Pearson Physics Level 30
Unit VIII Atomic Physics: Chapter 16
Solutions

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Example 16.1 Practice Problems

1. ***Given***
   $Z = 12$
   $A = 24$
   Required
   neutron number (N)
   Analysis and Solution
   Since $A = Z + N$,
   $$\begin{aligned} N &= A - Z \\ &= 24 - 12 \\ &= 12 \end{aligned}$$
   Paraphrase
   There are 12 neutrons in a nucleus of $^{24}_{12}\text{Mg}$.
2. ***Given***
   $Z = 92$
   $N = 146$
   Required
   atomic mass number (A)
   Analysis and Solution
   Since $A = Z + N$,
   $$\begin{aligned} A &= 92 + 146 \\ &= 238 \end{aligned}$$
   Paraphrase
   The atomic mass number of the uranium atom is 238.

Concept Check

All three nuclei have the same atomic number, $Z = 6$, but different atomic weights and numbers of neutrons.

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Example 16.2 Practice Problems

1. ***Given***
   $r = 5$ fm $= 5 \times 10^{-15}$ m
   Required
   gravitational force (F_g)

Analysis and Solution

Use the equation $F_g = \frac{Gm_1m_2}{r^2}$, where m_1 and m_2 represent the mass of each proton, 1.67×10^{-27} kg.

$$F_g = \frac{\left(6.67 \times 10^{-11}\ \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right)\left(1.67 \times 10^{-27}\ \text{kg}\right)^2}{\left(5 \times 10^{-15}\ \text{m}\right)^2}$$
$$= 7 \times 10^{-36}\ \text{N}$$

Paraphrase

The gravitational force between two protons that are 5 fm apart is 7×10^{-36} N.

2. ***Given***

r = 5 fm = 5×10^{-15} m

Required

electrostatic force (F_e)

Analysis and Solution

Use the equation $F_e = \frac{kq_1q_2}{r^2}$, where q_1 and q_2 represent the charge on each proton, 1.60×10^{-19} C.

$$F_e = \frac{\left(8.99 \times 10^{9}\ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)\left(1.60 \times 10^{-19}\ \text{C}\right)^2}{\left(5 \times 10^{-15}\ \text{m}\right)^2}$$
$$= 9\ \text{N}$$

Paraphrase

The electrostatic force that two protons exert on each other when they are 5 fm apart is 9 N.

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Example 16.3 Practice Problems

1. ***Given***

m = 0.221 u

Required

energy equivalent in eV (E)

Analysis and Solution

Use the equation $E = mc^2$, where m = 0.221 u, 1 u = 1.66×10^{-27} kg, and 1 eV = 1.60×10^{-19} J.

$$E = \left(0.221\ \cancel{\text{u}}\right)\left(1.66\times10^{-27}\ \frac{\text{kg}}{\cancel{\text{u}}}\right)\left(3.00\times10^{8}\ \frac{\text{m}}{\text{s}}\right)^2$$
$$= \frac{3.30\times10^{-11}\ \cancel{\text{J}}}{1.60\times10^{-19}\ \frac{\cancel{\text{J}}}{\text{eV}}}$$
$$= 206\ \text{MeV}$$

Paraphrase

A mass of 0.221 u is equivalent to 206 MeV.

2. ***Given***

E = 250 MeV

Required

mass equivalent (m)

Analysis and Solution

Use the equation $E = mc^2$.

$$m = \frac{E}{c^2}$$
$$= \frac{250\times10^{6}\ \cancel{\text{eV}}}{\left(3.00\times10^{8}\ \frac{\text{m}}{\text{s}}\right)^2} \times \frac{1.60\times10^{-19}\ \frac{\text{J}}{\cancel{\text{eV}}}}{1.66\times10^{-27}\ \frac{\text{kg}}{\text{u}}}$$
$$= 0.268\ \text{u}$$

Paraphrase

250 MeV is equivalent to a mass of 0.268 u.

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Concept Check

Start with the expression $\Delta m = Zm_{{}^1_1\text{H}} + Nm_{\text{neutron}} - m_{\text{atom}}$. Note that a hydrogen atom consists of one proton and one electron. So, $Zm_{{}^1_1\text{H}}$ = Z(mass of proton + mass of electron). Also, m_{atom} = (mass of nucleus + mass of Z electrons), because there are Z electrons in a neutral atom. Putting these equations together gives:

Z(mass of proton + mass of electron) + Nm_{neutron} – (mass of nucleus + mass of Z electrons). The electron masses subtract, leaving $\Delta m = Zm_{\text{proton}} + Nm_{\text{neutron}} - m_{\text{nucleus}}$.

Example 16.4 Practice Problems

1. ***Given***

m_{Na} = 22.989 769 u

Required

mass defect (Δm)

Analysis and Solution

From ${}^{23}_{11}\text{Na}$, A = 23 and Z = 11.

Use the equation $N = A - Z$ to find the number of neutrons, then use the equation $\Delta m = Zm_{^1_1\text{H}} + Nm_{\text{neutron}} - m_{\text{atom}}$, where $m_{^1_1\text{H}} = 1.007\ 825\ \text{u}$ and $m_{\text{neutron}} = 1.008\ 665\ \text{u}$.

Solution

$$\begin{aligned} N &= A - Z \\ &= 23 - 11 \\ &= 12 \end{aligned}$$

$$\begin{aligned} \Delta m &= Zm_{^1_1\text{H}} + Nm_{\text{neutron}} - m_{\text{atom}} \\ &= 11(1.007\ 825\ \text{u}) + 12(1.008\ 665\ \text{u}) - 22.989\ 769\ \text{u} \\ &= 0.200\ 286\ \text{u} \end{aligned}$$

Paraphrase

The mass defect for the sodium nucleus is 0.200 286 u.

2. ***Given***

the sodium atom, $^{23}_{11}\text{Na}$

Required

binding energy (E_b)

Analysis and Solution

Use Δm from question 1 and the conversion factor 1 u = 931.4943 MeV.

Substitute into the equation $E_b = \Delta m \times c^2$.

$$\begin{aligned} E_b &= \left(0.200\ 286\ \cancel{\text{u}}\right)\left(931.5\ \frac{\text{MeV}}{\cancel{\text{u}}}\right) \\ &= 186.57\ \text{MeV} \end{aligned}$$

Paraphrase

The binding energy of the sodium 23 nucleus is 186.57 MeV.

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16.1 Check and Reflect

Knowledge

1. To find the number of neutrons, *N*, and the number of protons, *Z*, use the equation $A = Z + N$.

(a) $^{90}_{38}\text{Sr}$

$A = 90$ and $Z = 38$, so

$$\begin{aligned} N &= A - Z \\ &= 90 - 38 \\ &= 52 \end{aligned}$$

$^{90}_{38}\text{Sr}$ has 38 protons and 52 neutrons.

(b) $^{13}_{6}\text{C}$

$A = 13$ and $Z = 6$, so

$$\begin{aligned} N &= A - Z \\ &= 13 - 6 \\ &= 7 \end{aligned}$$

$^{13}_{6}\text{C}$ has 6 protons and 7 neutrons.

(c) ${}^{56}_{26}\text{Fe}$

A = 56 and Z = 26, so

$$
\begin{aligned}
N &= A - Z \\
&= 56 - 26 \\
&= 30
\end{aligned}
$$

${}^{56}_{26}\text{Fe}$ has 26 protons and 30 neutrons.

(d) ${}^{1}_{1}\text{H}$

A = 1 and Z = 1, so

$$
\begin{aligned}
N &= A - Z \\
&= 1 - 1 \\
&= 0
\end{aligned}
$$

${}^{1}_{1}\text{H}$ has 1 proton and no neutrons.

2. $\dfrac{1.6\times10^{-10}\ \cancel{\text{J}}}{1.60\times10^{-19}\ \dfrac{\cancel{\text{J}}}{\text{eV}}} = 1.0\times10^{9}\ \text{eV} = 1.0\ \text{GeV}$

3. 1 u = 931.5 MeV so

$$
\begin{aligned}
0.25\ \text{u} &= 0.25\times931.5\ \text{MeV} \\
&= 233\ \text{MeV}
\end{aligned}
$$

4. ***Given***

E = 5.00 GJ

Required

mass (m)

Analysis and Solution

Use the equation $E = mc^2$.

$$
\begin{aligned}
m &= \frac{E}{c^2} \\
&= \frac{5.00\times10^{9}\ \text{J}}{(3.00\times10^{8}\ \text{m/s})^2} \\
&= 5.56\times10^{-8}\ \text{kg}
\end{aligned}
$$

Paraphrase

The quantity of mass that is converted to 5.00 GJ of energy is 5.56×10^{-8} kg .

5. Isotopes are atoms that have the same atomic number but different neutron numbers. They are chemically very similar but do not have the same atomic mass.

6. A stable nucleus is bound together by the strong nuclear force. Binding energy is the energy required to separate all the protons and neutrons in a nucleus and move them infinitely far apart. The mass equivalence of this binding energy, calculated using Einstein's equation, $E = mc^2$, accounts for the slightly lower mass of a stable nucleus than that given by $Zm_{\text{proton}} + Nm_{\text{neutron}}$.

Applications

7. ***Given***

${}^{22}_{10}\text{Ne}$, m_{atom} = 21.991 385 u

Required

binding energy (E_{b})

Analysis and Solution
First calculate the number of neutrons using $A = N + Z$.
Then use the equation $\Delta m = Zm_{\text{H}} + Nm_{\text{neutron}} - m_{\text{atom}}$ to calculate the mass difference.
Use your answer for the mass difference and the equation $E_{\text{b}} = \Delta m \times c^2$
(or use 1 u = 931.4943 MeV) to calculate the binding energy.
Solution
From ${}^{22}_{10}\text{Ne}, Z = 10, A = 22$

$$\begin{aligned} N &= A - Z \\ &= 22 - 10 \\ &= 12 \end{aligned}$$

$$\begin{aligned} \Delta m &= 10(1.007\,825 \text{ u}) + 12(1.008\,665 \text{ u}) - 21.991\,385 \text{ u} \\ &= 0.190\,845 \text{ u} \end{aligned}$$

$$\begin{aligned} E_{\text{b}} &= \Delta m \times c^2 \\ &= 0.190\,845 \cancel{\text{u}} \times 931.5 \frac{\text{MeV}}{\cancel{\text{u}}} \\ &= 177.772 \text{ MeV} \end{aligned}$$

Paraphrase
The binding energy for neon-22 is 177.772 MeV.

8. (a) ***Given***
${}^{40}_{19}\text{K}$, $m_{\text{atom}} = 39.963\,998$ u
Required
mass defect (Δm)
Analysis and Solution
Use the equation $A = N + Z$ to calculate the number of neutrons.
From ${}^{40}_{19}\text{K}$, A = 40 and Z = 19, so

$$\begin{aligned} N &= A - Z \\ &= 40 - 19 \\ &= 21 \end{aligned}$$

Use $\Delta m = Zm_{\text{H}} + Nm_{\text{neutron}} - m_{\text{atom}}$ to find the binding energy.

$$\begin{aligned} \Delta m &= 19(1.007\,825 \text{ u}) + 21(1.008\,665 \text{ u}) - 39.963\,998 \text{ u} \\ &= 0.366\,642 \text{ u} \end{aligned}$$

Paraphrase
Potassium-40 has a mass defect of 0.366 642 u.

(b) ***Given***
$\Delta m = 0.366\,642$ u (from part (a))
Required
binding energy per nucleon $\left(\frac{E_{\text{b}}}{A}\right)$
Analysis and Solution
Convert the mass defect to binding energy using the equation $E_{\text{b}} = \Delta m \times c^2$.

$$\begin{aligned} E_{\text{b}} &= (0.366\,642 \text{ u})(931.5 \text{ MeV/u}) \\ &= 341.527 \text{ MeV} \end{aligned}$$

Divide by atomic mass number to obtain the binding energy per nucleon, $\frac{E_b}{A}$.

