \text{N}$$

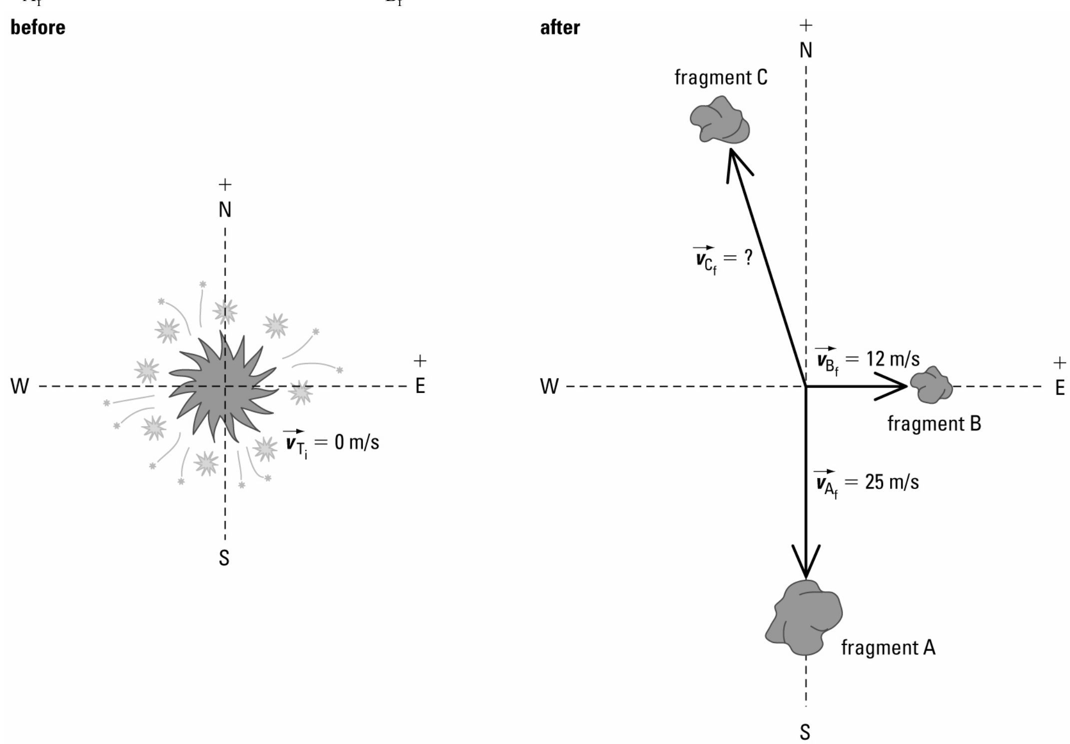
Paraphrase

Since $2.67\text{m}_\text{T} > 118$, the crate will not slide.

68. ***Given***

$m_\text{A} = 8.5$ g $\quad m_\text{B} = 5.6$ g $\quad m_\text{C} = 6.7$ g

$\vec{v}_{\text{A}_\text{f}} = 25$ m/s [S] $\quad \vec{v}_{\text{B}_\text{f}} = 12$ m/s [E]



Required

final velocity of fragment C ($\vec{v}_{\text{C}_\text{f}}$)

Analysis and Solution

Choose the original firecracker and all three fragments as an isolated system. The firecracker has an initial velocity of zero. So the system has an initial momentum of zero.

$$\vec{p}_{\text{sys}_\text{i}} = 0$$

The momentum of each fragment is in the same direction as its velocity. Calculate the momentum of fragments A and B.

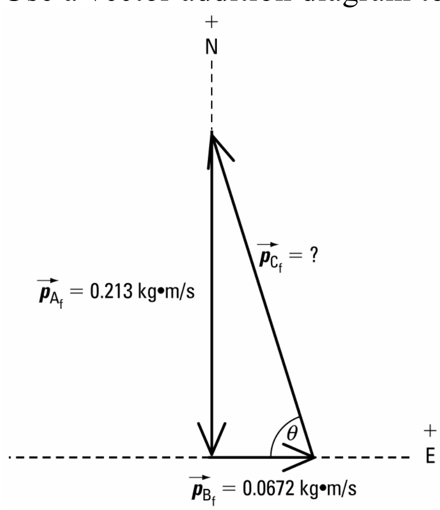
$$\begin{aligned} p_{\text{A}_\text{f}} &= m_\text{A} v_{\text{A}_\text{f}} \\ &= \left(8.5 \not{g} \times \frac{1\text{ kg}}{1000 \not{g}}\right)(25\text{ m/s}) \\ &= 0.213\text{ kg}\bullet\text{m/s} \\ \vec{p}_{\text{A}_\text{f}} &= 0.213\text{ kg}\bullet\text{m/s [S]} \end{aligned}$$

$$\begin{aligned} p_{\text{B}_\text{f}} &= m_\text{B} v_{\text{B}_\text{f}} \\ &= \left(5.6 \not{g} \times \frac{1\text{ kg}}{1000 \not{g}}\right)(12\text{ m/s}) \\ &= 0.0672\text{ kg}\bullet\text{m/s} \\ \vec{p}_{\text{B}_\text{f}} &= 0.0672\text{ kg}\bullet\text{m/s [E]} \end{aligned}$$

Apply the law of conservation of momentum to the system.

$$\begin{aligned} \vec{p}_{\text{sys}_\text{i}} &= \vec{p}_{\text{sys}_\text{f}} \\ 0 &= \vec{p}_{\text{A}_\text{f}} + \vec{p}_{\text{B}_\text{f}} + \vec{p}_{\text{C}_\text{f}} \end{aligned}$$

Use a vector addition diagram to determine the momentum of fragment C.



From the figure above, careful measurements give p_{C_f} = 0.22 kg•m/s and θ = 72.5° N of W.

Divide the momentum of fragment C by its mass to find the velocity.

$$p_{\text{C}_\text{f}} = m_\text{C} v_{\text{C}_\text{f}}$$

$$v_{C_f} = \frac{p_{C_f}}{m_C}$$

$$= \frac{0.22\ \text{kg}\bullet\frac{\text{m}}{\text{s}}}{(6.7\ \text{g})\left(\frac{1\ \text{kg}}{1000\ \text{g}}\right)}$$

$$= 33\ \text{m/s}$$

$$\vec{v}_{C_f} = 33\ \text{m/s [72.5° N of W]}$$

Paraphrase

The velocity of fragment C will be 33 m/s [72.5° N of W] immediately after the explosion.

69. (a) ***Given***

$m_A = 1.2 \times 10^{-21}$ kg

$\vec{v}_{A_i} = 9.2$ m/s [E]

$\vec{v}_{A_f} = 3.3$ m/s [45.0° W of N]

$m_B = 1.2 \times 10^{-21}$ kg

$\vec{v}_{B_i} = 8.5$ m/s [N]

Required

final speed of buckyball B if collision is elastic ($\vec{v}_{B_f}$)

Analysis and Solution

Choose both buckyballs as an isolated system.
If the collision is elastic, the total kinetic energy of the system is conserved.

$$E_{k_i} = E_{k_f}$$

$$\frac{1}{2}m_A\left(v_{A_i}\right)^2 + \frac{1}{2}m_B\left(v_{B_i}\right)^2 = \frac{1}{2}m_A\left(v_{A_f}\right)^2 + \frac{1}{2}m_B\left(v_{B_f}\right)^2$$

$$\left(\frac{m_A}{m_B}\right)\left(v_{A_i}\right)^2 + \left(v_{B_i}\right)^2 = \left(\frac{m_A}{m_B}\right)\left(v_{A_f}\right)^2 + \left(v_{B_f}\right)^2$$

$$\left(v_{B_f}\right)^2 = \left(v_{B_i}\right)^2 + \left(\frac{m_A}{m_B}\right)\left\{\left(v_{A_i}\right)^2 - \left(v_{A_f}\right)^2\right\}$$

$$\left(v_{B_f}\right)^2 = (8.5\ \text{m/s})^2 + \left(\frac{1.2\times10^{-21}\ \text{kg}}{1.2\times10^{-21}\ \text{kg}}\right)\{(9.2\ \text{m/s})^2 - (3.3\ \text{m/s})^2\}$$

$$= 1.5 \times 10^2\ \text{m}^2/\text{s}^2$$

$$v_{B_f} = 12\ \text{m/s}$$

Paraphrase

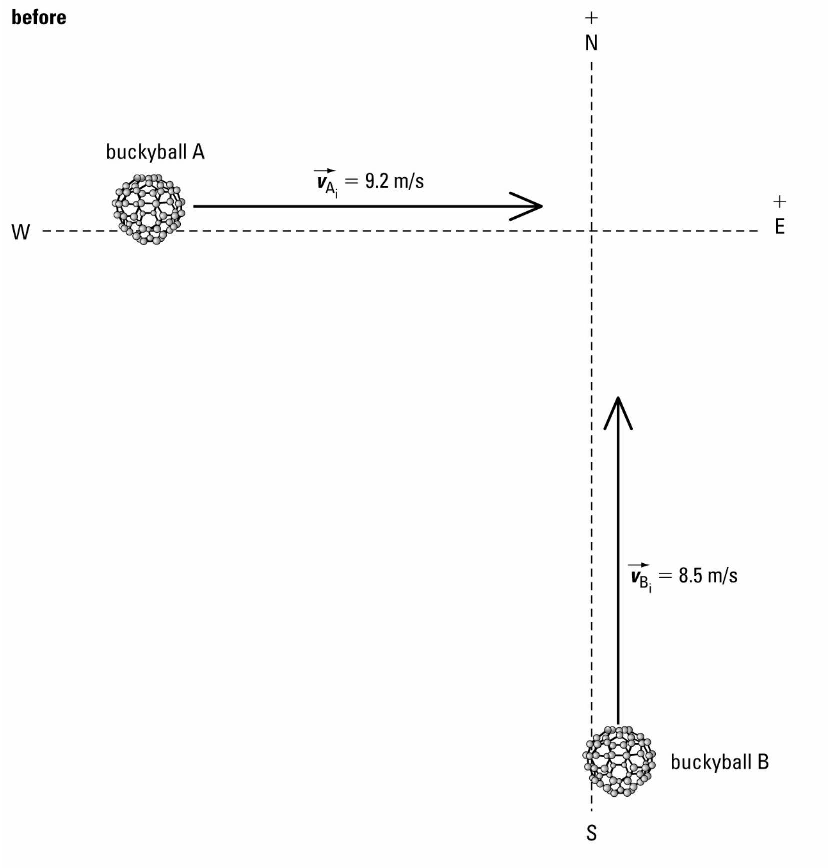
If the collision between the buckyballs is elastic, the speed of buckyball B will be 12 m/s immediately after the interaction.

