Pearson Physics Level 30
Unit V Momentum and Impulse: Chapter 9
Solutions

Student Book page 449

Concept Check

(a) If the mass is doubled but the velocity remains the same, the magnitude of the momentum will double.

$$\begin{aligned} p &= mv \\ &= (2m)v \\ &= 2(mv) \end{aligned}$$

(b) If the velocity is reduced to $\frac{1}{3}$ of its original magnitude, the magnitude of the momentum will be reduced to $\frac{1}{3}$.

$$\begin{aligned} p &= mv \\ &= m\left(\frac{1}{3}v\right) \\ &= \left(\frac{1}{3}\right)(mv) \end{aligned}$$

(c) If the direction of the velocity changes from [E] to [W], the direction of the momentum changes from [E] to [W]. However, the magnitude of the momentum remains the same.

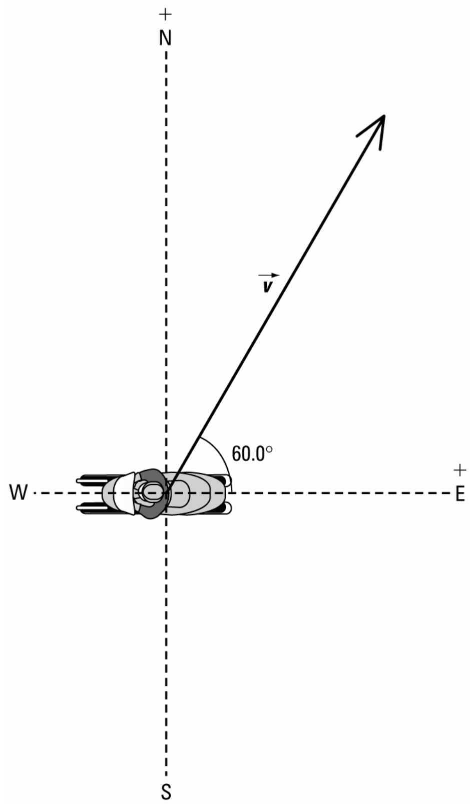
Student Book page 451

Example 9.1 Practice Problems

1. (a) ***Given***

m_g = 65 kg $\qquad$ $\vec{v}$ = 11.5 m/s [60.0° N of E]

m_s = 535 kg



Required
momentum of system ($\vec{p}$)

Analysis and Solution
The girl and snowmobile are a system because they move together as a unit. Find the total mass of the system.

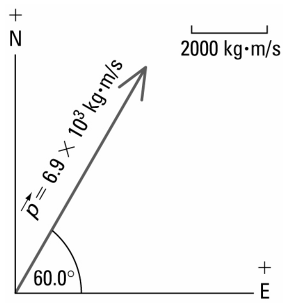
$$
\begin{aligned}
m_T &= m_g + m_s \\
&= 65 \text{ kg} + 535 \text{ kg} \\
&= 600 \text{ kg}
\end{aligned}
$$

The momentum of the system is in the direction of the velocity of the system. So use the scalar form of $\vec{p} = m\vec{v}$ to find the magnitude of the momentum.

$$
\begin{aligned}
p &= m_T v \\
&= (600 \text{ kg})(11.5 \text{ m/s}) \\
&= 6.9 \times 10^3 \text{ kg•m/s}
\end{aligned}
$$

Paraphrase
The momentum of the girl-snowmobile system is 6.9×10^3 kg•m/s [60.0° N of E].

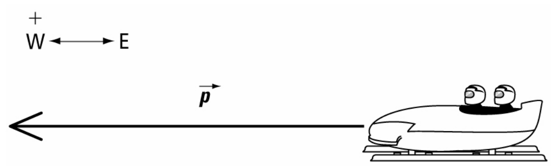
(b)



2. ***Given***

m_T = 390 kg

$\vec{p}$ = 4.68 × 10^3 kg•m/s [W]



Required

velocity of sled ($\vec{v}$)

Analysis and Solution

The momentum of the system is in the direction of the velocity of the system. So use the scalar form of $\vec{p} = m\vec{v}$ to find the speed.

$$p = m_T v$$

$$v = \frac{p}{m_T}$$

$$= \frac{4.68 \times 10^3 \ \text{kg} \cdot \frac{\text{m}}{\text{s}}}{390 \ \text{kg}}$$

$$= 12.0 \text{ m/s}$$

Paraphrase

The velocity of the sled is 12.0 m/s [W].