From ${}^{40}_{19}\text{K}$, $A = 40$, so

$$\begin{aligned}\frac{E_b}{A} &= \frac{341.527\ \text{MeV}}{40} \\ &= 8.53818\ \text{MeV/nucleon}\end{aligned}$$

Paraphrase

Potassium-40 has a binding energy per nucleon of 8.538 18 MeV.

9. To estimate the binding energy, read values from the graph. Note that the binding energy is given in energy per nucleon, so multiply by the atomic mass number to obtain the binding energy per nucleus.

(a) From the graph in Figure 16.4, ${}^{13}_{6}\text{C}$ has ~7.7 MeV/nucleon, so

$$\begin{aligned}E_b &= 13\ \text{nucleons} \times 7.7\ \text{MeV/nucleon} \\ &= 100\ \text{MeV}\end{aligned}$$

(b) ${}^{56}_{26}\text{Fe}$ has 8.5 MeV/nucleon, so

$$\begin{aligned}E_b &= 56\ \text{nucleons} \times 8.5\ \text{MeV/nucleon} \\ &= 476\ \text{MeV}\end{aligned}$$

(c) ${}^{238}_{92}\text{U}$ has 7.3 MeV/nucleon, so

$$\begin{aligned}E_b &= 238\ \text{nucleons} \times 7.3\ \text{MeV/nucleon} \\ &= 1737\ \text{MeV}\end{aligned}$$

10. MeV is an energy unit, so it is equivalent to joules. The units of MeV/c^2 are

$$\begin{aligned}\frac{\text{J}}{\left(\frac{\text{m}}{\text{s}}\right)^2} &= \frac{\text{N}\cdot\cancel{\text{m}}}{\frac{\text{m}^{\cancel{2}}}{\text{s}^2}} \\ &= \frac{\text{N}\cdot\text{s}^2}{\text{m}} \\ &= \frac{\text{kg}\cdot\frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}\cdot\cancel{\text{s}^2}}{\cancel{\text{m}}} \\ &= \text{kg}\end{aligned}$$

Extensions

11. (a) The strong nuclear force is the stronger of the two forces, by a factor of about 100 but, unlike the electrostatic force, it acts only over very short distances—a few femtometres at most—and acts on both protons and neutrons. The range of the electromagnetic force is infinite, but it acts only on charged particles, such as protons and electrons.

(b) The protons repel each other by the electromagnetic force and are attracted to each other and neutrons by the strong nuclear force. If the strong force acted over a larger distance, then much larger nuclei would exist. If the electromagnetic force were stronger, then it would be difficult to form large nuclei, especially those with many protons.

12. The nucleus is in a constant struggle between the strong nuclear force that binds it together and the electrostatic force that causes like charges to repel. If the electrostatic force were stronger than the strong force, then the nucleons making up the nucleus would disperse. Nuclei would be less stable, and very large nuclei would probably not exist because the strong nuclear force would not be able to overcome the repulsion of the protons in a large nucleus.

13. ***Given***

$r = r_0 A^{\frac{1}{3}}$, where $r_0 = 1.20$ fm

Required

radius of the nucleus of ${}^{90}_{38}\text{Sr}$ atom (r)

distance between adjacent nucleons in ${}^{90}_{38}\text{Sr}$ nucleus (d)

Analysis and Solution

From ${}^{90}_{38}\text{Sr}$, $A = 90$. Substitute this value into the equation for nuclear radius.

The radius of the nucleus of the strontium-90 atom is:

$$\begin{aligned} r &= (1.20 \text{ fm})90^{\frac{1}{3}} \\ &\approx 5.4 \text{ fm} \end{aligned}$$

Using r, determine the volume of the nucleus using the equation $V = \frac{4}{3}\pi r^3$.

The volume of this nucleus is:

$$\begin{aligned} V &= \frac{4}{3}\pi(5.4 \text{ fm})^3 \\ &= \frac{4}{3}\pi(5.4 \times 10^{-15} \text{ m})^3 \\ &= 6.6 \times 10^{-43} \text{ m}^3 \end{aligned}$$

Divide this answer by A to get an approximate volume per nucleon.

The volume occupied by each nucleon is:

$$\begin{aligned} \frac{V}{A} &= \frac{6.6 \times 10^{-43} \text{ m}^3}{90} \\ &= 7.3 \times 10^{-45} \text{ m}^3 \end{aligned}$$

Take the cube root of this value to obtain the approximate size of the nucleon.

The approximate radius of the space occupied by each nucleon is:

$$\begin{aligned} \left(7.3 \times 10^{-45} \text{ m}^3\right)^{\frac{1}{3}} &= 1.9 \times 10^{-15} \text{ m} \\ &= 1.9 \text{ fm} \end{aligned}$$

Paraphrase

Each nucleon occupies a volume of about 7.3×10^{-45} m^3, which means that the centres of the nucleons are, on average, about 1.9 fm apart. This number sets an upper limit on the possible size of the individual nucleons. The actual size of the protons and neutrons must be less than this value.

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Concept Check

Apply the right-hand rule. The alpha particle deflects in the upward direction and is therefore positively charged. The beta particle deflects downward and is therefore negatively charged. The gamma ray appears to be neutral because it is not deflected.

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Example 16.5 Practice Problems

1. ${}^{212}_{84}\text{Po} \rightarrow {}^{210}_{86}\text{Rn} + {}^{4}_{2}\alpha$
Charge: $84 \neq 86 + 2$
Atomic mass number: $212 \neq 210 + 4$
This equation violates the laws of conservation of charge and of atomic mass number. Therefore, this decay process is impossible.

2. ${}^{233}_{91}\text{Pa} \rightarrow {}^{233}_{92}\text{U} + {}^{0}_{-1}\beta$
Charge: $91 = 92 - 1$
Atomic mass number: $233 = 233$
This equation is correct—both mass number and charge are conserved. Therefore, this decay process is possible.

3. ${}^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + {}^{1}_{1}\text{H}$
Charge: $6 \neq 7 + 1$
Atomic mass number: $14 \neq 14 + 1$
This equation violates the laws of conservation of charge and of atomic mass number. Therefore, this decay process is impossible.

Concept Check

The conservation law applies to nucleons. Electrons are not nucleons.

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Example 16.6 Practice Problems

All alpha decay processes fit the pattern ${}^{A}_{Z}\text{X} \rightarrow {}^{A-4}_{Z-2}\text{Y} + {}^{4}_{2}\alpha$.

1. For thorium-230, the daughter element is ${}^{230-4}_{90-2}\text{Y} = {}^{226}_{88}\text{Y}$.
On the periodic table, the element with $Z = 88$ is radium.
${}^{230}_{90}\text{Th} \rightarrow {}^{226}_{88}\text{Ra} + {}^{4}_{2}\alpha$

2. For uranium-238, the daughter element is ${}^{238-4}_{92-2}\text{Y} = {}^{234}_{90}\text{Y}$.
On the periodic table, the element with $Z = 90$ is thorium.
${}^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\alpha$

3. For polonium-214, the daughter element is ${}^{214-4}_{84-2}\text{Y} = {}^{210}_{82}\text{Y}$.
On the periodic table, the element with $Z = 82$ is lead.
${}^{214}_{84}\text{Po} \rightarrow {}^{210}_{82}\text{Pb} + {}^{4}_{2}\alpha$

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Concept Check

Alpha decay will only occur if the nucleus can achieve a lower energy by emitting an alpha particle. The alpha particle removes energy from the nucleus, so ΔE must be positive. Use the mass-energy equivalence to understand this concept. The mass-energy of the parent nucleus equals the mass-energy of the daughter nucleus and the alpha particle, plus the kinetic energy of these two particles, according to the equation

$$m_{\text{parent}}c^2 = m_{\text{daughter}}c^2 + m_\alpha c^2 + \Delta E$$

where ΔE is the kinetic energy of the daughter nucleus and of the alpha particle.
An example of negative ΔE is the decay of titanium into calcium:

$$^{48}_{22}\text{Ti} \rightarrow {}^{44}_{20}\text{Ca} + {}^{4}_{2}\alpha + \Delta E$$

The atomic mass of titanium is 47.947 946 u, and that of calcium is 43.955 482 u. When the mass of calcium is combined with the mass of the alpha particle (4.002 603 u), the remaining energy (ΔE) is –9.4445 MeV.

Example 16.7 Practice Problems

1. ***Given***

$^{230}_{90}\text{Th}$

Required

energy released using α -decay (ΔE)

Analysis and Solution

From $^{A}_{Z}\text{X} \rightarrow {}^{A-4}_{Z-2}\text{Y} + {}^{4}_{2}\alpha$, the α-decay process for thorium is $^{230}_{90}\text{Th} \rightarrow {}^{226}_{88}\text{Ra} + {}^{4}_{2}\alpha$ (see Example 16.6 Practice Problem 1).
Then use the equation $\Delta E = (m_{\text{parent}} - m_{\text{daughter}} - m_\alpha)c^2$ to determine the energy released during α -decay.

$$\begin{aligned}\Delta E &= (230.033\,134 \text{ u} - 226.025\,410 \text{ u} - 4.002\ 603 \text{ u})c^2\\ &= 5.121 \times 10^{-3} \text{ u} \times \frac{1.49 \times 10^{-10} \text{ J}}{\text{u}}\\ &= 7.63 \times 10^{-13} \text{ J} \quad \text{or} \quad 4.77 \text{ MeV}\end{aligned}$$

Paraphrase

A thorium nucleus releases 7.63×10^{-13} J or 4.77 MeV of energy when it undergoes α-decay.

2. ***Given***

$^{238}_{92}\text{U}$

Required

energy released using α -decay (ΔE)

Analysis and Solution

Determine the decay process of uranium-238 to find the daughter nucleus.
From Example 16.6 Practice Problem 2, the decay equation for uranium-238 is $^{238}_{92}\text{U} \rightarrow {}^{234}_{90}\text{Th} + {}^{4}_{2}\alpha$, so the daughter nucleus is thorium-234.
Use the equation $\Delta E = (m_{\text{parent}} - m_{\text{daughter}} - m_\alpha)c^2$ to determine the energy released during α -decay.

$$\Delta E = (m_{\text{parent}} - m_{\text{daughter}} - m_{\alpha})c^2$$
$$= (238.050\ 788 - 234.043\ 601 - 4.002\ 603)c^2$$
$$= 4.584 \times 10^{-3}\ \text{u} \times \frac{1.49 \times 10^{-10}\ \text{J}}{\text{u}}$$
$$= 6.83 \times 10^{-13}\ \text{J}$$
$$= 4.27\ \text{MeV}$$

Paraphrase

Uranium releases 6.83×10^{-13} J or 4.27 MeV of energy when it undergoes α-decay.

3. ***Given***

$^{214}_{84}\text{Po}$

Required

energy released using α -decay (ΔE)

Analysis and Solution

Determine the decay process of polonium-214 to find the daughter nucleus.
From Example 16.6 Practice Problem 3, the decay equation for polonium-214 is $^{214}_{84}\text{Po} \rightarrow {}^{210}_{82}\text{Pb} + {}^{4}_{2}\alpha$, so the daughter nucleus is lead-210.

Then use the equation $\Delta E = (m_{\text{parent}} - m_{\text{daughter}} - m_{\alpha})c^2$ to determine the energy released during α -decay.

$$\Delta E = (213.995\ 201 - 209.984\ 189 - 4.002\ 603)c^2$$
$$= 8.409 \times 10^{-3}\ \text{u} \times \frac{1.49 \times 10^{-10}\ \text{J}}{\text{u}}$$
$$= 1.25 \times 10^{-12}\ \text{J}$$
$$= 7.83\ \text{MeV}$$

Paraphrase

When polonium undergoes α-decay, it releases 1.25×10^{-12} J or 7.83 MeV of energy.

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Concept Check

Free neutrons are unstable and decay into a proton and an electron. (The typical lifetime of a neutron is about 900 s). Since a neutron can be considered as being composed of a proton and an electron, its mass is slightly higher than the combined masses of an electron and a proton.

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Example 16.8 Practice Problems

1. Use the pattern $^{A}_{Z}\text{X} \rightarrow {}^{A}_{Z+1}\text{X} + {}^{0}_{-1}\beta$. The atomic number must increase by one without changing the atomic mass number.