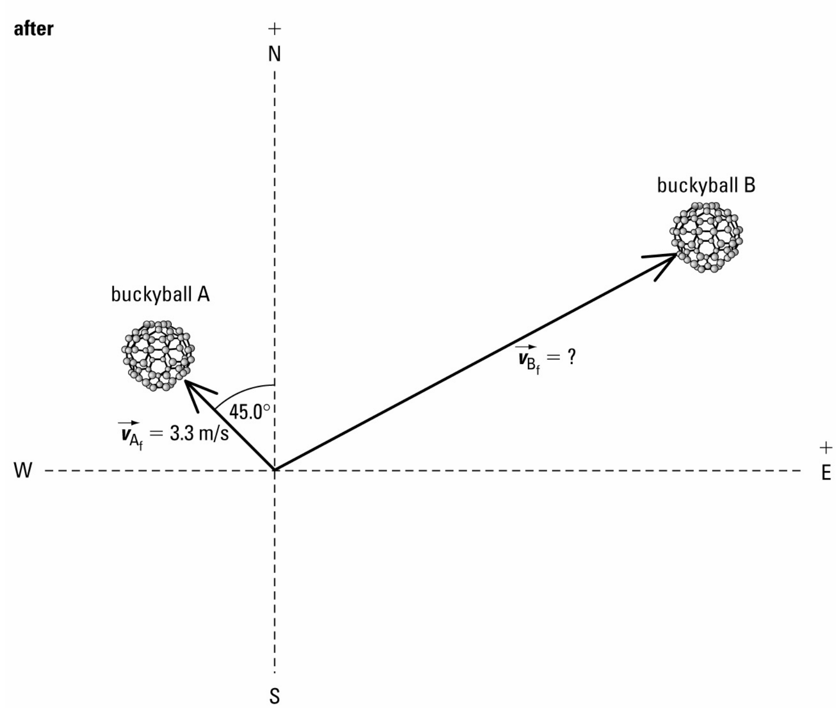
(b) ***Given***

$m_A = 1.2 \times 10^{-21}$ kg $\qquad$ $m_B = 1.2 \times 10^{-21}$ kg

$\vec{v}_{A_i} = 9.2$ m/s [E] $\qquad$ $\vec{v}_{B_i} = 8.5$ m/s [N]

$\vec{v}_{A_f} = 3.3$ m/s [45.0° W of N]



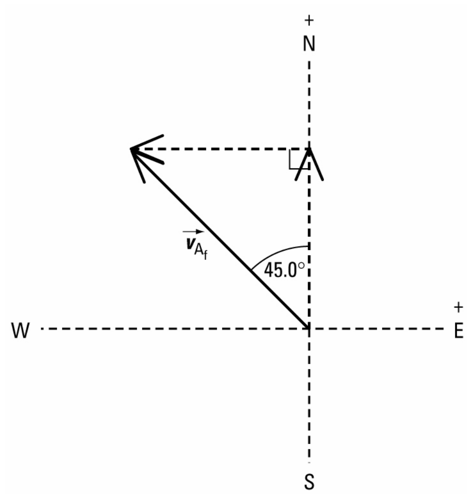


Required

direction of buckyball B ($\vec{v}_{B_f}$)

Analysis and Solution

Choose both buckyballs as an isolated system.
Resolve all velocities into east and north components.



Vector	East component	North component
$\vec{v}_{A_i}$	9.2 m/s	0
$\vec{v}_{B_i}$	0	8.5 m/s
$\vec{v}_{A_f}$	–(3.3 m/s)(sin 45.0°)	(3.3 m/s)(cos 45.0°)

Apply the law of conservation of momentum to the system in the east and north directions.

E direction

$$
\begin{aligned}
p_{\text{sys}_{\text{i}E}} &= p_{\text{sys}_{\text{f}E}} \\
p_{\text{A}_{\text{i}E}} + p_{\text{B}_{\text{i}E}} &= p_{\text{A}_{\text{f}E}} + p_{\text{B}_{\text{f}E}} \\
m_{\text{A}} v_{\text{A}_{\text{i}E}} + 0 &= m_{\text{A}} v_{\text{A}_{\text{f}E}} + m_{\text{B}} v_{\text{B}_{\text{f}E}} \\
\left(\frac{m_{\text{A}}}{m_{\text{B}}}\right) v_{\text{A}_{\text{i}E}} &= \left(\frac{m_{\text{A}}}{m_{\text{B}}}\right) v_{\text{A}_{\text{f}E}} + v_{\text{B}_{\text{f}E}} \\
v_{\text{B}_{\text{f}E}} &= \left(\frac{m_{\text{A}}}{m_{\text{B}}}\right)(v_{\text{A}_{\text{i}E}} - v_{\text{A}_{\text{f}E}}) \\
&= \left(\frac{1.2\times10^{-21}\ \cancel{\text{kg}}}{1.2\times10^{-21}\ \cancel{\text{kg}}}\right)\{9.2\text{ m/s} - [-(3.3\text{ m/s})(\sin 45.0°)]\} \\
&= 11.5\text{ m/s}
\end{aligned}
$$

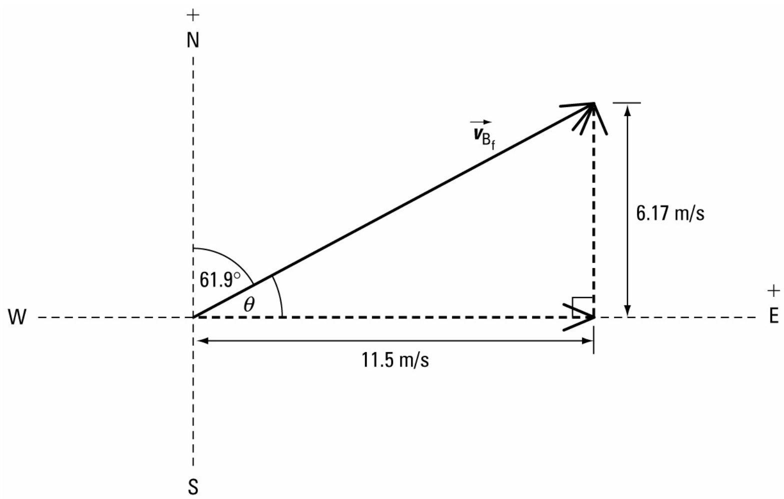
N direction

$$
\begin{aligned}
p_{\text{sys}_{\text{i}N}} &= p_{\text{sys}_{\text{f}N}} \\
p_{\text{A}_{\text{i}N}} + p_{\text{B}_{\text{i}N}} &= p_{\text{A}_{\text{f}N}} + p_{\text{B}_{\text{f}N}} \\
0 + m_{\text{B}} v_{\text{B}_{\text{i}N}} &= m_{\text{A}} v_{\text{A}_{\text{f}N}} + m_{\text{B}} v_{\text{B}_{\text{f}N}} \\
v_{\text{B}_{\text{i}N}} &= \left(\frac{m_{\text{A}}}{m_{\text{B}}}\right) v_{\text{A}_{\text{f}N}} + v_{\text{B}_{\text{f}N}} \\
v_{\text{B}_{\text{f}N}} &= v_{\text{B}_{\text{i}N}} - \left(\frac{m_{\text{A}}}{m_{\text{B}}}\right) v_{\text{A}_{\text{f}N}} \\
&= 8.5\text{ m/s} - \left(\frac{1.2\times10^{-21}\ \cancel{\text{kg}}}{1.2\times10^{-21}\ \cancel{\text{kg}}}\right)(3.3\text{ m/s})(\cos 45.0°) \\
&= 6.17\text{ m/s}
\end{aligned}
$$

Use the tangent function to find the direction of $\vec{v}_{\text{B}_{\text{f}}}$.

$$
\begin{aligned}
\tan\theta &= \frac{\text{opposite}}{\text{adjacent}} \\
&= \frac{6.17\ \cancel{\frac{\text{m}}{\text{s}}}}{11.5\ \cancel{\frac{\text{m}}{\text{s}}}} \\
&= 0.5365 \\
\theta &= \tan^{-1}(0.5365) \\
&= 28.2°
\end{aligned}
$$

From the figure below, this angle is between $\vec{v}_{B_f}$ and the east direction. So the direction of $\vec{v}_{B_f}$ measured from north is 90.0° – 28.2° = 61.8°.

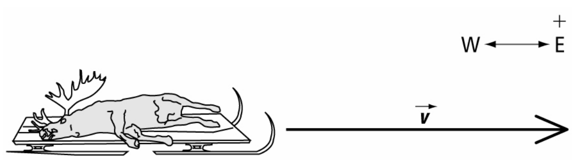


Paraphrase

The direction of buckyball B will be 61.8° E of N immediately after the collision.