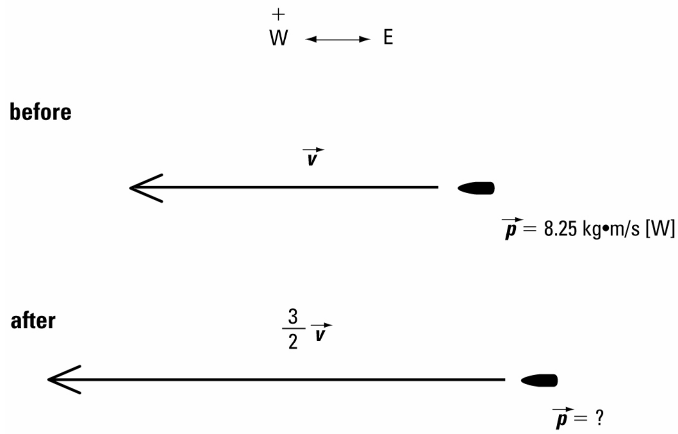
Student Book page 452

Example 9.2 Practice Problems

1. ***Analysis and Solution***

From the equation $\vec{p} = m\vec{v}$, $p \propto m$ and $p \propto v$.

The figure below represents the situation of the problem.



$p \propto \frac{3}{4} m$ and $p \propto \frac{3}{2} v$

Calculate the factor change of p.

$\frac{3}{4} \times \frac{3}{2} = \frac{9}{8}$

Calculate p.

$$\frac{9}{8} p = \frac{9}{8} \times (8.25 \text{ kg•m/s})$$
$$= 9.28 \text{ kg•m/s}$$

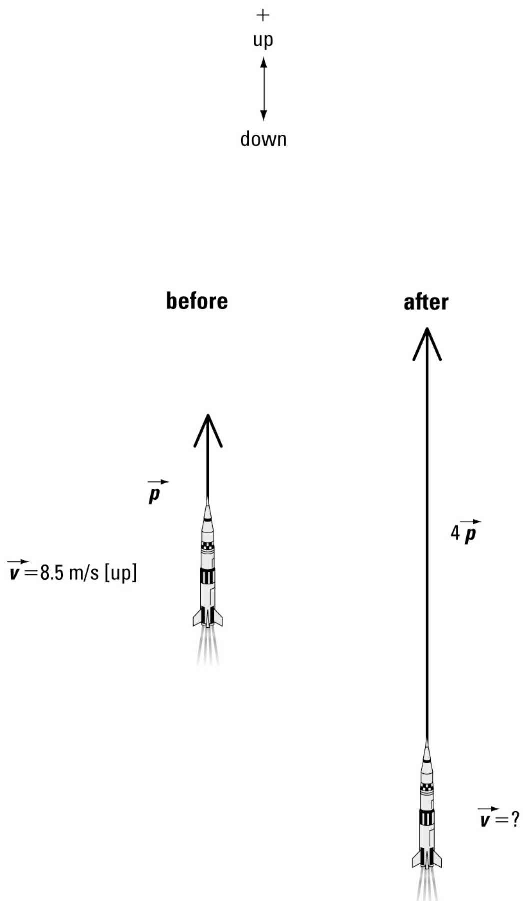
The new momentum will be 9.28 kg•m/s [W].

2. ***Analysis and Solution***

Write the equation $\vec{p} = m\vec{v}$ as $\vec{v} = \frac{\vec{p}}{m}$.

From the equation $\vec{v} = \frac{\vec{p}}{m}$, $v \propto \frac{1}{m}$ and $v \propto p$.

The figure below represents the situation of the problem.