(a) $^{228}_{88}\text{Ra} \rightarrow {}^{228}_{89}\text{Ac}^{+} + {}^{0}_{-1}\beta$

(b) $^{212}_{82}\text{Pb} \rightarrow {}^{212}_{83}\text{Bi}^{+} + {}^{0}_{-1}\beta$

(c) $^{208}_{81}\text{Tl} \rightarrow {}^{208}_{82}\text{Pb}^{+} + {}^{0}_{-1}\beta$

Example 16.9 Practice Problems

1. (a) ***Given***

cobalt-60 nucleus

Required

products of β^{-} decay

Analysis and Solution

Identify the correct structure for cobalt-60 and then apply the β^{-} decay pattern,

$^{A}_{Z}\text{X} \rightarrow {}_{Z+1}^{A}\text{X} + {}^{0}_{-1}\beta + \overline{\nu}$.

$^{60}_{27}\text{Co} \rightarrow {}^{60}_{28}\text{Ni}^{+} + {}^{0}_{-1}\beta$

Paraphrase

The β^{-} decay of cobalt-60 produces a nickel ion, $^{60}_{28}\text{Ni}^{+}$.

(b) ***Given***

cobalt-60 nucleus

$^{60}_{27}\text{Co} \rightarrow {}^{60}_{28}\text{Ni}^{+} + {}^{0}_{-1}\beta$ (from part (a))

Required

energy released by β^{-} decay (ΔE)

Analysis and Solution

The energy released is equivalent to the difference between the mass of the parent atom and the total mass of the decay products. Use the equation $\Delta m = m_{\text{parent}} - m_{\text{products}}$, then convert this value to energy using Einstein's mass-energy equivalence principle.

$$\begin{aligned}
\Delta m &= m_{\text{parent}} - m_{\text{products}} \\
&= m_{^{60}_{27}\text{Co}} - \left(m_{^{60}_{28}\text{Ni}^{+}} + m_{^{0}_{-1}\beta}\right) \\
&= m_{^{60}_{27}\text{Co}} - m_{^{60}_{28}\text{Ni}^{+}} \\
&= 59.933\ 817\ \text{u} - 59.930\ 786\ \text{u} \\
&= 0.003031\ \text{u}
\end{aligned}$$

$$\begin{aligned}
\Delta E &= 0.003\ 031\ \text{u} \times \frac{931.5\ \text{MeV}}{1\ \text{u}} \\
&= 2.82\ \text{MeV}
\end{aligned}$$

Paraphrase

The β^{-} decay of a cobalt-60 nucleus releases 2.82 MeV of energy.

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Concept Check

The discrepancy in the beta decay that led to the discovery of the neutrino was a small amount of missing energy (hence mass through the mass-energy equivalence). Because charge is conserved in this decay, the neutrino does not change the charge. Therefore, it must be neutral.

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Example 16.10 Practice Problems

1. (a) During β^+ decay, the atomic number decreases by 1. When the atomic number of thallium-81 decreases by 1, it becomes $Z - 1 = 81 - 1 = 80$, which is the atomic number for mercury-80.

(b) Use the general pattern ${}^{A}_{Z}\text{X} \rightarrow {}^{A}_{Z-1}\text{Y} + {}^{0}_{1}\beta + \nu$ for β^+ decay: ${}^{202}_{81}\text{Tl} \rightarrow {}^{202}_{80}\text{Hg} + {}^{0}_{1}\beta + \nu$

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Concept Check

Gamma decays cannot be shown as paths on a decay series graph because gamma decay does not change either the atomic number (Z) or the atomic mass number (A) of an atom. Gamma decay is the result of nucleons moving to lower-energy states and emitting gamma-ray photons.

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16.2 Check and Reflect

Knowledge

1. The three basic radioactive decay processes are:
- alpha decay, where the nucleus emits an alpha particle and its atomic number changes from Z to $Z - 2$
- beta decay:
  - (a) β^- decay, where the nucleus emits an electron and an antineutrino, and its atomic number increases by 1
  - (b) β^+ decay, where the nucleus emits a positron and a neutrino, and its atomic number decreases by 1
- gamma decay, where the nucleus drops from an excited state to a lower energy state and emits a gamma-ray photon

2. The heaviest stable isotopes tend to have an equal number of protons and neutrons ($Z = N$), so the ratio is 1:1.

3. (a) Use the pattern ${}^{A}_{Z}\text{X} \rightarrow {}^{A-4}_{Z-2}\text{Y} + {}^{4}_{2}\alpha$ to get ${}^{234}_{91}\text{Pa} \rightarrow {}^{230}_{89}\text{Ac} + {}^{4}_{2}\alpha$.

(b) The parent nucleus is protactinium and the daughter nucleus is actinium.

4. (a) Since Z increase from 6 to 7, this type of beta decay must be β^- decay.

(b) ${}^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + {}^{0}_{-1}\beta + \bar{\nu}$

5. (a) Since Z decreases by 1, the decay must be β^+ decay.

(b) ${}^{22}_{11}\text{Na} \rightarrow {}^{22}_{10}\text{Ne} + {}^{0}_{1}\beta + \nu$

6. The emission of the beta particle leaves the daughter nucleus in an excited state. The gamma-ray photon is emitted as the daughter nucleus drops into a lower energy state.

7. Of the three main radioactive decay processes, gamma rays usually have the greatest penetrating power.

Applications

8. ***Given***

$^{22}_{11}\text{Na}$ decays to produce $^{22}_{10}\text{Ne}$.

$m_{\text{Na}} = 21.994\ 436$ u

$m_{\text{Ne}} = 21.991\ 385$ u

Required

energy released by the decay process (ΔE)

Analysis and Solution

Find the mass defect, Δm, using the equation $\Delta m = m_{\text{parent}} - m_{\text{products}}$, and convert it to its energy equivalent by using the conversion factor 1 u = 931.5 MeV.

$$\begin{aligned}\Delta m &= m_{\text{parent}} - m_{\text{products}} \\ &= m_{^{22}_{11}\text{Na}} - \left(m_{^{22}_{10}\text{Ne}} + m_{\text{e}}\right) \\ &= m_{^{22}_{11}\text{Na}} - m_{^{22}_{10}\text{Ne}} \\ &= 21.994\ 436\ \text{u} - 21.991\ 385\ \text{u} \\ &= 0.003051\ \text{u}\end{aligned}$$

$$\begin{aligned}\Delta E &= 0.003\ 051\ \text{u} \times \frac{931.5\ \text{MeV}}{1\ \text{u}} \\ &= 2.84\ \text{MeV}\end{aligned}$$

Paraphrase

The decay of sodium into neon liberates 2.84 MeV of energy.

9. Yes, atomic number can increase during nuclear decay. A β^- decay causes atomic number to increase by one. For example, carbon-14 can decay to nitrogen-14, according to the decay process $^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + {}^{0}_{-1}\beta + \overline{\nu}$.

10. You receive about 300 μSv of radiation from natural sources each year and about 75 μSv from medical and dental sources. Hence, you receive on average about four times as much radiation from natural background sources than from medical procedures.

11.

Decay process	Decay type	Parent element	Daughter element
(a) $^{232}_{90}\text{Th} \rightarrow {}^{228}_{88}\text{Ra*} + {}^{4}_{2}\alpha$	alpha decay	thorium	radium
(b) $^{22}_{11}\text{Na} \rightarrow {}^{22}_{10}\text{Ne} + {}^{0}_{+1}\beta + \nu$	β^+ decay	sodium	neon
(c) $^{228}_{88}\text{Ra*} \rightarrow {}^{228}_{88}\text{Ra} + \gamma$	gamma decay	—	—
(d) $^{228}_{88}\text{Ra} \rightarrow {}^{228}_{89}\text{Ac} + {}^{0}_{-1}\beta + \overline{\nu}$	β^- decay	radium	actinium
(e) $^{228}_{89}\text{Ac} \rightarrow {}^{228}_{90}\text{Th} + {}^{0}_{-1}\beta + \overline{\nu}$	β^- decay	actinium	thorium
(f) $^{228}_{90}\text{Th} \rightarrow {}^{224}_{88}\text{Ra} + {}^{4}_{2}\alpha$	alpha decay	thorium	radium
(g) $^{0}_{1}\text{p} \rightarrow {}^{0}_{0}\text{n} + {}^{0}_{+1}\beta + \nu$	β^+ decay	proton	neutron

Extensions

12. (a) The absorption of an electron causes a decrease in the atomic number. The electron combines with a proton in the nucleus to form a neutron.

(b) ${}^{A}_{Z}\text{X} + {}^{0}_{-1}\beta \rightarrow {}^{A}_{Z-1}\text{Y} + \nu$

(c) Electron capture is similar to beta decay because both processes change an atom from one element to another. However, during electron capture, an electron is absorbed, whereas during beta decay, an electron (or a positron) is emitted.

13. One possible experiment would be to bombard a nucleus with neutrons in an attempt to excite the nucleus and cause it to emit gamma-ray photons when the nucleus returns to the ground state. If the nucleus behaves in a manner analogous to the atom, you should see distinct energies given off in the form of gamma-ray photons of specific wavelengths, similar to the bright line spectrum seen in electron transitions.

14. Radon is a natural radioactive decay product of radium. Since radium occurs naturally, so does radon. Concrete contains trace amounts of radium and hence can also produce trace amounts of radon. Since radon is a gas, it can migrate through the cement (or soil). Furthermore, because it is heavier than air, radon can accumulate in a basement. Such accumulation is a health concern because radon is itself radioactive—it undergoes alpha decay. If you inhale radon, you run an increased risk of absorbing an alpha emitter into your lungs or bloodstream, which poses an increased risk of cancer.

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Example 16.11 Practice Problems

1. ***Given***

$\lambda = 4.1 \times 10^{-9}\ \text{s}^{-1}$

$N = 1.01 \times 10^{22}$

Required

activity (A)

Analysis and Solution

Use the equation $A = -\lambda N$ and substitute the given values.

$$\begin{aligned} A &= -(4.1\times10^{-9}\ \text{s}^{-1})(1.01\times10^{22}) \\ &= -4.1\times10^{13}\ \text{s}^{-1} \\ &= -4.1\times10^{13}\ \text{Bq} \end{aligned}$$

The negative sign means that the number of cobalt-60 nuclei is decreasing.

Paraphrase

The sample has a decay rate of 4.1×10^{13} Bq.

2. ***Given***

$N = 5.00 \times 10^{20}$ atoms

$A = 2.50 \times 10^{12}$ Bq

Required

the decay constant (λ)

Analysis and Solution

Use the equation $A = -\lambda N$ (the negative sign signifies decay).

$$\lambda = -\frac{A}{N}$$
$$= -\frac{2.50\times10^{12}\ \text{Bq}}{5.00\times10^{20}\ \text{atoms}}$$
$$= -5.00\times10^{-9}\ \text{s}^{-1}$$

Paraphrase

The decay constant for this sample is 5.00×10^{-9} s^{-1}.

Student Book page 813

Example 16.12 Practice Problems

1. ***Given***

$t_{1/2} = 1.6$ s

Required

time required for 99% of astatine in a sample to decay (t)

Analysis and Solution

Use the equation $N = N_0\left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$. Re-write as $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$ and let $\alpha = \frac{t}{t_{1/2}}$:

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\alpha}$$

Since 99% of the sample has undergone decay, $\frac{N}{N_0} = 0.01$.

$$0.01 = \left(\frac{1}{2}\right)^{\alpha}$$

Choose values for α and plug them into your calculation until $\left(\frac{1}{2}\right)^{\alpha} = 0.01$.

$\alpha = 6.64$

Solve for t:

$$6.64 = \frac{t}{t_{1/2}}$$

$$t = (6.64)\left(t_{1/2}\right)$$
$$= (6.64)(1.6\ \text{s})$$
$$= 10.6\ \text{s}$$

Paraphrase

It takes only about 11 s for 99% of a sample of astatine-218 to undergo radioactive decay.