70. (a) ***Given***

$\vec{p} = 3.87 \times 10^3$ kg•m/s [E]
m = 650 kg



Required

velocity of moose-sled system ($\vec{v}$)

Analysis and Solution

The momentum of the system is in the direction of its velocity. So use the scalar form of $\vec{p} = m\vec{v}$ to find the speed.

$$p = mv$$

$$v = \frac{p}{m}$$

$$= \frac{3.87 \times 10^3 \ \text{kg} \bullet \frac{\text{m}}{\text{s}}}{650 \ \text{kg}}$$

$$= 5.95 \text{ m/s}$$

$$\vec{v} = 5.95 \text{ m/s [E]}$$

Paraphrase

The velocity of the moose-sled system is 5.95 m/s [E].

(b) ***Given***

F_f = 1400 N

$\vec{v}_i$ = 5.954 m/s [E] from part (a)

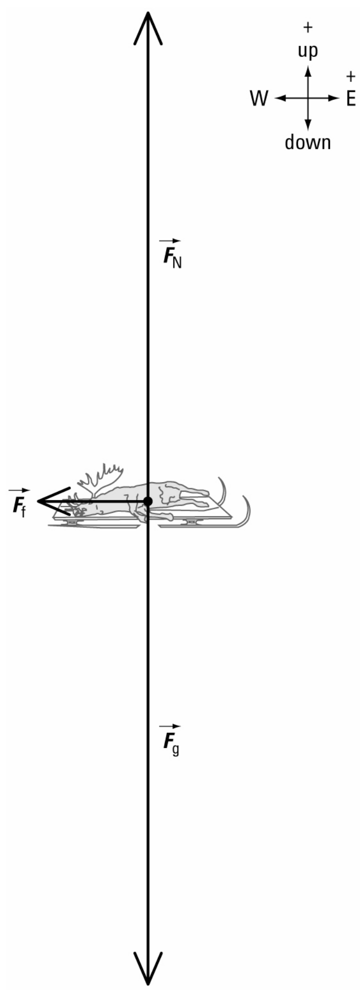
$\vec{v}_f$ = 0 m/s

Required

minimum time interval to stop (Δt)

Analysis and Solution

Draw a free-body diagram for the moose-sled system.



The system is not accelerating up or down. So in the vertical direction, F_{net_v} = 0 N.

In order to keep the moose-sled system from running into the snowmobile, the only force acting on the system must be the force of friction. So in the horizontal direction, F_{net_h} = 1400 N.

Write equations to find the net force on the system in both the horizontal and vertical directions.

horizontal direction

$$\vec{F}_{\text{net}_\text{h}} = \vec{F}_{\text{f}_\text{kinetic}}$$

$$F_{\text{net}_\text{h}} = F_{\text{f}_\text{kinetic}}$$

$$= -1400 \text{ N}$$

vertical direction

$$\vec{F}_{\text{net}_\text{v}} = \vec{F}_\text{N} + \vec{F}_\text{g}$$

$$F_{\text{net}_\text{v}} = 0$$

Calculations in the vertical direction are not required in this problem.

$\vec{F}_{\text{net}_\text{h}}$ acting for the minimum time interval provides the required impulse to stop the system.

Impulse is numerically equal to $m\Delta\vec{v}$ or $m(\vec{v}_\text{f} - \vec{v}_\text{i})$. Calculate the time interval.

$$\vec{F}_{\text{net}_\text{h}}\Delta t = m(\vec{v}_\text{f} - \vec{v}_\text{i})$$

$$(-1400 \text{ N})\Delta t = m(0 - 5.954 \text{ m/s})$$

$$\Delta t = \frac{m(-5.954 \text{ m/s})}{-1400 \text{ N}}$$

$$= \frac{(650 \cancel{\text{kg}})\left(-5.954 \dfrac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)}{-1400 \cancel{\text{kg}} \bullet \dfrac{\cancel{\text{m}}}{\text{s}^{\cancel{2}}}}$$

$$= 2.76 \text{ s}$$

Paraphrase

The minimum time for the driver to stop the snowmobile is 2.76 s.

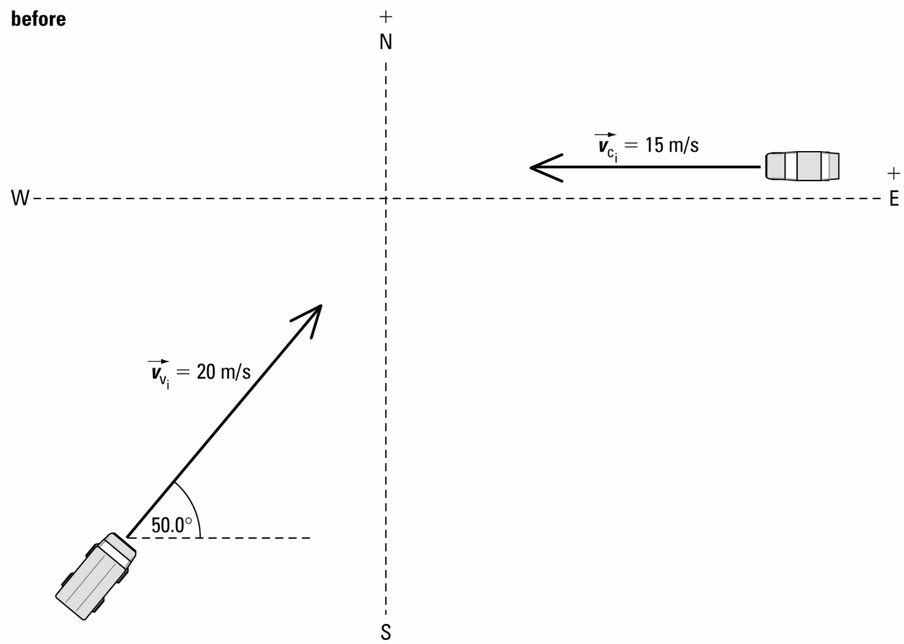
71. ***Given***

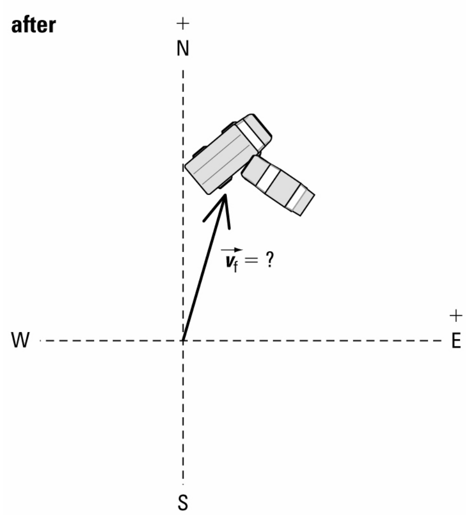
$m_\text{c} = 940$ kg

$\vec{v}_{\text{c}_\text{i}} = 15$ m/s [W]

$m_\text{v} = 1680$ kg

$\vec{v}_{\text{v}_\text{i}} = 20$ m/s [50.0° N of E]



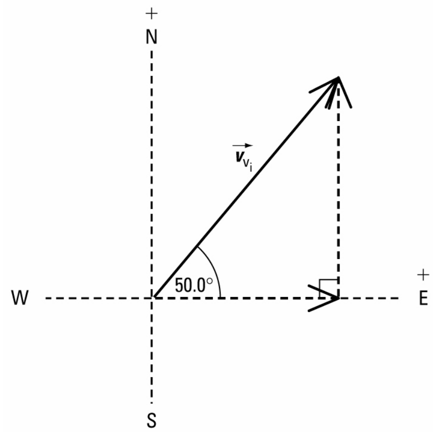


Required

final velocity of centre of mass ($\vec{v}_f$)

Analysis and Solution

Choose the car and the van as an isolated system.
The car and van stick together after collision. So both vehicles have the same final velocity.
Resolve all velocities into east and north components.



Vector	East component	North component
$\vec{v}_{c_i}$	–15 m/s	0
$\vec{v}_{v_i}$	(20 m/s)(cos 50.0°)	(20 m/s)(sin 50.0°)

Apply the law of conservation of momentum to the system in the east and north directions.

E direction

$$p_{sys_{iE}} = p_{sys_{fE}}$$

$$p_{c_{iE}} + p_{v_{iE}} = p_{sys_{fE}}$$

$$m_c v_{c_{iE}} + m_v v_{v_{iE}} = (m_c + m_v) v_{f_E}$$

$$v_{f_E} = \left(\frac{1}{m_c + m_v}\right)(m_c v_{c_{iE}} + m_v v_{v_{iE}})$$

$$v_{f_E} = \left(\frac{1}{940 \text{ kg} + 1680 \text{ kg}}\right)\{(940 \text{ kg})(-15 \text{ m/s}) + (1680 \text{ kg})(20 \text{ m/s})(\cos 50.0°)\}$$

$$= 2.86 \text{ m/s}$$

N direction

$$p_{sys_{iN}} = p_{sys_{fN}}$$

$$p_{c_{iN}} + p_{v_{iN}} = p_{sys_{fN}}$$

$$0 + m_v v_{v_{iN}} = (m_c + m_v) v_{f_N}$$

$$v_{f_N} = \left(\frac{m_v}{m_c + m_v}\right) v_{v_{iN}}$$

$$= \left(\frac{1680 \text{ kg}}{940 \text{ kg} + 1680 \text{ kg}}\right)(20 \text{ m/s})(\sin 50.0°)$$

$$= \left(\frac{1680 \not{\text{kg}}}{2620 \not{\text{kg}}}\right)(20 \text{ m/s})(\sin 50.0°)$$

$$= 9.82 \text{ m/s}$$

Use the Pythagorean theorem to find the magnitude of $\vec{v}_f$.

$$v_f = \sqrt{\left(v_{f_E}\right)^2 + \left(v_{f_N}\right)^2}$$

$$= \sqrt{(2.86\text{ m/s})^2 + (9.82\text{ m/s})^2}$$

$$= 10\text{ m/s}$$

Use the tangent function to find the direction of $\vec{v}_f$.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

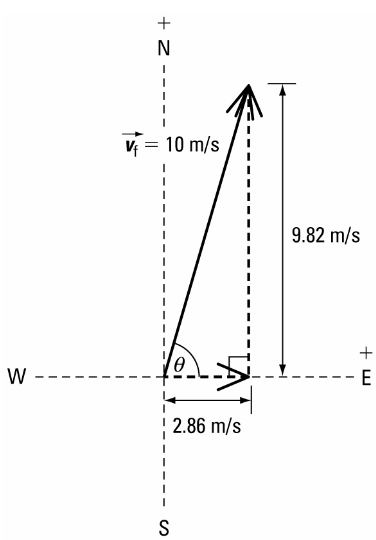
$$= \frac{9.82\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}}{2.86\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}}$$

$$= 3.433$$

$$\theta = \tan^{-1}(3.433)$$

$$= 73.8°$$

From the figure below, this angle is between $\vec{v}_f$ and the east direction.



$$\vec{v}_f = 10\text{ m/s [73.8° N of E]}$$

Paraphrase

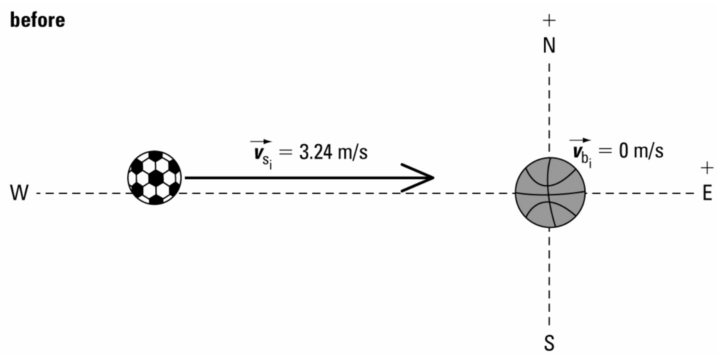
The velocity of the centre of mass of both vehicles will be 10 m/s [73.8° N of E] immediately after the collision.

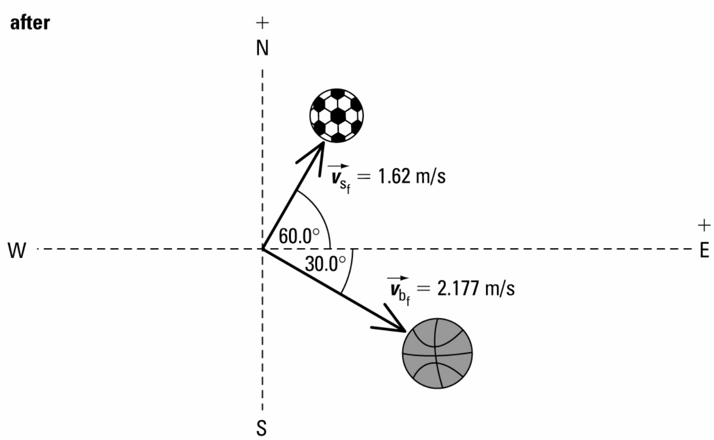
72. ***Given***

$m_s = 0.450$ kg

$\vec{v}_{s_i} = 3.24$ m/s [E] $\qquad \vec{v}_{b_i} = 0$ m/s

$\vec{v}_{s_f} = 1.62$ m/s [60.0° N of E] $\qquad \vec{v}_{b_f} = 2.177$ m/s [30.0° S of E]



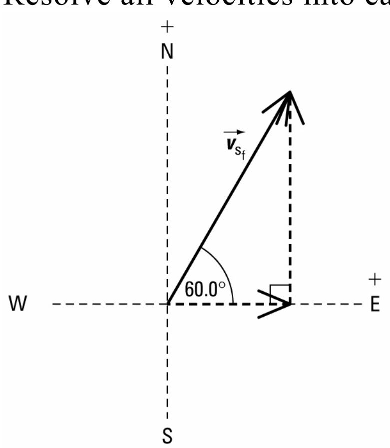


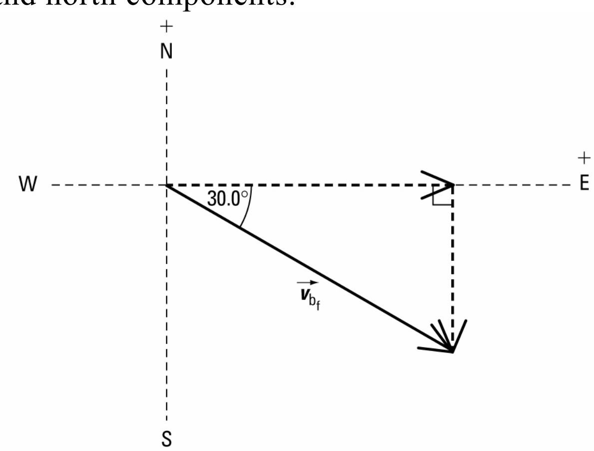
Required

mass of basketball (m_b)

Analysis and Solution

Choose the soccer ball and basketball as an isolated system.
Resolve all velocities into east and north components.





Vector	East component	North component
$\vec{v}_{s_i}$	3.24 m/s	0
$\vec{v}_{s_f}$	(1.62 m/s)(cos 60.0°)	(1.62 m/s)(sin 60.0°)
$\vec{v}_{b_f}$	(2.177 m/s)(cos 30.0°)	–(2.177 m/s)(sin 30.0°)

The basketball has an initial velocity of zero. So its initial momentum is zero.

$$\vec{p}_{b_i} = 0$$

Apply the law of conservation of momentum to the system in the north direction.

N direction

$$\begin{aligned}
p_{sys_{iN}} &= p_{sys_{fN}} \\
p_{s_{iN}} + p_{b_{iN}} &= p_{s_{fN}} + p_{b_{fN}} \\
m_s v_{s_{iN}} + 0 &= m_s v_{s_{fN}} + m_b v_{b_{fN}} \\
0 + 0 &= m_s v_{s_{fN}} + m_b v_{b_{fN}} \\
m_b v_{b_{fN}} &= -m_s v_{s_{fN}} \\
m_b &= -m_s \left(\frac{v_{s_{fN}}}{v_{b_{fN}}} \right) \\
&= -(0.450 \text{ kg}) \left(\frac{\left(1.62 \frac{\text{m}}{\text{s}}\right) \sin 60.0^\circ}{\left(-2.177 \frac{\text{m}}{\text{s}}\right) \sin 30.0^\circ} \right) \\
&= 0.580 \text{ kg}
\end{aligned}$$

Paraphrase

The mass of the basketball is 0.580 kg.

73. ***Given***

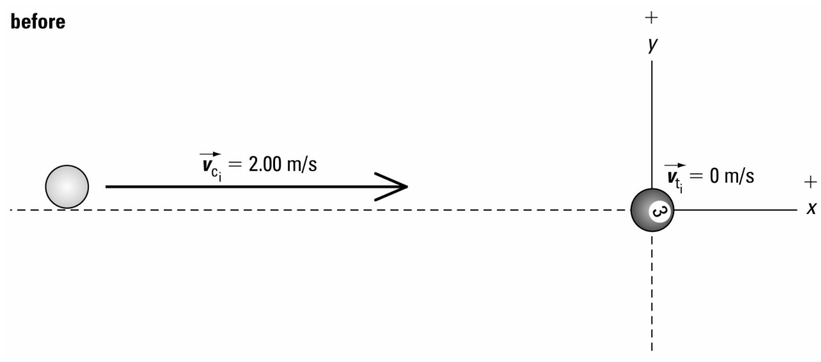
$m_c = 0.160$ kg

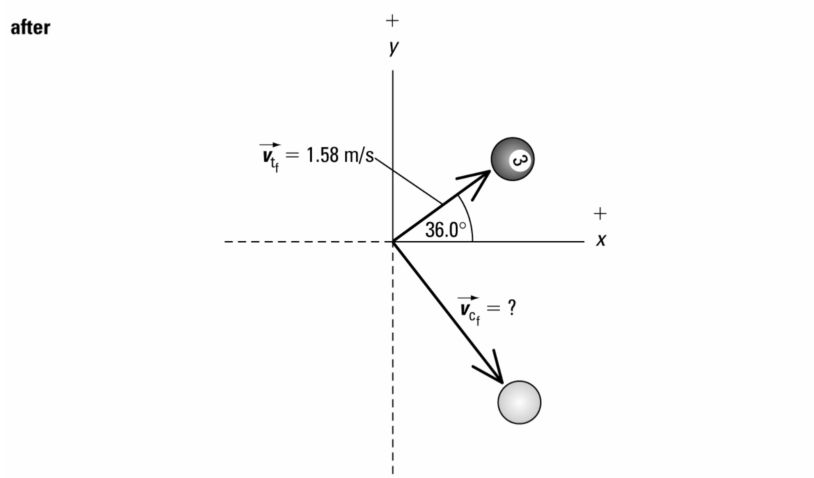
$\vec{v}_{c_i} = 2.00$ m/s [0°]

$m_t = 0.160$ kg

$\vec{v}_{t_i} = 0$ m/s

$\vec{v}_{t_f} = 1.58$ m/s [36.0°]



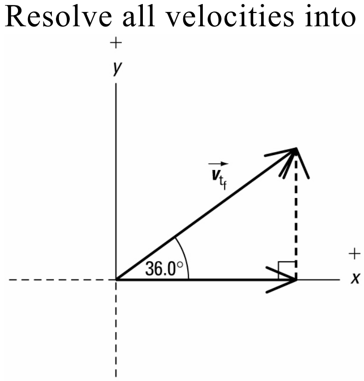


Required

final velocity of cue ball ($\vec{v}_{c_f}$)

Analysis and Solution

Choose the cue ball and three-ball as an isolated system.
Resolve all velocities into x and y components.