2. ***Given***

$t_{1/2} = 1600$ years

$t = 8000$ years

Required

the percentage of a sample of radium-226 remaining after 8000 years $\left(\frac{N}{N_0}\right)$

Analysis and Solution

Use the equation $N = N_0\left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$.

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

$$\begin{aligned}\frac{N}{N_0} &= \left(\frac{1}{2}\right)^{\frac{8000}{1600}}\\ &= \left(\frac{1}{2}\right)^{5}\\ &= \frac{1}{32}\\ &= 3.125\%\end{aligned}$$

Paraphrase

After 8000 years, only 3.125% of the original radium-226 sample will remain.

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Example 16.13 Practice Problems

1. ***Given***

$t_{1/2} = 29.1$ years

$t = 100$ years

Required

proportion of the sample remaining after 100 years $\left(\frac{N}{N_0}\right)$

Analysis and Solution

Use the equation $N = N_0\left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$.

$$\begin{aligned}\frac{N}{N_0} &= \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}\\ &= \left(\frac{1}{2}\right)^{\frac{100}{29.1}}\\ &= \left(\frac{1}{2}\right)^{3.44}\\ &= 0.092\\ &= 9.2\%\end{aligned}$$

Paraphrase

After 100 years, 9.2% of the original amount of strontium-90 remains.

2. ***Given***

$t_{\frac{1}{2}}$ = 12.3 years

m = 100 mg

t = 5.0 years

Required

proportion of tritium remaining after 5.0 years $\left(\frac{N}{N_0}\right)$

Analysis and Solution

Use the equation $N = N_0\left(\frac{1}{2}\right)^{\frac{t}{t_{\frac{1}{2}}}}$.

$$\begin{aligned}\frac{N}{N_0} &= \left(\frac{1}{2}\right)^{\frac{t}{t_{\frac{1}{2}}}}\\ &= \left(\frac{1}{2}\right)^{\frac{5.0}{12.3}}\\ &= \left(\frac{1}{2}\right)^{0.41}\\ &= 0.75\\ &= 75\%\end{aligned}$$

75% of 100 mg is 75 mg.

Paraphrase

After 5.0 years, 75%, or 75 mg, of tritium will remain.

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Concept Check

Most materials weakly absorb beta radiation. The amount of absorption depends on the thickness of the absorber. Therefore, by measuring the amount of radiation emitted by a known source of beta radiation, the thickness of the absorber can be determined. Gamma radiation can easily penetrate most materials, and it takes very large thickness differences to produce an appreciable difference in the absorption of gamma rays. Because of its great penetrating quality, gamma radiation can be used to provide a detailed image of the interior of such objects as metal pipes and panels, to help reveal flaws or cracks in structures.

16.3 Check and Reflect

Knowledge

1. For each half-life, one-half of a sample undergoes decay while the other half remains. So, after four half-lives, there remains $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{16}$ of the original amount of radioactive material.

2. ***Given***

$\lambda = 5.8 \times 10^{-12}\ \text{s}^{-1}$
$N = 6.4 \times 10^{23}$

Required

activity of the sample (A)

Analysis and Solution

Substitute the given values into the equation $A = -\lambda N$.

$$\begin{aligned} A &= -\lambda N \\ &= -\left(5.8\times10^{-12}\ \text{s}^{-1}\right)\left(6.4\times10^{23}\right) \\ &= -3.7\times10^{12}\ \text{s}^{-1} \end{aligned}$$

Paraphrase

The radioactive sample decays at a rate of 3.7×10^{12} atoms/s.

3. Since both samples have the same mass, assume that they have roughly the same number of atoms. The first sample has a half-life that is much shorter than that of the second sample. Therefore, the first sample must undergo more radioactive decays per second, and hence have a greater activity, than the second sample.

Applications

4. ***Given***

$\frac{N}{N_0} = \frac{1}{16}$

$t_{1/2} = 3.0 \times 10^5$ years

Required

the age of the rock sample (t)

Analysis and Solution

Use the equation $N = N_0\left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$.

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

$$\begin{aligned} \frac{1}{16} &= \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}} \\ &= \left(\frac{1}{2}\right)^{4} \end{aligned}$$

$$\frac{t}{t_{1/2}} = 4$$

$$t = 4\left(t_{1/2}\right)$$
$$= 4(3.0 \times 10^5 \text{ years})$$
$$= 1.2 \times 10^6 \text{ years}$$

Paraphrase

The rock sample is approximately 1.2 million years old.

5. ***Given***

$t_{1/2}$ = 2.6 h

t = 24 h

Required

the proportion of the tracer remaining in a patient after 24 hours $\left(\frac{N}{N_0}\right)$

Analysis and Solution

Use the equation $N = N_0\left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$.

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$
$$= \left(\frac{1}{2}\right)^{\frac{24}{2.6}}$$
$$= \left(\frac{1}{2}\right)^{9.23}$$
$$= 1.7 \times 10^{-3}$$

Paraphrase

After 24 hours, only 1.7×10^{-3} (0.17%) of the original quantity of the tracer remains.

6. ***Given***

$\frac{N}{N_0}$ = 25% = $\frac{1}{4}$

Required

the age of the arrow (t)

Analysis and Solution

Assume that the arrow has one-quarter of its original carbon-14 content.

Use the equation $N = N_0\left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$.

Look up the half-life of carbon-14 in a source. It is 5730 years.

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

$$\frac{1}{4} = \left(\frac{1}{2}\right)^2$$

$$\frac{t}{t_{1/2}} = 2$$

$$\begin{aligned} t &= 2\left(t_{1/2}\right) \\ &= 2(5730 \text{ years}) \\ &= 11\,460 \text{ years} \end{aligned}$$

The arrow appears to be two half-lives old.

Paraphrase

The arrow is approximately 11 500 years old.

7. ***Given***

A = 2.5 MBq

$t_{1/2}$ = 12 h = 0.5 d

Required

the activity of the sample one week (7 d) later (A)

Analysis and Solution

The activity depends on the number of radioactive nuclei present ($A = -\lambda N$), but N depends on time, according to the equation $N = N_0\left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$. Therefore,

$$A = -\lambda N_0\left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

Let the original activity be $A_0 = -\lambda N_0$.

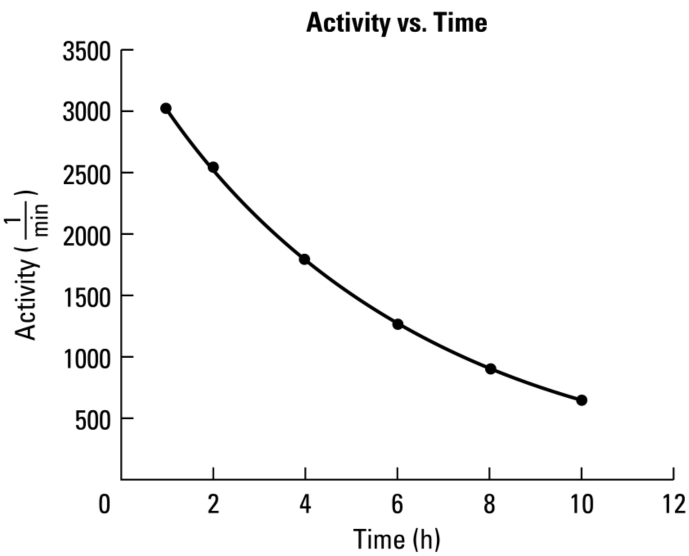
$$\begin{aligned} A &= A_0\left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}} \\ &= (2.5\times10^6 \text{ Bq})\left(\frac{1}{2}\right)^{\frac{7}{0.5}} \\ &= 153 \text{ Bq} \end{aligned}$$

Paraphrase

After one week, the activity will have dropped from 2.5×10^6 Bq to just 153 Bq.

8.

Time (h)	Activity (decays/min)
1	3027
2	2546
4	1800
6	1273
8	900
10	636



(a) The activity drops from 3000 decays/min to 1500 decays/min in about 5 h, so the half-life is about 5 h.

(b) ***Given***

$A = 3027$ at $t = 1$ h

$t_{1/2} = 5$ h

Required

activity at $t = 0$ (A_0)

Analysis and Solution

From question 7, $A = A_0\left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$.

$$A_0 = \frac{A}{\left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}}$$

$$= \frac{3027}{\left(\frac{1}{2}\right)^{\frac{1}{5}}}$$

$$= 3477$$

Paraphrase

At $t = 0$, the activity of the sample is 3477 decays/min.

Extensions

9. Carbon-14 is useless for dating extremely old artefacts because it has a half-life of only 5730 years. A 65-million-year-old object would be about 11 284 carbon-14 half-lives old! If you started with one mole of carbon-14, then after 11 284 half-lives, there would be no carbon-14 atoms left! You should be suspicious because carbon-14 could not have been used to date this bone.

10.

For	Against
Irradiation kills bacteria, thereby prolonging the shelf-life of foods and allowing them to be transported longer distances without spoiling.	Irradiation may cause changes to the genetic structure of food. The major argument against irradiation is that its long-term effects are unknown.

11. (a) Depleted uranium is uranium left over from spent fuel rods or from decommissioned nuclear weapons.

(b) Uranium is a very heavy, dense metal. Its high density makes it very effective at piercing armour. It is also useful as ballast because it takes up less room while still providing much needed weight (due to its high density).

(c) There are at least three major concerns with the use of depleted uranium.

(i) Depleted uranium is a heavy metal and therefore presents the same kinds of health concerns that other, including non-radioactive, heavy metals pose for health, including nervous system disorders.

(ii) Depleted uranium is still radioactive. Exposure to it poses the same health risks as exposure to any radioactive substance does.

(iii) Especially when used in weaponry, depleted uranium can be physically changed by impact into a dust which, when airborne and inhaled, poses an even greater health risk.

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Concept Check

If the binding energy per nucleon increases, then the nucleons are more tightly bound to each other. In order for nucleons to bind more tightly, they must each lose energy. Consequently, a nuclear reaction that increases the binding energy per nucleon releases energy. As an analogy, imagine that you dropped your backpack while hiking and it rolled down the side of an embankment, ending up 100 m below you. Your backpack is now more tightly bound to Earth because it is closer to it, and it will take even more energy to bring it back to your elevation.

Binding energy represents the energy needed to separate nucleons from each other. The nucleus with the highest binding energy per nucleon is iron, which makes it the most stable of nuclei.

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Example 16.14 Practice Problems

1. ***Given***

$^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{94}_{40}\text{Zr} + {}^{139}_{52}\text{Te} + 3\,{}^{1}_{0}\text{n}$

Required

energy released (ΔE)

Analysis and Solution
First calculate the net change in mass resulting from the reaction.
$m_\mathrm{i} = {}^{235}_{92}\mathrm{U}$
$= 235.044 \text{ u}$
Final mass (the net number of neutrons produced is two):
$m_\mathrm{f} = {}^{94}_{40}\mathrm{Zr} + {}^{139}_{52}\mathrm{Te} + 2\,{}^{1}_{0}\mathrm{n}$
$= 93.906 \text{ u} + 138.935 \text{ u} + 2(1.009 \text{ u})$
$= 234.859 \text{ u}$
$m_\mathrm{i} - m_\mathrm{f} = 235.044 \text{ u} - 234.859 \text{ u}$
$= 0.185 \text{ u}$
Use mass-energy equivalence to calculate the energy released. Use the equation $\Delta E = (m_\mathrm{i} - m_\mathrm{f})c^2$ or determine ($m_\mathrm{i} - m_\mathrm{f}$) in atomic mass units, u, and multiply by 931.5 MeV/u.

$$\begin{aligned}\Delta E &= (m_\mathrm{i} - m_\mathrm{f})c^2 \\ &= 0.185 \text{ u} \times \frac{931.5 \text{ MeV}}{1 \text{ u}} \\ &= 172 \text{ MeV}\end{aligned}$$

Paraphrase
The nuclear fission of U-235 into tellurium and zirconium releases 172 MeV of energy.