Vector	*x* component	*y* component
$\vec{v}_{c_i}$	2.00 m/s	0
$\vec{v}_{t_f}$	(1.58 m/s)(cos 36.0°)	(1.58 m/s)(sin 36.0°)

The three-ball has an initial velocity of zero. So its initial momentum is zero.

$$\vec{p}_{t_i} = 0$$

Apply the law of conservation of momentum to the system in the *x* and *y* directions.

x direction

$$\begin{aligned}
p_{sys_{ix}} &= p_{sys_{fx}} \\
p_{c_{ix}} + p_{t_{ix}} &= p_{c_{fx}} + p_{t_{fx}} \\
m_c v_{c_{ix}} + 0 &= m_c v_{c_{fx}} + m_t v_{t_{fx}} \\
v_{c_{ix}} &= v_{c_{fx}} + \left(\frac{m_t}{m_c}\right) v_{t_{fx}} \\
v_{c_{fx}} &= v_{c_{ix}} - \left(\frac{m_t}{m_c}\right) v_{t_{fx}} \\
&= 2.00\text{ m/s} - \left(\frac{0.160\ \cancel{\text{kg}}}{0.160\ \cancel{\text{kg}}}\right)(1.58\text{ m/s})(\cos 36.0°) \\
&= 0.7218\text{ m/s}
\end{aligned}$$

y direction

$$\begin{aligned}
p_{sys_{iy}} &= p_{sys_{fy}} \\
p_{c_{iy}} + p_{t_{iy}} &= p_{c_{fy}} + p_{t_{fy}} \\
m_c v_{c_{iy}} + 0 &= m_c v_{c_{fy}} + m_t v_{t_{fy}} \\
0 + 0 &= m_c v_{c_{fy}} + m_t v_{t_{fy}} \\
v_{c_{fy}} &= -\left(\frac{m_t}{m_c}\right) v_{t_{fy}} \\
&= -\left(\frac{0.160\ \cancel{\text{kg}}}{0.160\ \cancel{\text{kg}}}\right)(1.58\text{ m/s})(\sin 36.0°) \\
&= -0.9287\text{ m/s}
\end{aligned}$$

Use the Pythagorean theorem to find the magnitude of $\vec{v}_{c_f}$.

$$v_{c_f} = \sqrt{(v_{c_{fx}})^2 + (v_{c_{fy}})^2}$$

$$= \sqrt{(0.7218 \text{ m/s})^2 + (-0.9287 \text{ m/s})^2}$$

$$= 1.18 \text{ m/s}$$

Use the tangent function to find the direction of $\vec{v}_{c_f}$.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{0.9287 \frac{\text{m}}{\text{s}}}{0.7218 \frac{\text{m}}{\text{s}}}$$

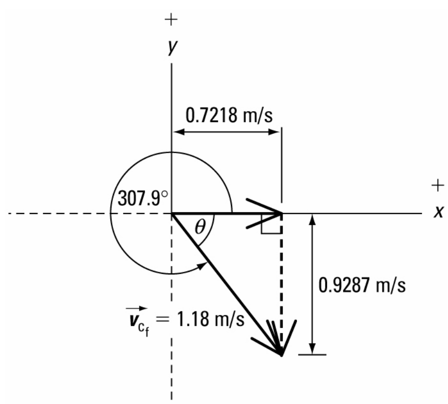
$$= 1.287$$

$$\theta = \tan^{-1}(1.287)$$

$$= 52.1°$$

From the figure below, this angle is between $\vec{v}_{c_f}$ and the positive x-axis. So the direction of $\vec{v}_{c_f}$ measured *counterclockwise* from the positive x-axis is

360° – 52.1° = 307.9°.



$$\vec{v}_{c_f} = 1.18 \text{ m/s } [307.9°]$$

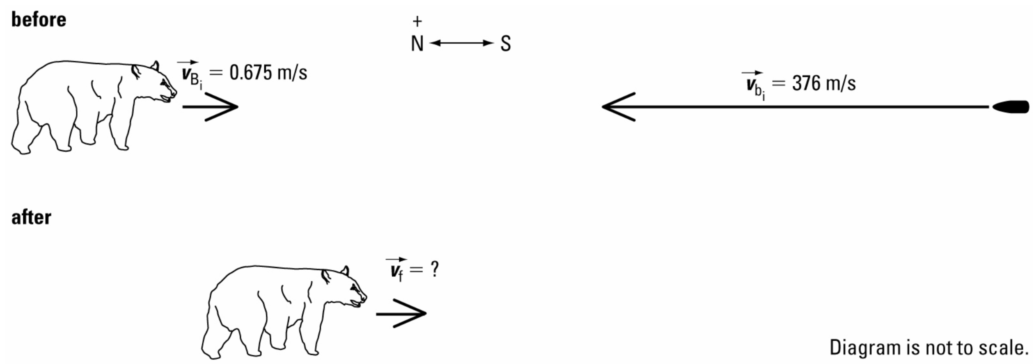
Paraphrase

The velocity of the cue ball will be 1.18 m/s [307.9°] immediately after the collision.

74. ***Given***

$m_B = 250$ kg $m_b = 2.27 \times 10^{-2}$ kg

$\vec{v}_{B_i} = 0.675$ m/s [S] $\vec{v}_{b_i} = 376$ m/s [N]



Required

final velocity of bear ($\vec{v}_{B_f}$)

Analysis and Solution

Choose the bear and the bullet as an isolated system.
The bullet becomes embedded in the bear. So the bear and bullet have the same final velocity.
Apply the law of conservation of momentum to the system.

$$\vec{p}_{sys_i} = \vec{p}_{sys_f}$$

$$\vec{p}_{B_i} + \vec{p}_{b_i} = \vec{p}_{sys_f}$$

$$m_B \vec{v}_{B_i} + m_b \vec{v}_{b_i} = (m_B + m_b)\, \vec{v}_f$$

$$\vec{v}_f = \left(\frac{1}{m_B + m_b}\right)(m_B \vec{v}_{B_i} + m_b \vec{v}_{b_i})$$

$$= \left(\frac{1}{250 \text{ kg} + 2.27 \times 10^{-2} \text{ kg}}\right)\{(250 \text{ kg})(-0.675 \text{ m/s}) + (2.27 \times 10^{-2} \text{ kg})(+376 \text{ m/s})\}$$

$$= -0.641 \text{ m/s}$$

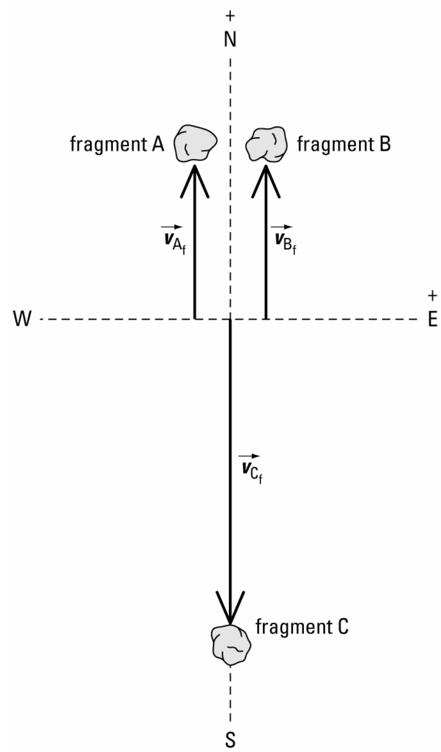
$$\vec{v}_f = 0.641 \text{ m/s [S]}$$

Paraphrase and Verify

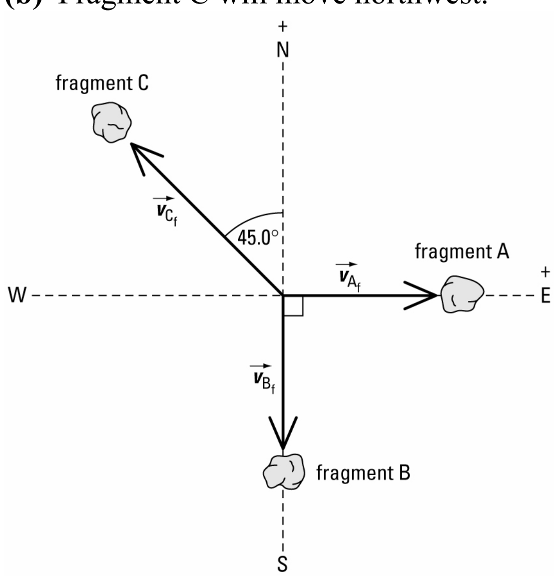
The velocity of the bear was 0.641 m/s [S] immediately after being shot.
Since the bear was still moving toward the hunter, the hunter's claim is false.

75. The direction of fragment C is determined by applying the law of conservation of momentum.

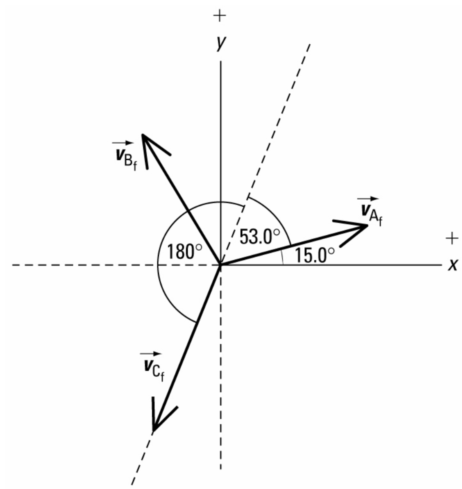
(a) Fragment C will move south.