2. ***Analysis and Solution***
${}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{87}_{35}\mathrm{Br} + {}^{146}_{57}\mathrm{La} + ?\,{}^{1}_{0}\mathrm{n}$
To determine the number of neutrons released, balance the atomic mass numbers on both sides of the equation:
$235 + 1 = 87 + 146 + ?$
$? = 3$
Therefore,
${}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{87}_{35}\mathrm{Br} + {}^{146}_{57}\mathrm{La} + 3\,{}^{1}_{0}\mathrm{n}$

3. ***Given***
${}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{87}_{35}\mathrm{Br} + {}^{146}_{57}\mathrm{La} + 3\,{}^{1}_{0}\mathrm{n}$
Required
energy released (ΔE)
Analysis and Solution
Use the equation $\Delta E = (m_\mathrm{i} - m_\mathrm{f})c^2$ or convert ($m_\mathrm{i} - m_\mathrm{f}$) into MeV.
The masses are:
$m_\mathrm{i} = {}^{235}_{92}\mathrm{U}$
$= 235.044 \text{ u}$
$m_\mathrm{f} = {}^{87}_{35}\mathrm{Br} + {}^{146}_{57}\mathrm{La} + 2\,{}^{1}_{0}\mathrm{n}$ (Two net neutrons are released.)
$= 86.921 \text{ u} + 145.926 \text{ u} + 2(1.009 \text{ u})$
$= 234.865 \text{ u}$
$m_\mathrm{i} - m_\mathrm{f} = 235.044 \text{ u} - 234.865 \text{ u}$
$= 0.179 \text{ u}$

$$
\begin{aligned}
\Delta E &= (m_\mathrm{i} - m_\mathrm{f})c^2 \\
&= 0.179 \text{ u} \times \frac{931.5 \text{ MeV}}{1 \text{ u}} \\
&= 167 \text{ MeV}
\end{aligned}
$$

Paraphrase

This reaction releases 167 MeV of energy.

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Example 16.15 Practice Problems

1. (a) ***Given***

E = 1600 MJ

d = 500 km

Required

the mass of gasoline (m)

Analysis and Solution

The chemical energy in gasoline is 4.4 × 10^7 J/kg (from Example 16.15).
Divide the given energy (1600 MJ) by this value.

$$\frac{1600 \times 10^6 \ \cancel{\text{J}}}{4.4 \times 10^7 \ \frac{\cancel{\text{J}}}{\text{kg}}} = 36 \text{ kg}$$

Paraphrase

It takes 36 kg of gasoline to provide 1600 MJ of energy.

(b) ***Given***

E = 1600 MJ

d = 500 km

Required

mass of U-235 required to deliver the same amount of energy (m)

Analysis and Solution

Do the same calculation as in part (a) using the energy content for U-235:
7.11 × 10^{13} J/kg.

$$\frac{1600 \times 10^6 \ \cancel{\text{J}}}{7.11 \times 10^{13} \ \frac{\cancel{\text{J}}}{\text{kg}}} = 2.2 \times 10^{-5} \text{ kg} = 22 \text{ mg}$$

Paraphrase

To travel 500 km in an average family car would require only 22 mg of uranium-235!

Student Book page 821

Concept Check

The small nuclear dimensions mean that the de Broglie wavelengths of nucleons are much smaller than the de Broglie wavelengths of electrons in the atom. Since momentum depends inversely on the wavelength, nucleons have much higher momenta, and hence higher energy, than do the electrons in the atom. The particle-in-a-box example from

Chapter 14 (Example 14.12) could be used here as an analogy. Nucleons are confined to much smaller "boxes", and hence must have greater energies.

Another reason why nuclear reactions are more energetic has to do with the strong nuclear force, which operates only in atomic nuclei. It takes a great deal of energy to overcome the strong force. Also, the electric potential energy of protons in a nucleus is enormous because the protons are packed so tightly together. Changing nuclear structure involves manipulating these forces, which can lead to enormous energy changes.

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Example 16.16 Practice Problems

1. (a) ***Given***

$P = 1.6 \times 10^{25}$ W

Required

number of helium nuclei produced per second

Analysis and Solution

Use the results of Example 16.16. Every fusion of four hydrogen atoms to a helium nucleus releases 24.67 MeV. Convert this value to joules and then divide power (J/s) by energy to determine the number of helium nuclei formed each second.

24.67×10^{6} eV $\times$ 1.60×10^{-19} J/eV = 3.947×10^{-12} J

$$\begin{aligned}\text{Rate} &= \frac{1.6 \times 10^{25}\ \text{J/s}}{3.947 \times 10^{-12}\ \text{J}} \\ &= 4.1 \times 10^{36}\ \text{s}^{-1}\end{aligned}$$

Paraphrase

The star produces 4.1×10^{36} new helium nuclei each second by nuclear fusion.

(b) Given

t = 4 billion years

Required

mass of helium produced (m)

Analysis and Solution

Multiply the result in (a) by the time (4 billion years) to get the total number of the nuclei formed.

$$t = 4\ \text{billion}\ \cancel{\text{years}} \times \frac{365\ \cancel{\text{days}}}{\cancel{\text{year}}} \times \frac{24\ \cancel{\text{h}}}{\cancel{\text{day}}} \times \frac{3600\ \text{s}}{\cancel{\text{h}}} = 1.26 \times 10^{17}\ \text{s}$$

The number of helium nuclei produced = $(4.05 \times 10^{36}\ \text{s}^{-1})(1.26 \times 10^{17}\ \text{s})$
$= 5.1 \times 10^{53}$

Multiply by the mass of a helium nucleus, 6.65×10^{-27} kg, to obtain the total mass of helium produced.

The mass of helium produced = $(5.1 \times 10^{53}$ nuclei$)(6.65 \times 10^{-27}$ kg/nucleus$)$
$= 3.4 \times 10^{27}$ kg

Paraphrase

After 4 billion years, this star will have produced 3.4×10^{27} kg of helium.

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Concept Check

For fusion to occur, protons must be close enough so that the strong nuclear force can cause them to fuse. In order to come sufficiently close, the protons must first overcome the repulsive electrostatic force between them, called the Coulomb barrier. To penetrate the Coulomb barrier, they must have very high kinetic energy, that is, they must be moving very fast, which is the same as stating that they must be at a very high temperature. For this reason, fusion reactions can occur only at extremely high temperatures.

Student Book page 824

Concept Check

Tritium is a radioactive isotope of hydrogen. If inhaled, it poses a serious health concern because tritium undergoes beta decay. If absorbed into the body, tritium becomes a long-term, low-level radiation source that can cause cellular damage, including genetic mutation.

16.4 Check and Reflect

Knowledge

1. (a) The total number of nucleons must add up to 235. There are 140 nucleons in the Xe nucleus, plus two more neutrons, so

$$\begin{aligned} A &= 235 - 140 - 2 \\ &= 93 \end{aligned}$$

Also, charge must be conserved, so

$$92 = 54 + Z$$
$$Z = 38$$

From the periodic table, the missing element is strontium-93, ${}^{93}_{38}\text{Sr}$.

(b) This reaction is a spontaneous fission reaction because a larger nucleus splits into two smaller fragments.

2. In order for energy to be released during a nuclear reaction, the binding energy per nucleon must increase. The release of energy means that the nucleons in the products of a nuclear reaction are more tightly bound per nucleon than the nucleons in the parent nucleus.

3. The combined binding energy of the iron and silicon nuclei is

492 MeV + 237 MeV = 729 MeV

Because this value is greater than the binding energy of the final nucleus, 718 MeV, this reaction requires energy, so energy will not be released.

4. (a) Heavy elements ($A > 120$) will release energy in fission reactions because they can split into nuclei that each have binding energies per nucleon that are higher than the binding energies of the parent nucleus.

(b) Light elements ($A < 50$) that can combine to form a nucleus with a higher binding energy per nucleon than the original nuclei are most likely to undergo fusion.

5. (a) Use the alpha decay pattern, ${}^{A}_{Z}\text{X} \rightarrow {}^{A-4}_{Z-2}\text{Y} + {}^{4}_{2}\alpha$. Aluminium-27 is ${}^{27}_{13}\text{Al}$, so the decay equation is ${}^{27}_{13}\text{Al} \rightarrow {}^{A-4}_{Z-2}\text{Y} + {}^{4}_{2}\alpha$.

$A - 4 = 23$

$Z - 2 = 11$

From the periodic table, this element is sodium.

(b) ${}^{27}_{13}\text{Al} \rightarrow {}^{23}_{11}\text{Na} + {}^{4}_{2}\alpha$

Applications

6. (a) ${}^{4}_{2}\text{He} + {}^{16}_{8}\text{O} \rightarrow {}^{20}_{10}\text{Ne} + \gamma$

This fusion reaction produces neon-20. Since the neon-20 nucleus will be in an excited state, a gamma ray will be emitted.

(b) ***Given***

helium-4

oxygen-16

Required

energy released (ΔE)

Analysis and Solution

Using the nuclear process equation, compare final and initial masses.

${}^{4}_{2}\text{He} + {}^{16}_{8}\text{O} \rightarrow {}^{20}_{10}\text{Ne} + \gamma$

$$\begin{aligned} m_\text{i} &= m_\text{He} + m_\text{O} \\ &= 4.002\,603 \text{ u} + 15.994\,915 \text{ u} \end{aligned}$$

$$\begin{aligned} m_\text{f} &= m_\text{Ne} \\ &= 19.992\,440 \text{ u} \end{aligned}$$

$$\begin{aligned} m_\text{i} - m_\text{f} &= 4.002\,603 \text{ u} + 15.994\,915 \text{ u} - 19.992\,440 \text{ u} \\ &= 0.005\,078 \text{ u} \end{aligned}$$

Use the equation $\Delta E = (m_\text{i} - m_\text{f})c^2$ to calculate the energy released.

$$\begin{aligned} \Delta E &= (m_\text{i} - m_\text{f})c^2 \\ &= 0.005\,078 \text{ u} \times \frac{931.5 \text{ MeV}}{\text{u}} \\ &= 4.73 \text{ MeV} \end{aligned}$$

Paraphrase

The fusion reaction of helium-4 and oxygen-16 releases 4.73 MeV of energy.

7. (a) ***Given***

Deuterium $\left({}^{2}_{1}\text{H}\right)$ and tritium $\left({}^{3}_{1}\text{H}\right)$ fuse to form helium $\left({}^{4}_{2}\text{He}\right)$.

Required

particle emitted

Analysis and Solution

Use the laws of conservation of atomic mass number and of charge.

${}^{2}_{1}\text{H} + {}^{3}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + ?$

Since there is no change in charge but a change in atomic mass number, a neutron has been emitted. The fusion equation is therefore

${}^{2}_{1}\text{H} + {}^{3}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + {}^{1}_{0}\text{n}$

Paraphrase

This reaction emits a neutron.

(b) ***Given***

Deuterium $\left({}^{2}_{1}\mathrm{H}\right)$ and tritium $\left({}^{3}_{1}\mathrm{H}\right)$ fuse to form helium $\left({}^{4}_{2}\mathrm{He}\right)$.

Required

energy released (ΔE)

Analysis and Solution

Use the equation $\Delta E = (m_\mathrm{i} - m_\mathrm{f})c^2$.

From ${}^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n}$,

$$\begin{aligned} m_\mathrm{i} &= {}^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \\ &= 2.014\ 102\ \mathrm{u} + 3.016\ 049\ \mathrm{u} \end{aligned}$$

$$\begin{aligned} m_\mathrm{f} &= {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n} \\ &= 4.002\ 603\ \mathrm{u} + 1.008\ 665\ \mathrm{u} \end{aligned}$$

$$\begin{aligned} m_\mathrm{i} - m_\mathrm{f} &= 2.014\ 102\ \mathrm{u} + 3.016\ 049\ \mathrm{u} - (4.002\ 603\ \mathrm{u} + 1.008\ 665\ \mathrm{u}) \\ &= 0.018\ 883\ \mathrm{u} \end{aligned}$$

$$\begin{aligned} \Delta E &= (m_\mathrm{i} - m_\mathrm{f})c^2 \\ &= 0.018\ 883\ \mathrm{u} \times \frac{931.5\ \mathrm{MeV}}{\mathrm{u}} \\ &= 17.6\ \mathrm{MeV} \end{aligned}$$

Paraphrase

The reaction produces 17.6 MeV of energy.

8. (a) ***Given***

P = 700 MW

efficiency = 27%

average energy release per fission = 200 MeV

Required

number of fission reactions per second

Analysis and Solution

First determine how much nuclear power must be generated to produce 700 MW of electrical power.

$$\begin{aligned} P \times 0.27 &= 700\ \mathrm{MW} \\ P &= 2.59 \times 10^{9}\ \mathrm{W} \\ &= 2.59 \times 10^{9}\ \mathrm{J/s} \end{aligned}$$

Divide the energy needed each second by 200 MeV per fission to get the number of U-235 nuclei needed each second.

$$\frac{2.59 \times 10^{9}\ \frac{\cancel{\mathrm{J}}}{\mathrm{s}}}{\left(200 \times 10^{6}\ \cancel{\mathrm{eV}}\right)\left(1.60 \times 10^{-19}\ \frac{\cancel{\mathrm{J}}}{\cancel{\mathrm{eV}}}\right)} = 8.10 \times 10^{19}\ \mathrm{s}^{-1}$$

Paraphrase

The number of nuclei that undergo fission each second when the reactor is running at full power is 8.10×10^{19}.

(b) ***Given***

A nuclear reactor produces 700 MW of electrical power. The average energy release per fission is 200 MeV and CANDU is 27% efficient.

Required
mass of U-235 used per year (m)
Analysis and Solution
Multiply the answer in (a) by the total time (one year) and by the mass per U-235 nucleus. In one year, the number of fission reactions that occur is

$$\left(8.10 \times 10^{19}\ \frac{1}{\cancel{\text{s}}}\right)\left(365.25\ \frac{\cancel{\text{d}}}{\text{a}}\right)\left(86\ 400\ \frac{\cancel{\text{s}}}{\cancel{\text{d}}}\right) = 2.556 \times 10^{27}\ \text{a}^{-1}$$

The mass of U-235 used is:

$$\begin{aligned} m &= \left(2.556 \times 10^{27}\right)\left(235.044\ \text{u}\right)\left(1.660\ 539 \times 10^{-27}\ \frac{\text{kg}}{\text{u}}\right) \\ &= 998\ \text{kg} \end{aligned}$$

Paraphrase
The CANDU reactor will consume a little less than one tonne of U-235 in a year to produce an electrical power output of 700 MW. The assumption made is that only pure U-235 is used. In fact, the U-235 content is much smaller, so a considerably greater mass of nuclear fuel must be used.

Extensions

9. (a) ${}^{56}_{26}\text{Fe} + {}^{56}_{26}\text{Fe} \rightarrow {}^{112}_{52}\text{Te}$
The element formed is tellurium-112.

(b) ***Given***
tellurium-112
Required
Show that this reaction absorbs energy.
Analysis and Solution
Compare the initial and final masses of the reactants and product.

$$\begin{aligned} m_\text{i} &= m_{{}^{56}_{26}\text{Fe}} + m_{{}^{56}_{26}\text{Fe}} \\ &= 55.934\ 938\ \text{u} + 55.934\ 938\ \text{u} \end{aligned}$$

$$\begin{aligned} m_\text{f} &= m_{{}^{112}_{52}\text{Te}} \\ &= 111.917\ 010\ \text{u} \end{aligned}$$

Calculate the energy using the equation $\Delta E = (m_\text{i} - m_\text{f})c^2$.

$$\begin{aligned} \Delta E &= (m_\text{i} - m_\text{f})c^2 \\ &= (55.934\ 938\ \text{u} + 55.934\ 938\ \text{u} - 111.917\ 010\ \text{u})c^2 \\ &= -0.047\ 134\ \text{u} \times \frac{931.5\ \text{MeV}}{\text{u}} \\ &= -43.9\ \text{MeV} \end{aligned}$$

A negative answer means that energy is absorbed.
Paraphrase
The fusion of two iron-56 nuclei results in the formation of tellurium-112, which is heavier than the initial two nuclei. Therefore, this reaction absorbs 43.9 MeV of energy.

(c) Stars similar to our Sun do not have sufficient mass to reach the core temperatures and densities needed to cause the production of iron, let alone the fusion of iron. The production of elements that are heavier than iron occurs during supernova explosions. The Sun is not massive enough to undergo a supernova explosion.

10. (a) The major radioactive waste isotopes produced by nuclear reactors, and their half-lives, are listed in the table below.

Waste Isotope	**Half-life**
cesium-137	30 years
strontium-90	30 years
uranium-235	700 million years
plutonium-240	24 000 years

(b) A current short-term method of storing nuclear waste is the use of on-site, sealed containers. Highly radioactive materials are stored in water-filled storage tanks. A long-term storage method is vitrification, where radioactive material is stored in a glass-like matrix as "logs" that can be deposited in deep wells bored in a stable landmass such as the Canadian Shield.

One proposed long-term disposal method, called subductive waste disposal, would place radioactive waste in subduction zones along active undersea faults. The radioactive material would be subducted or absorbed in the magma layers of Earth's mantle.

11.

Coal power (risk/benefits)	Nuclear power (risk/benefits)
Benefit: Coal provides a relatively low-cost energy source that is available in many areas of the world and is easy to mine. Production of power from coal is an old and proven technology.	**Risk:** Nuclear power is a complex and costly technology to implement. Incidents such as Three Mile Island in the U.S. and Chernobyl in the Ukraine remind us that this method of power generation is potentially dangerous. Nuclear accidents can pollute large regions of a country with wastes that have half-lives of tens to thousands of years.
Risk: Burning coal is a major source of greenhouse gases, which pose an environmental risk.	**Benefit:** Nuclear power is a “clean” energy source with few direct pollutants introduced into the environment (not including waste disposal, uranium mining, or a potential nuclear accident).
Risk: Coal combustion releases pollutants into the atmosphere, including naturally occurring radioactive isotopes. Without costly “scrubbing” of exhaust gas from a coal generation plant, more radioactive material is introduced into the environment than direct emissions from a nuclear power plant.	**Risk:** Disposal of spent fuel rods and radioactive waste is a major environmental concern for which no fool-proof long-term method currently exists.
Benefit: Coal is a safe energy source in politically unstable regions of the world.	**Risk:** Nuclear power generation poses a number of potential risks in politically unstable regions of the world. Acts of terrorism could lead to sabotage of nuclear facilities. As well, nuclear reactors can be modified to become “breeder reactors” for producing weapons-grade fissionable material.
Risk: Coal is a non-renewable resource that requires large-scale pit or sub-surface mining. Both methods present environmental hazards.	**Benefit:** Although uranium is also a non-renewable resource, it is many times more efficient as an energy source than coal combustion.

Chapter 16 Review

Knowledge

1. False: A hydrogen nucleus contains no neutrons!
2. Usually true: The exception is normal hydrogen, where $A = Z$.
3. Elements with the same atomic number but different neutron numbers are isotopes.
4. These nuclei are isotopes of uranium: Each atom contains the same number of electrons and protons but different numbers of neutrons.
5. For ${}^{115}_{55}\text{Cs}$, $A = Z + N$.

$N = 115 - 55 = 60$ neutrons
$Z = 55$ protons

6. 1 eV = 1.6×10^{-19} J so 50 MeV = $(50 \times 10^{6}\ \text{eV})(1.6 \times 10^{-19}\ \text{J/eV})$
$= 8.0 \times 10^{-12}$ J

7. Use the equation $E = mc^2$.

$$E = (0.001\ \text{kg})(3.00 \times 10^{8}\ \text{m/s})^2$$
$$= 9 \times 10^{13}\ \text{J}$$

One gram of matter is equivalent to $9 \times 10^{13}\ \text{J}$ of energy.

8. 1 u = 1.66×10^{-27} kg

The energy equivalent of this mass is

$$E = mc^2$$
$$= \left(1.66 \times 10^{-27}\ \text{kg}\right)\left(3.00 \times 10^{8}\ \text{m/s}\right)^2$$
$$= 1.49 \times 10^{-10}\ \text{J}$$
$$2.3\ \text{u} = (2.3)\left(1.49 \times 10^{-10}\ \text{J}\right)$$
$$= 3.44 \times 10^{-10}\ \text{J}$$

The energy equivalent of 2.3 u is $3.44 \times 10^{-10}\ \text{J}$.

9. Use the conversion 1 u = 931.5 MeV.

$$\frac{300\ \text{MeV}}{931.5\ \frac{\text{MeV}}{\text{u}}} = 0.322\ \text{u}$$

10. Use binding energy as the energy equivalent of the mass defect.

Since 1 u = 931.5 MeV,

$$0.022\ \text{u} \times 931.5\ \frac{\text{MeV}}{\text{u}} = 20.5\ \text{MeV}$$

11. False: Alpha decay decreases the atomic number by 4.
12. **(a)** Beta particles can be negative (electrons) or positive (positrons).
(b) An alpha particle is a helium nucleus, so its charge is +2.
(c) A gamma ray is a high-energy photon, so it has no charge.
13. **(a)** Alpha decay emits a helium nucleus, which removes two neutrons and two protons from the nucleus.

(b) β^- decay appears as the emission of an electron and the appearance of a new proton in the nucleus. β^+ decay appears as the emission of a positron and the creation of a new neutron in the nucleus.

(c) Gamma decay does not change the number of nucleons. It represents the change in internal energy of the nucleus.

14. ${}^{43}_{19}\text{K} \rightarrow {}^{43}_{20}\text{Ca} + {}^{0}_{-1}\beta + \bar{\nu}$

This equation is an example of β^- decay due to the emission of an electron and an antineutrino. The parent nucleus is potassium and the daughter element is calcium.

15. Gamma radiation is the most penetrating; beta radiation is the least penetrating.

16. The most important evidence involves the energy of radioactive decay products. Typical radioactive decays are measured in hundreds of thousands to millions of electron volts. Atomic processes are many orders of magnitude less energetic.

17. Radon exposure averages approximately 200 μSv/year, compared to 73 μSv/year for dental X rays. Of these two sources, radon exposure is much more significant because radon is a gas that, when inhaled, can be absorbed into body tissue. Radon decays via alpha particle emission and remains in the body for long periods of time, in close proximity to vital organs.

18. ***Given***

$N = 1.5 \times 10^{20}$

$\lambda = 1.2 \times 10^{-12}\ \text{s}^{-1}$

Analysis and Solution

Use the definition for activity,

$$A = \frac{\Delta N}{\Delta t} = -\lambda N$$

$$\begin{aligned} A &= -(1.2 \times 10^{-12}\ \text{s}^{-1})(1.5 \times 10^{20}) \\ &= -180\ \text{MBq} \end{aligned}$$

19. ***Given***

$N_0 = 5.0 \times 10^{20}$

$t_{1/2} = 1.5$ h

$t = 6$ h

Required

the number of nuclei remaining after 6 h (N)

Analysis and Solution

The number of radioactive nuclei has decreased by exactly one-half in 1.5 h:

$$\frac{5.0 \times 10^{20}}{2.5 \times 10^{20}} = 2$$

So, the half-life, $t_{1/2}$, of the sample is 1.5 h. Since 6 h = 4 × 1.5 h, the sample will decay by an additional 4 half-lives.

The number of radioactive nuclei remaining after 6 h is:

$$\left(2.5 \times 10^{20}\right)\left(\frac{1}{2}\right)^4 = 1.6 \times 10^{19}$$

Alternatively, use the equation $N = N_0\left(\frac{1}{2}\right)^{\frac{t}{t_{½}}}$.

$$N = \left(2.5 \times 10^{20}\right)\left(\frac{1}{2}\right)^{\frac{6}{1.5}}$$
$$= 1.6 \times 10^{19}$$

Paraphrase

The sample has a half-life of 1.5 h so, after 6 h, it will have decayed from 5.0×10^{20} to 1.6×10^{19} nuclei.