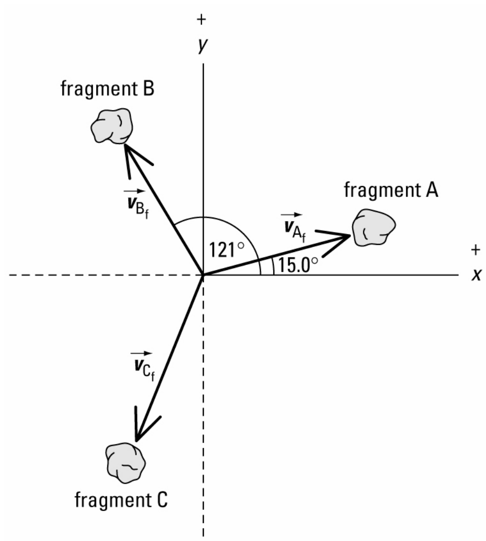
(b) Fragment C will move northwest.



(c) Since fragments A and B do not move along the *x* and *y* axes, find the angle between $\vec{v}_{A_f}$ and $\vec{v}_{B_f}$. Then divide it by 2 to get the line along which fragment C will move (see dashed line in figure below).



Then apply the law of conservation of momentum to find the direction along the dashed line.



Fragment C will move 15° + 53° + 180° = 248°.

Extensions

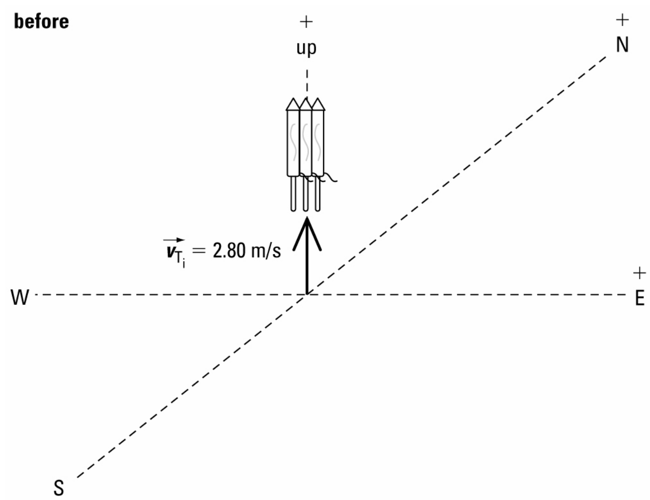
76. A Pelton wheel causes the water to reverse direction. This results in twice the usual change in momentum of the water from a standard water wheel. Since the impulse provided to the water using a Pelton wheel is approximately double that using a standard water wheel, the Pelton wheel is dramatically more efficient.

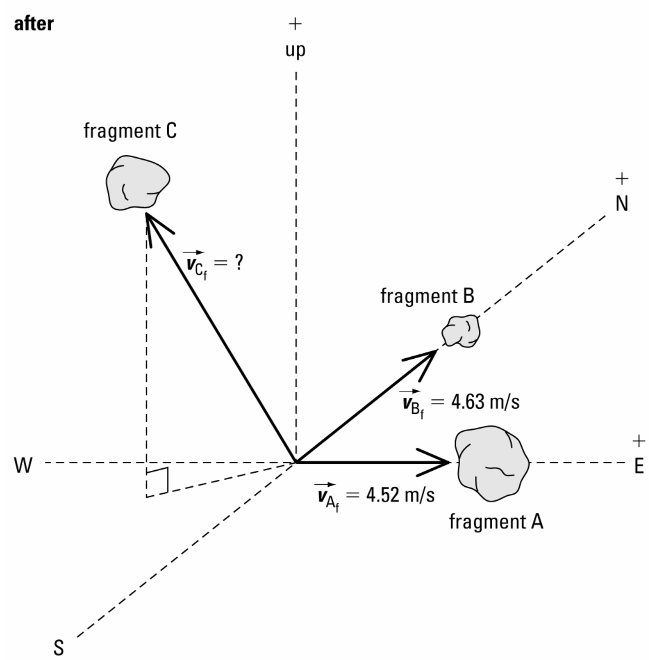
77. ***Given***

$\vec{v}_{T_i}$ = 2.80 m/s [up]

m_A = 0.210 kg $\quad$ m_B = 0.195 kg $\quad$ m_C = 0.205 kg

$\vec{v}_{A_f}$ = 4.52 m/s [E] $\quad$ $\vec{v}_{B_f}$ = 4.63 m/s [N]





Required

final velocity of fragment C ($\vec{v}_{C_f}$)

Analysis and Solution

Choose the original fireworks bundle and all three fragments as an isolated system.

Since the original fireworks bundle has an upward velocity, this problem is three-dimensional.

Resolve all velocities into up, east, and north components.

Vector	Up component	East component	North component
$\vec{v}_{T_i}$	2.80 m/s	0	0
$\vec{v}_{A_f}$	0	4.52 m/s	0
$\vec{v}_{B_f}$	0	0	4.63 m/s

Apply the law of conservation of momentum to the system in the up, east, and north directions.

Up direction

$$
\begin{aligned}
p_{\text{sys}_{iU}} &= p_{\text{sys}_{fU}} \\
p_{\text{sys}_{iU}} &= p_{A_{fU}} + p_{B_{fU}} + p_{C_{fU}} \\
(m_A + m_B + m_C)\,v_{T_{iU}} &= 0 + 0 + m_C\,v_{C_{fU}} \\
v_{C_{fU}} &= \left(\frac{m_A + m_B + m_C}{m_C}\right) v_{T_{iU}} \\
&= \left(\frac{0.210\text{ kg} + 0.195\text{ kg} + 0.205\text{ kg}}{0.205\text{ kg}}\right)(2.80\text{ m/s}) \\
&= \left(\frac{0.610\ \cancel{\text{kg}}}{0.205\ \cancel{\text{kg}}}\right)(2.80\text{ m/s}) \\
&= 8.332\text{ m/s}
\end{aligned}
$$

E direction

$$
\begin{aligned}
p_{\text{sys}_{iE}} &= p_{\text{sys}_{fE}} \\
p_{\text{sys}_{iE}} &= p_{A_{fE}} + p_{B_{fE}} + p_{C_{fE}} \\
0 &= m_A\,v_{A_{fE}} + 0 + m_C\,v_{C_{fE}} \\
m_C\,v_{C_{fE}} &= -m_A\,v_{A_{fE}} \\
v_{C_{fE}} &= -\left(\frac{m_A}{m_C}\right) v_{A_{fE}} \\
&= -\left(\frac{0.210\ \cancel{\text{kg}}}{0.205\ \cancel{\text{kg}}}\right)(4.52\text{ m/s}) \\
&= -4.630\text{ m/s}
\end{aligned}
$$

N direction

$$
\begin{aligned}
p_{\text{sys}_{iN}} &= p_{\text{sys}_{fN}} \\
p_{\text{sys}_{iN}} &= p_{A_{fN}} + p_{B_{fN}} + p_{C_{fN}} \\
0 &= 0 + m_B\,v_{B_{fN}} + m_C\,v_{C_{fN}}
\end{aligned}
$$

$$m_C v_{C_{fN}} = -m_B v_{B_{fN}}$$

$$v_{C_{fN}} = -\left(\frac{m_B}{m_C}\right) v_{B_{fN}}$$

$$= -\left(\frac{0.195 \text{ kg}}{0.205 \text{ kg}}\right)(4.63 \text{ m/s})$$

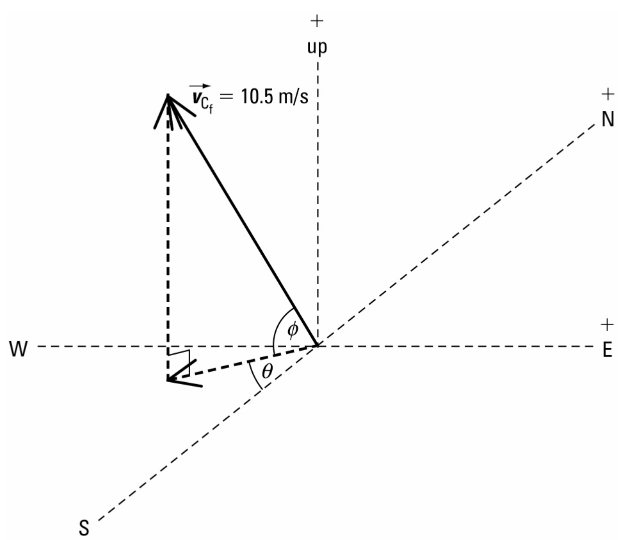
$$= -4.404 \text{ m/s}$$

Use the Pythagorean theorem to find the magnitude of $\vec{v}_{C_f}$.

$$v_{C_f} = \sqrt{\left(v_{C_{fU}}\right)^2 + \left(v_{C_{fE}}\right)^2 + \left(v_{C_{fN}}\right)^2}$$

$$= \sqrt{(8.332 \text{ m/s})^2 + (-4.630 \text{ m/s})^2 + (-4.404 \text{ m/s})^2}$$

$$= 10.5 \text{ m/s}$$



Use the tangent function to find the direction of $\vec{v}_{C_f}$ in the east-north plane.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{4.404 \frac{\text{m}}{\text{s}}}{4.630 \frac{\text{m}}{\text{s}}}$$

$$= 0.9512$$

$$\theta = \tan^{-1}(0.9512)$$

$$= 43.6°$$

From the figure above, this angle is between the projection of $\vec{v}_{C_f}$ in the east-north plane and the west direction. So the direction of $\vec{v}_{C_f}$ measured from south is 90.0° – 43.6° = 46.4°.