20. The half-life of carbon-14 is 5730 years. For this reason, carbon-14 is usually useful for dating items that are no older than 50 000–60 000 years. Ages of most rock samples are measured in millions of years.

21. Fission produces energy when a heavy nucleus splits into two smaller nuclei that have greater binding energy per nucleon than the original nucleus. This process is most effective for the most massive nuclei, with $A > 200$.

22. The fusion of hydrogen nuclei into helium is the most important energy source in stars.

23. The steps in the proton-proton chain are:
i) hydrogen + hydrogen produces deuterium
ii) hydrogen + deuterium produces helium-3
iii) helium-3 + helium-3 produces helium-4 plus 2 protons

Applications

24. (a) ***Given***

$^{4}_{2}\text{He}$

Required

binding energy per nucleon (E_b)

Analysis and Solution

Use $E_b = \Delta mc^2$, where $\Delta m = Zm_H + Nm_{neutron} - m_{atom}$, Z = atomic number, N = neutron number, m_H = mass of the hydrogen atom, $m_{neutron}$ = mass of neutron, and
m_{atom} = mass of the atom.

From $^{4}_{2}\text{He}$, $A = 4$, $Z = 2$

$N = A - Z$
$= 4 - 2$
$= 2$

$m_{^{4}_{2}\text{He}} = 4.002\ 603\ \text{u}$

$\Delta m = Zm_H + Nm_{neutron} - m_{atom}$
$= 2(1.007\ 825\ \text{u}) + 2(1.008\ 665\ \text{u}) - 4.002\ 603\ \text{u}$
$= 0.030\ 377\ \text{u}$

Use atomic mass units and multiply by 931.5 MeV/u to obtain the energy in MeV. Students will need to look up the masses.

$E_b = \Delta mc^2$
$$= 0.030\ 377\ \text{u} \times \frac{931.5\ \text{MeV}}{\text{u}}$$
$$= 28.296\ \text{MeV}$$

Divide this number by the atomic number to find the binding energy per nucleon.

$$\frac{28.296\text{ MeV}}{4} = 7.074\text{ MeV}$$

Paraphrase

The binding energy per nucleon in a helium atom is 7.074 MeV.

(b) ***Given***

$^{28}_{14}\text{Si}$

Required

binding energy per nucleon (E_b)

Analysis and Solution

Use $E_b = \Delta mc^2$, where $\Delta m = Zm_H + Nm_{neutron} - m_{atom}$, Z = atomic number, N = neutron number, m_H = mass of the hydrogen atom, $m_{neutron}$ = mass of neutron, and

m_{atom} = mass of the atom.

Use atomic mass units and multiply by 931.5 MeV/u to obtain the energy in MeV. Students will need to look up the masses.

From $^{28}_{14}\text{Si}$, $A = 28$, $Z = 14$

$N = A - Z$

$= 28 - 14$

$= 14$

$m_{^{28}_{14}\text{Si}} = 27.976\ 927\text{ u}$

$\Delta m = Zm_H + Nm_{neutron} - m_{atom}$

$= 14(1.007\ 825\text{ u}) + 14(1.008\ 665\text{ u}) - 27.976\ 927\text{ u}$

$= 0.253\ 933\text{ u}$

Use atomic mass units and multiply by 931.5 MeV/u to obtain the energy in MeV. Students will need to look up the masses.

$E_b = \Delta mc^2$

$$= 0.253\ 933\text{ u} \times \frac{931.5\text{ MeV}}{\text{u}}$$

$= 236.539\text{ MeV}$

Divide this number by the atomic number to find the binding energy per nucleon.

$$\frac{236.539\text{ MeV}}{28} = 8.448\text{ MeV}$$

Paraphrase

The binding energy per nucleon in a silicon atom is 8.448 MeV.

(c) ***Given***

$^{58}_{26}\text{Fe}$

Required

binding energy per nucleon (E_b)

Analysis and Solution

Use $E_b = \Delta mc^2$, where $\Delta m = Zm_H + Nm_{neutron} - m_{atom}$, Z = atomic number, N = neutron number, m_H = mass of the hydrogen atom, $m_{neutron}$ = mass of neutron, and m_{atom} = mass of the atom.

From ${}^{58}_{26}\mathrm{Fe}$, $A = 58$, $Z = 26$

$N = A - Z$
$= 58 - 26$
$= 32$

$m_{{}^{58}_{26}\mathrm{Fe}} = 57.933\ 276\ \mathrm{u}$

$\Delta m = Zm_{\mathrm{H}} + Nm_{\mathrm{neutron}} - m_{\mathrm{atom}}$
$= 26(1.007\ 825\ \mathrm{u}) + 32(1.008\ 665\ \mathrm{u}) - 57.933\ 276\ \mathrm{u}$
$= 0.547\ 454\ \mathrm{u}$

Use atomic mass units and multiply by 931.5 MeV/u to obtain the energy in MeV. Students will need to look up the masses.

$E_{\mathrm{b}} = \Delta mc^2$
$= 0.547\ 454\ \mathrm{u} \times \frac{931.5\ \mathrm{MeV}}{\mathrm{u}}$
$= 509.953\ \mathrm{MeV}$

Divide this number by the atomic number to find the binding energy per nucleon.

$$\frac{509.953\ \mathrm{MeV}}{58} = 8.792\ \mathrm{MeV}$$

Paraphrase
The binding energy per nucleon in an iron atom is 8.792 MeV.

(d) ***Given***
${}^{235}_{92}\mathrm{U}$

Required
binding energy per nucleon (E_{b})

Analysis and Solution
Use $E_{\mathrm{b}} = \Delta mc^2$, where $\Delta m = Zm_{\mathrm{H}} + Nm_{\mathrm{neutron}} - m_{\mathrm{atom}}$, Z = atomic number, N = neutron number, m_{H} = mass of the hydrogen atom, m_{neutron} = mass of neutron, and
m_{atom} = mass of the atom.

From ${}^{235}_{92}\mathrm{U}$, $A = 235$, $Z = 92$

$N = A - Z$
$= 235 - 92$
$= 143$

$m_{{}^{235}_{92}\mathrm{U}} = 235.043\ 930\ \mathrm{u}$

$\Delta m = Zm_{\mathrm{H}} + Nm_{\mathrm{neutron}} - m_{\mathrm{atom}}$
$= 92(1.007\ 825\ \mathrm{u}) + 143(1.008\ 665\ \mathrm{u}) - 235.043\ 930\ \mathrm{u}$
$= 1.915\ 065\ \mathrm{u}$

Use atomic mass units and multiply by 931.5 MeV/u to obtain the energy in MeV. Students will need to look up the masses.

$E_{\mathrm{b}} = \Delta mc^2$
$= 1.915\ 065\ \mathrm{u} \times \frac{931.5\ \mathrm{MeV}}{\mathrm{u}}$
$= 1783.883\ \mathrm{MeV}$

Divide this number by the atomic number to find the binding energy per nucleon.

$$\frac{1783.883\text{ MeV}}{235} = 7.591\text{ MeV}$$

Paraphrase

The binding energy per nucleon in a uranium atom is 7.591 MeV. It is useful to note that iron has the greatest binding energy of the four nuclei.

25. (a) ***Given***

$^{52}_{26}\text{Fe}$

Required

β^+ decay process

Analysis and Solution

Use the basic form for β^+ decay:

$^{A}_{Z}\text{X} \rightarrow {}^{A}_{Z-1}\text{Y} + {}^{0}_{1}\beta + \nu$

$^{52}_{26}\text{Fe} \rightarrow {}^{52}_{25}\text{Y} + {}^{0}_{1}\beta + \nu$

The element with $A = 52$ and $Z = 25$ is manganese. Therefore,

$^{52}_{26}\text{Fe} \rightarrow {}^{52}_{25}\text{Mn} + {}^{0}_{1}\beta + \nu$

Paraphrase

The β^+ decay of $^{52}_{26}\text{Fe}$ converts iron into manganese.

(b) To demonstrate charge conservation, note that $26 = 25 + {}^{0}_{1}\beta = 25 + 1 = 26$. The atomic mass number remains at 52. β^+ decay turns a proton into a neutron and yields a positron in the process. Charge and atomic mass number are therefore conserved.

26. (a) ***Given***

α-decay to produce lead-208

Required

the parent nucleus

Analysis and Solution

Use the standard form for α-decay.

$^{A}_{Z}\text{X} \rightarrow {}^{A-4}_{Z-2}\text{Y} + {}^{4}_{2}\alpha$

Let $A - 4 = 208$ and $Z - 2 = 82$.

$A = 212$, $Z = 84$

Therefore, the parent nucleus is polonium-212.

Paraphrase

Polonium-212 decays into lead-208 by emitting an alpha particle.

(b) ***Given***

α-decay to produce lead-208

Required

kinetic energy of alpha particle (E_k)

Analysis and Solution

Compare the binding energies of the parent and daughter nuclei. The difference will be approximately the energy of the α-particle.

$\Delta E = \left(m_{^{212}_{84}\text{Po}} - m_{^{208}_{82}\text{Pb}} - m_{^{4}_{2}\text{He}}\right)c^2$ or convert to MeV using 1 u = 931.5 MeV.

$$\Delta E = (211.988\ 868 - 207.976\ 652 - 4.002\ 603)\text{u} \times \frac{931.5\text{ MeV}}{\text{u}}$$

$= 8.95\ \text{MeV}$

Paraphrase

The approximate kinetic energy of the alpha particle is 8.95 MeV.

27. (a) ***Given***

sodium is converted into neon

Required

decay equation

Analysis and Solution

Write both sodium and neon in atomic form and inspect to determine what decay process has occurred.

sodium = $^{22}_{11}\text{Na}$

neon = $^{22}_{10}\text{Ne}$

Sodium must convert into a neon isotope. Since the atomic number decreases by 1, this process must be β^+ decay.

$^{22}_{11}\text{Na} \rightarrow {}^{22}_{10}\text{Ne} + {}^{0}_{1}\beta + \nu$

Paraphrase

The complete decay process for the transmutation of $^{22}_{11}\text{Na}$ into $^{22}_{10}\text{Ne}$ is

$^{22}_{11}\text{Na} \rightarrow {}^{22}_{10}\text{Ne} + {}^{0}_{1}\beta + \nu$.

(b) ***Given***

sodium is converted into neon

Required

energy released (ΔE)

Analysis and Solution

Using the decay process equation, compare the mass of the parent nucleus with those of the daughter nucleus and decay products to determine the mass defect and energy released.

$^{22}_{11}\text{Na} \rightarrow {}^{22}_{10}\text{Ne} + {}^{0}_{1}\beta + \nu$

To determine energy given off, compare masses of parent and daughter nuclei.

$$\begin{aligned}\Delta m &= (m_{\text{Na}} - m_{\text{Ne}} - m_{\text{e}}) \\ &= (21.994\ 436 - 21.991\ 385 - 0.000\ 549)\text{u} \\ &= 0.002\ 502\ \text{u}\end{aligned}$$

$$\begin{aligned}\Delta E &= \Delta mc^2 \\ &= 0.002\ 502\ \text{u} \times \frac{931.5\ \text{MeV}}{\text{u}} \\ &= 2.33\ \text{MeV}\end{aligned}$$

Paraphrase

The energy released during this decay is 2.33 MeV.

28. ***Given***

Artifacts contain $\frac{1}{4}$ the original amount of carbon-14.

Required

approximate age of artifacts

Analysis and Solution

The half-life, $t_{1/2}$, of carbon-14 is 5730 years.

Use the equation $N = N_0\left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$.

$$\frac{N}{N_0} = \frac{1}{4} = \left(\frac{1}{2}\right)^2$$

$$\frac{t}{t_{1/2}} = 2$$

$$t = 2t_{1/2}$$

$$\begin{aligned} t &= 2(5730 \text{ years}) \\ &= 11\,460 \text{ years} \end{aligned}$$

Paraphrase

The oldest archaeological sites in Alberta appear to be approximately 11 500 years old.