Use the sine function to find the direction of $\vec{v}_{C_f}$ above the east-north plane.

$$\begin{aligned}
\sin\phi &= \frac{\text{opposite}}{\text{hypotenuse}} \\
&= \frac{8.332\ \frac{\text{m}}{\text{s}}}{10.50\ \frac{\text{m}}{\text{s}}} \\
&= 0.7935 \\
\phi &= \sin^{-1}(0.7935) \\
&= 52.5°
\end{aligned}$$

From the figure above, this angle is between $\vec{v}_{C_f}$ and the projection of $\vec{v}_{C_f}$ in the east-north plane.

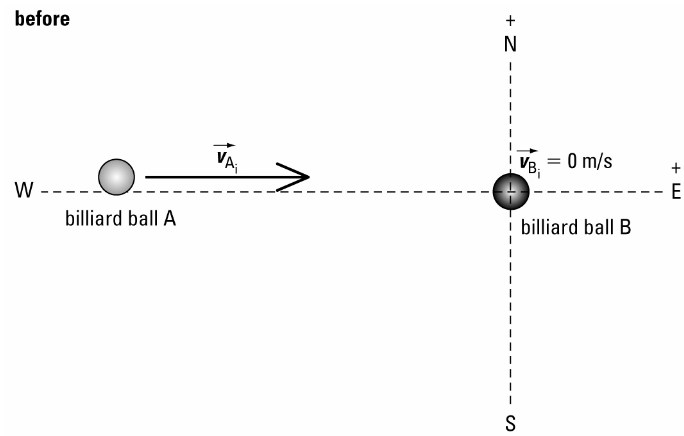
$$\vec{v}_{C_f} = 10.5 \text{ m/s [46.4° W of S and 52.5° up]}$$

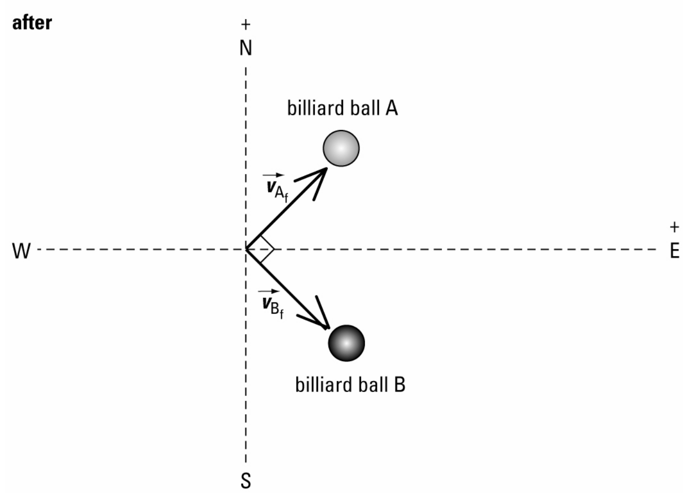
Paraphrase

The velocity of fragment C will be 10.5 m/s [46.4° W of S and 52.5° up] immediately after the explosion.

78. Students can find information on space shuttles and improvements to rocket engines using the links at www.pearsoned.ca/school/physicssource. Students can also search the Internet using the keywords “2005, new, current, recent, etc.”

79. The figure below shows a representation of the situation.





At the instant the billiard balls collide, only one point on the surface of ball A is in contact with ball B. If you were to draw a tangent through this point of contact, you would find that the impulsive forces act perpendicular to the tangent and these forces act along the line joining the centres of both balls.

Evidence of the direction of the impulsive forces is in the direction of motion of both balls after collision. For ball B, since $\vec{p}_{B_i}$ = 0 before collision and $\vec{p}_{B_f}$ is directed SE after collision, ball B experienced a change in momentum in the SE direction.

From Newton's third law, if ball A exerts an impulsive force SE on ball B, then ball B exerts an impulsive force NW of equal magnitude on ball A. Since $\vec{p}_{A_i}$ is directed east and the impulsive force acts NW, the resultant change in momentum for ball A is NE.

The impulsive forces acting perpendicular to the tangent through the point of contact is true no matter in what directions the balls were initially moving in.

80. ***Given***

m_c = 2200 kg

$\vec{v}_{c_i} = v_{c_i}$ m/s [W]

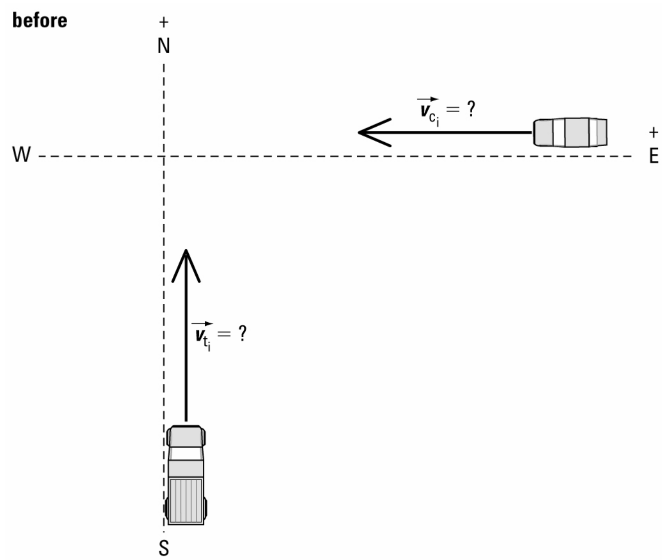
d = 20 m

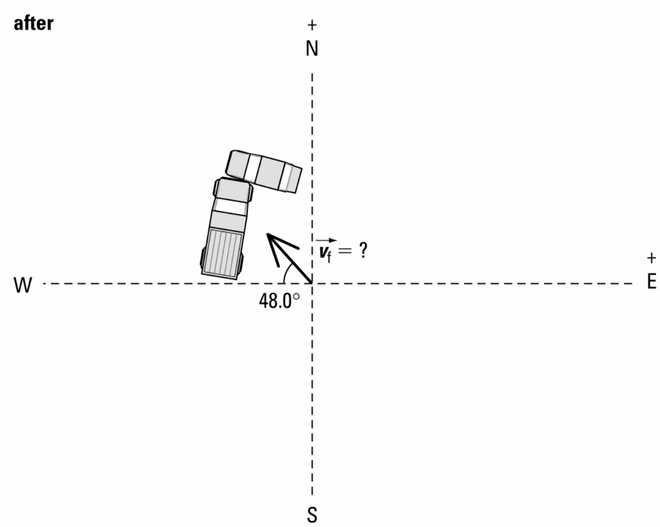
m_t = 2500 kg

$\vec{v}_{t_i} = v_{t_i}$ m/s [N]

$\vec{v}_f = v_f$ m/s [48.0° N of W]

μ_k = 0.78





Required

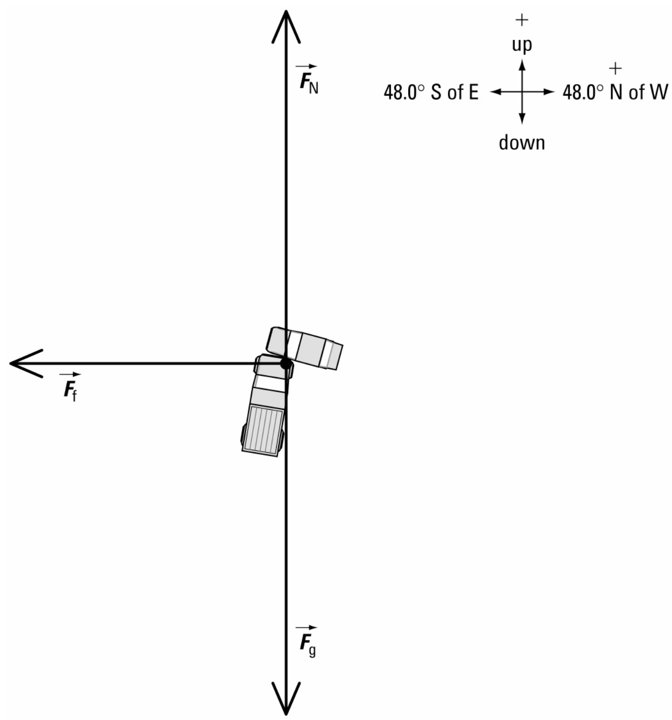
initial speeds of car and truck (v_{c_i} and v_{t_i})

Analysis and Solution

The car and truck move together as a unit after collision. So calculate the total mass.

$$\begin{aligned} m_T &= m_c + m_t \\ &= 2200\ \text{kg} + 2500\ \text{kg} \\ &= 4700\ \text{kg} \end{aligned}$$

Draw a free-body diagram for the car-truck system after collision.



The system is not accelerating up or down. So in the vertical direction, $F_{net_v} = 0$ N.

Write equations to find the net force on the system in both the horizontal and vertical directions.

horizontal direction

$$\vec{F}_{net_h} = \vec{F}_{f_{kinetic}}$$
$$F_{net_h} = F_{f_{kinetic}}$$
$$m_T a = -\mu_k F_N$$

vertical direction

$$\vec{F}_{net_v} = \vec{F}_N + \vec{F}_g$$
$$F_{net_v} = F_N + F_g$$
$$0 = F_N + (-m_T g)$$
$$0 = F_N - m_T g$$
$$F_N = m_T g$$

Substitute $F_N = m_T g$ into the equation for F_{net_h}.

$$\cancel{m_T}\, a = -\mu_k \cancel{m_T}\, g$$
$$a = -(0.78)(9.81 \text{ m/s}^2)$$
$$= -7.65 \text{ m/s}^2$$

Since the car-truck system skids to a stop, calculate the velocity of the system at the start of the skid.