29. (a) ***Given***

$\lambda = 1.98 \times 10^{-11}\ \text{s}^{-1}$

$A = 0.10$ MBq

Required

number of radium atoms (N)

Analysis and Solution

Use the equation $A = -\lambda N$.

$$\begin{aligned} N &= -\frac{A}{\lambda} \\ &= -\frac{0.10 \times 10^6\ \text{Bq}}{1.98 \times 10^{-11}\ \text{s}^{-1}} \\ &= -5.05 \times 10^{15} \end{aligned}$$

(The negative sign indicates that the number of atoms is decreasing.)

Paraphrase

The clock dial contains 5.05×10^{15} atoms of radium-226.

(b) ***Given***

5.05×10^{15} atoms of radium-226 (from part (a))

Required

mass of radium (m)

Analysis and Solution

The mass of a radium atom is 226 u, where 1 u = $1.660\,539 \times 10^{-27}$ kg. Multiply the mass per atom by the number of atoms.

$$\begin{aligned} m &= (226\ \text{u})\left(1.660\,539 \times 10^{-27}\ \frac{\text{kg}}{\text{u}}\right)\left(5.05 \times 10^{15}\right) \\ &= 1.90 \times 10^{-9}\ \text{kg} \end{aligned}$$

Paraphrase

The clock dial contains 1.90×10^{-9} kg of radium.

(c) ***Given***

$t_{1/2}$ = 1600 years

t = 5000 years

$\lambda = 1.98 \times 10^{-11}\ \text{s}^{-1}$

5.05×10^{15} atoms of radium-226 (from part (a))

Required

activity after 5000 years (A)

Analysis and Solution

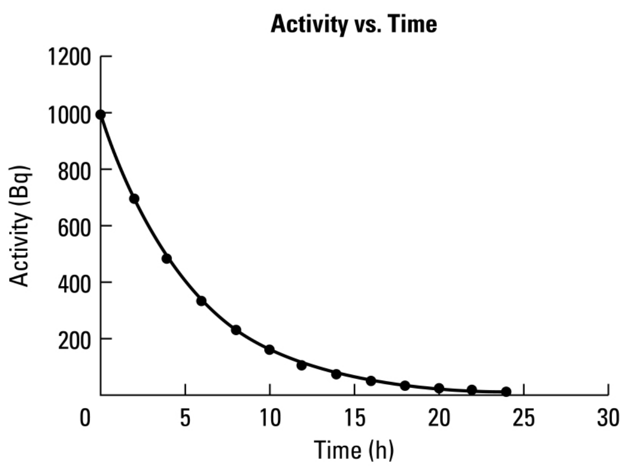
Use the equation $A = -\lambda N$, where $N = N_0\left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$.

$$\begin{aligned} A &= -\lambda N \\ &= -\lambda N_0\left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}} \\ &= -\left(1.98\times10^{-11}\ \text{s}^{-1}\right)(5.05 \times 10^{15})\left(\frac{1}{2}\right)^{\frac{5000}{1600}} \\ &= -1.15\times10^{4}\ \text{Bq} \end{aligned}$$

Paraphrase

After 5000 years, the activity of the dials will have dropped by about a factor of 10 to about 11 500 Bq.

30.



(a) By inspection, when t = 5 h, the activity of the sample is 400 Bq.

(b) To estimate the half-life, note how long it takes the activity to drop from 400 Bq to 200 Bq. At t = 5 h, the activity is 400 Bq. At t = 8 h, the activity is 200 Bq. It has taken approximately 3 h for the activity to drop by a factor of one-half, so the half-life is approximately 3 h.

31. ***Given***

Three helium nuclei combine to form a carbon nucleus.

Required

energy released (ΔE)

Analysis and Solution

Calculate the mass difference between three helium nuclei and a carbon-12 nucleus. This mass defect represents the energy released by the reaction.

$$m_{He} = 4.002\ 603\ \text{u}$$

$$m_C = 12.000\ 000\ \text{u}$$

$$\begin{aligned}\Delta m &= 3(4.002\ 603\ \text{u}) - 12.000\ 000\ \text{u} \\ &= 0.007\ 809\ \text{u}\end{aligned}$$

$$\begin{aligned}\Delta E &= 0.007\ 809\ \text{u} \times \frac{931.5\ \text{MeV}}{\text{u}} \\ &= 7.27\ \text{MeV}\end{aligned}$$

Paraphrase

The triple-α process that converts helium into carbon releases 7.27 MeV of energy.

32. (a) The key advantage of polonium over a chemical fuel is that you will obtain a much higher energy yield per unit mass by using nuclear decay rather than energy conversion through chemical means.

(b) ***Given***

P = 20 W
t = 14.5 years
efficiency = 15%
fuel = polonium-208
polonium-208 decays into lead-204
$\lambda = 7.57 \times 10^{-9}\ \text{s}^{-1}$
$t_{1/2}$ = 2.9 years

Required

mass of polonium-208 (m)

Analysis and Solution

Since ${}^{208}_{84}\text{Po}$ decays into ${}^{204}_{82}\text{Pb}$, the pattern is that of an alpha decay—the atomic mass numbers and the atomic numbers of the two elements differ by a helium nucleus.

m_{Po} = 207.981 246 u
m_{Pb} = 203.973 044 u
m_α = 4.002 603 u

$$\begin{aligned}\Delta m &= m_{Po} - m_{Pb} - m_\alpha \\ &= 207.981\ 246\ \text{u} - 203.973\ 044\ \text{u} - 4.002\ 603\ \text{u} \\ &= 0.005\ 599\ \text{u}\end{aligned}$$

Determine the energy released during the alpha decay of polonium into lead.

$$\begin{aligned}\Delta E &= \Delta mc^2 \\ &= 0.005\ 599\ \text{u} \times \frac{931.5\ \text{MeV}}{\text{u}} \\ &= 5.22\ \text{MeV} \\ &= \left(5.22 \times 10^6\ \text{eV}\right)\left(1.60 \times 10^{-19}\ \frac{\text{J}}{\text{eV}}\right) \\ &= 8.35 \times 10^{-13}\ \text{J}\end{aligned}$$

Calculate the total number of decays needed to produce 20 W of electrical power. Remember that the energy conversion is only 15% efficient!

Since you need 20 J/s of electrical energy, you will need $\frac{20 \text{ J/s}}{0.15}$ = 133 J/s of thermal energy produced by the decay of polonium. Since each decay releases 8.35×10^{-13} J, you require

$$\frac{133 \frac{\cancel{\text{J}}}{\text{s}}}{8.35 \times 10^{-13} \cancel{\text{J}}} = 1.59 \times 10^{14} \text{ decays per second}$$

This value is the same as the activity of the polonium fuel after 14.5 years.
The activity is $A = -\lambda N$, where λ is the decay constant, 7.57×10^{-9} s^{-1}, and N is the total number of polonium atoms present. Solve for N:

$$\begin{aligned} N &= -\frac{A}{\lambda} \\ &= -\frac{1.59 \times 10^{14} \text{ s}^{-1}}{7.57 \times 10^{-9} \text{ s}^{-1}} \\ &= -2.10 \times 10^{22} \end{aligned}$$

Since you require 2.10×10^{22} atoms of polonium to be present after 14.5 years to ensure an electrical power output of 20 W, you must start with more polonium.
To determine the original number of polonium atoms required, N_0, use the equation

$$N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

$$\begin{aligned} N_0 &= \frac{N}{\left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}} \\ &= \frac{2.10 \times 10^{22}}{\left(\frac{1}{2}\right)^{\frac{14.5}{2.9}}} \\ &= 6.72 \times 10^{23} \end{aligned}$$

Divide by Avogadro's number to find the number of moles of polonium required:

$$\frac{6.72 \times 10^{23} \text{ atoms}}{6.02 \times 10^{23} \frac{\text{atoms}}{\text{mol}}} = 1.12 \text{ mol}$$

Since one mole of polonium has a mass of 207.8 g, the mass of polonium required is $1.12 \text{ mol} \times 207.8 \frac{\text{g}}{\text{mol}} = 233 \text{ g}$

Paraphrase

Powering the space probe for 14.5 years requires only 233 g of polonium. This mass is much less than that required if chemical fuels were used.

Extensions

33. Start with the atomic notation for all the nuclei involved. Make sure that charge and atomic mass number are conserved. Remember that most elements have isotopes.

To determine which reaction is more likely, look at the mass defects (differences between initial and final masses). The smallest mass defect will also represent the reaction that requires the least energy. This reaction will be the more probable one.

Solution 1

$^{14}_{7}\text{N} + ^{4}_{2}\text{He} \rightarrow ^{A1}_{8}\text{O} + ^{A2}_{1}\text{H}$

$A1$ and $A2$ need to be adjusted so that atomic mass number is conserved. A simple solution is to leave $A1$ as 16 (“normal” oxygen). Then, $A2 = 2$. The reaction is $^{14}_{7}\text{N} + ^{4}_{2}\text{He} \rightarrow ^{16}_{8}\text{O} + ^{2}_{1}\text{H}$, which means that deuterium is produced.

Solution 2

$^{14}_{7}\text{N} + ^{4}_{2}\text{He} \rightarrow ^{A1}_{8}\text{O} + ^{A2}_{1}\text{H}$

If $A1 = 17$ and $A2 = 1$, then the reaction is $^{14}_{7}\text{N} + ^{4}_{2}\text{He} \rightarrow ^{17}_{8}\text{O} + ^{1}_{1}\text{H}$.

To assess which reaction is more probable, note the following, where Mass 1 and Mass 2 refer to the two possible products given for Solutions 1 and 2, respectively.

Solution	**Mass N-14**	**Mass He-4**	**Mass 1**	**Mass 2**	**Δm (u)**	**ΔE (MeV)**
1	14.003 074	4.002 603	15.994 915	2.014 102	0.003 340	3.11
2	14.003 074	4.002 603	16.999 132	1.007 825	0.001 280	1.19

The more likely reaction is the one in Solution 2 because it requires less energy. An easy way to test this solution is to look for gamma rays. O-17 will be in an excited (nuclear) state and will decay via gamma-ray emission. Another test is to measure the charge-to-mass ratio for the “protons” created in the collision. In Solution 1, the “protons” are deuterons and the charge-to-mass ratio is one-half the ratio for normal protons (Solution 2).

Paraphrase

Two different isotopes of oxygen and hydrogen could be produced in this reaction. The more likely reaction is the lower-energy reaction, $^{14}_{7}\text{N} + ^{4}_{2}\text{He} \rightarrow ^{17}_{8}\text{O} + ^{1}_{1}\text{H}$.

34. The two most serious radioactive isotopes released in a nuclear blast are strontium-90 and cesium-137. Both of these isotopes have half-lives of approximately 30 years. They can be spread globally by high-altitude winds, eventually be taken up by plant matter through rainfall, and then enter the human food chain. The build-up of either of these isotopes in the human body is a serious medical concern and is directly linked to various forms of cancer.

35. Nucleosynthesis is the process in which heavier elements are produced from lighter elements in the central regions of stars. The two main processes responsible are nuclear fusion and the neutron capture process that occurs during a supernova explosion.

Some of the fusion reactions are:

proton + proton $\rightarrow$ He (proton-proton chain)
He + He + He $\rightarrow$ C (triple alpha)
He + C $\rightarrow$ O
C + p $\rightarrow$ N (Other p and C combinations yield many elements.)
O + O $\rightarrow$ Si (All other elements between O and Si are produced by various combinations of lighter nuclei.)
$^{28}Si + {}^{28}Si \rightarrow {}^{56}Ni + \gamma$ $^{56}Ni \rightarrow {}^{56}Co + {}^{0}_{1}\beta + \nu$ $^{56}Co \rightarrow {}^{56}Fe + {}^{0}_{1}\beta + \nu$

When iron production is reached (assuming the star is massive enough), any reaction after this point absorbs rather than releases energy. Smaller stars are unable to complete these sequences because reactions beyond the proton-proton chain require much higher temperatures. These temperatures can only be achieved if the mass of the star is high enough to compress the gas at its core to sufficiently high pressure and temperature. Our Sun will likely only reach the triple-alpha stage, where the core temperature climbs from 14×10^6 K to about 100×10^6 K.