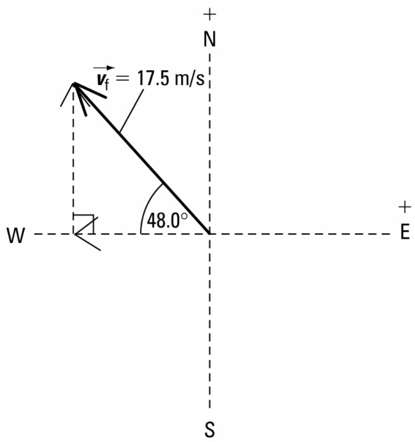
$$(v_f)^2 = (v_i)^2 + 2ad$$
$$0 = (v_i)^2 + 2ad$$
$$(v_i)^2 = -2ad$$

$$
\begin{aligned}
v_i &= \sqrt{-2ad} \\
&= \sqrt{-2(-7.65\ \text{m/s}^2)(20\ \text{m})} \\
&= 17.5\ \text{m/s}
\end{aligned}
$$

So the velocity of the system immediately after collision was 17.5 m/s [48.0° N of W].

Choose the car and the truck immediately before and immediately after collision as an isolated system.

Resolve all velocities into east and north components.



Vector	East component	North component
$\vec{v}_{c_i}$	$-v_{c_i}$	0
$\vec{v}_{t_i}$	0	v_{t_i}
$\vec{v}_f$	–(17.5 m/s)(cos 48.0°)	(17.5 m/s)(sin 48.0°)

Apply the law of conservation of momentum to the system in the east and north directions.

E direction

$$
\begin{aligned}
p_{\text{sys}_{iE}} &= p_{\text{sys}_{fE}} \\
p_{c_{iE}} + p_{t_{iE}} &= p_{\text{sys}_{fE}} \\
-m_c v_{c_{iE}} + 0 &= m_T v_{f_E} \\
v_{c_{iE}} &= -\left(\frac{m_T}{m_c}\right) v_{f_E} \\
&= -\left(\frac{4700\ \cancel{\text{kg}}}{2200\ \cancel{\text{kg}}}\right)\{-(17.5\ \text{m/s})(\cos 48.0°)\} \\
&= 25\ \text{m/s or } 90\ \text{km/h}
\end{aligned}
$$

N direction

$$p_{\text{sys}_{\text{i}N}} = p_{\text{sys}_{\text{f}N}}$$

$$p_{\text{c}_{\text{i}N}} + p_{\text{t}_{\text{i}N}} = p_{\text{sys}_{\text{f}N}}$$

$$0 + m_{\text{t}} v_{\text{t}_{\text{i}N}} = m_{\text{T}} v_{\text{f}_N}$$

$$v_{\text{t}_{\text{i}N}} = \left(\frac{m_{\text{T}}}{m_{\text{t}}}\right) v_{\text{f}_N}$$

$$= \left(\frac{4700\ \cancel{\text{kg}}}{2500\ \cancel{\text{kg}}}\right)(17.5\text{ m/s})(\sin 48.0°)$$

$$= 24\text{ m/s or }88\text{ km/h}$$

Paraphrase

Immediately before collision, the car had a speed of 90 km/h and the truck a speed of 88 km/h.

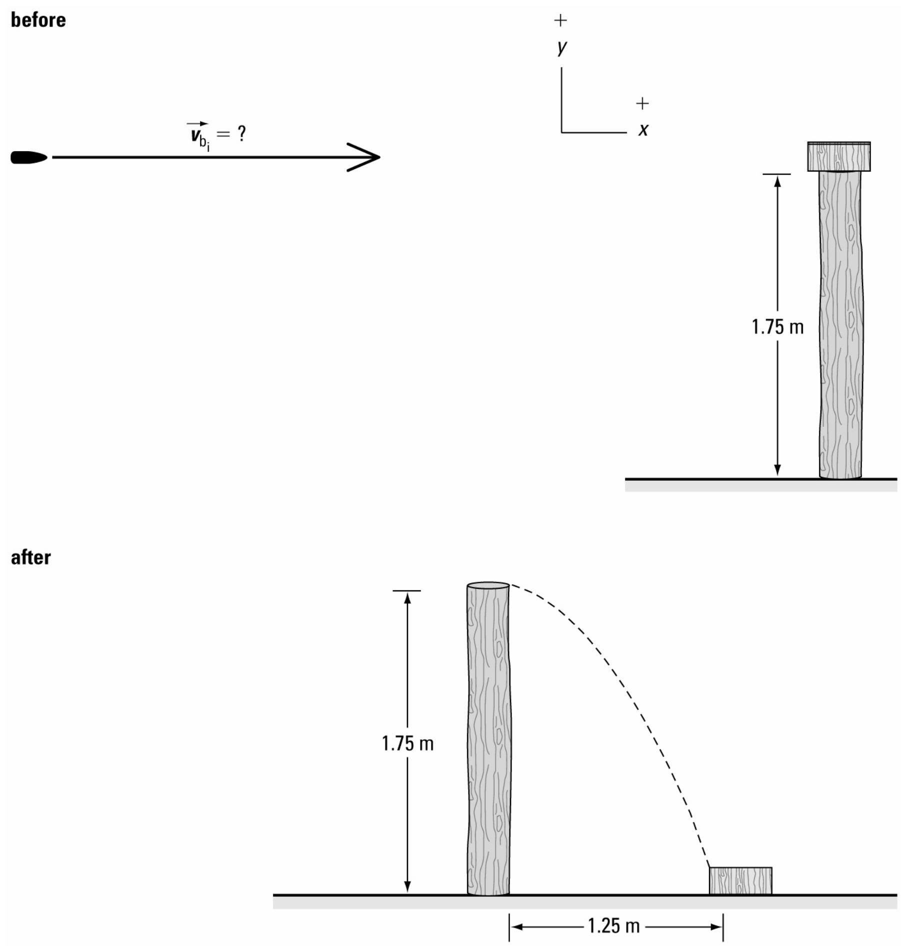
81. The links at www.pearsoned.ca/school/physicssource provide students with information on the latest research from the University of Calgary on running shoes and joint pain, research from the Medical Research Council of South Africa, and a podiatrist discussing pronation and how the design of running shoes can prevent injury.

The National Operating Committee on Standards for Athletic Equipment (NOCSAE) conducted a study to determine if wearing orthotics reduces bone strain when walking or running. The study included a variety of people wearing biomechanical orthotics, either semi-rigid polypropylene or soft Pelite, in Nike Airs or combat boots. The study found that for walking, there was no statistically significant difference between the pre- and post-run treadmill walking tension and compression strains, or strain rates with any combination of shoes and/or orthotics. For running, the use of these orthotics did not reduce strain during running and could be detrimental.

82. ***Given***

$m_{\text{w}} = 3.5$ kg $\quad m_{\text{b}} = 12$ g

$d_y = 1.75$ m $\quad d_x = 1.25$ m



Required

initial speed of bullet (v_{b_i})

Analysis and Solution

The bullet becomes embedded in the wood after collision. So calculate the total mass.

$$\begin{aligned} m_T &= m_w + m_b \\ &= 3.5 \text{ kg} + 12 \text{ g} \times \frac{1 \text{ kg}}{1000 \text{ g}} \\ &= 3.512 \text{ kg} \end{aligned}$$

The motion of the wood while it is falling is parabolic.

Write equations for the distance travelled in both the *x* and *y* directions to find the velocity of the wood-bullet system immediately after impact.

x direction

$$\begin{aligned} d_x &= v_x t \\ v_x &= \frac{d_x}{t} \end{aligned}$$

y direction

$$\begin{aligned} d_y &= -\frac{1}{2} g t^2 \\ t^2 &= -\frac{2d_y}{g} \end{aligned}$$

$$t = \sqrt{\frac{-2d_y}{g}}$$

$$= \sqrt{\frac{-2(1.75\ \cancel{\text{m}})}{-9.81\ \frac{\cancel{\text{m}}}{\text{s}^2}}}$$

$$= 0.597\ \text{s}$$

Substitute t = 0.597 s into the equation for v_x.

$$v_x = \frac{1.25\ \text{m}}{0.597\ \text{s}}$$

$$= 2.09\ \text{m/s}$$

So the velocity of the wood-bullet system immediately after impact was 2.09 m/s [0°].

Choose the wood and the bullet as an isolated system.

The wood has an initial velocity of zero. So its initial momentum is zero.

$$\vec{p}_{\text{w}_\text{i}} = 0$$

Apply the law of conservation of momentum to the system.

$$\vec{p}_{\text{sys}_\text{i}} = \vec{p}_{\text{sys}_\text{f}}$$

$$\vec{p}_{\text{b}_\text{i}} + \vec{p}_{\text{w}_\text{i}} = \vec{p}_{\text{sys}_\text{f}}$$

$$m_\text{b}\vec{v}_{\text{b}_\text{i}} + 0 = m_\text{T}\vec{v}_\text{f}$$

$$\vec{v}_{\text{b}_\text{i}} = \left(\frac{m_\text{T}}{m_\text{b}}\right)\vec{v}_\text{f}$$

$$= \left(\frac{3.512\ \cancel{\text{kg}}}{0.012\ \cancel{\text{kg}}}\right)(+2.09\ \text{m/s})$$

$$= +6.1 \times 10^2\ \text{m/s}$$

$$\vec{v}_{\text{b}_\text{i}} = 6.1 \times 10^2\ \text{m/s [0°]}$$

Paraphrase

The speed of the bullet immediately before impact was 6.1×10^2 m/s.

Consolidate Your Understanding

Students' answers in this section will vary greatly depending on their experiences, depth of understanding, and the depth of the research that they do into each of the topics.