$v \propto 4p$ and $v \propto \dfrac{1}{\left(\frac{1}{2}\right)m}$

Calculate the factor change of *v*.

$$4 \times \frac{1}{\left(\frac{1}{2}\right)} = 4 \times 2$$
$$= 8$$

Calculate *v*.

$$8v = 8 \times (8.5 \text{ m/s})$$
$$= 68 \text{ m/s}$$

The new velocity will be 68 m/s [up].

Student Book page 453

9.1 Check and Reflect

Knowledge

1. (a) Momentum is a vector quantity equal to the mass of an object times its velocity.
(b) The units of momentum are kilogram-metres per second (kg•m/s).

2. Momentum is a vector quantity because it has both magnitude and direction. It is the product of a scalar quantity (mass) and a vector quantity (velocity). The momentum of an object is in the same direction as the velocity of the object.

3. The general form of Newton's second law states that the net force acting on an object is equal to the rate of change of its momentum, $\vec{F}_{\text{net}} = \frac{\Delta \vec{p}}{\Delta t}$. From Unit II, Newton's second law states that net force is the product of mass and acceleration, $\vec{F}_{\text{net}} = m\vec{a}$. Acceleration is the rate of change of velocity, $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$. Momentum is equal to the product of mass and velocity, $\vec{p} = m\vec{v}$. If you substitute the definitions of acceleration and momentum into $\vec{F}_{\text{net}} = m\vec{a}$, you get $\vec{F}_{\text{net}} = \frac{\Delta \vec{p}}{\Delta t}$.

4. Stating Newton's second law in terms of momentum is more useful than stating it in terms of acceleration, because it can be applied to situations where either velocity or mass change, or *both* velocity and mass change. An example of a situation where both velocity and mass change occurs when a rocket takes off. The rocket accelerates upward (change in velocity) and fuel is consumed (change in mass).

5. An object has inertia because every object has mass. If an object is stationary, it has inertia but its momentum is zero. If the same object is in motion, it still has inertia but its momentum will not be zero. So, provided that the mass of the object does not change, the motion of the object does not change its inertia. However, the velocity of the object affects its momentum, since momentum is the product of mass and velocity.

6. (a) Three situations in which velocity is the dominant factor affecting the momentum of an object are:
- a bullet being fired by a rifle
- a golf ball being hit by a golf club
- a jet taking off

(b) Three situations in which mass is the dominant factor affecting the momentum of an object are:
- an avalanche moving down a hillside
- a whale swimming at a slow pace
- a bowling ball rolling down a bowling lane

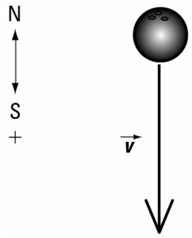
Applications

7. It would be an advantage to have the mass of the insect greater than the mass of the shell. Since the shell has mass, it has inertia. The greater the mass of the shell, the greater will be its inertia. So it will be harder for the insect to cause it to accelerate. However, if the shell has less mass than the insect, the insect will be able to accelerate the shell more easily.

8. ***Given***

$m = 6.0$ kg

$\vec{v} = 2.2$ m/s [S]



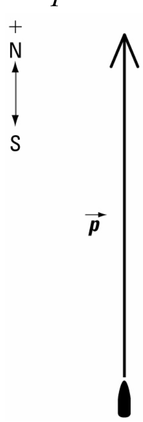
Required
momentum of bowling ball ($\vec{p}$)

Analysis and Solution
The momentum of the bowling ball is in the direction of its velocity. So use the scalar form of $\vec{p} = m\vec{v}$ to find the magnitude of the momentum.

$$\begin{aligned} p &= mv \\ &= (6.0 \text{ kg})(2.2 \text{ m/s}) \\ &= 13 \text{ kg•m/s} \end{aligned}$$

Paraphrase
The momentum of the bowling ball is 13 kg•m/s [S].

9. ***Given***
$m = 75$ g
$\vec{p} = 9.00$ kg•m/s [N]



Required
velocity of bullet ($\vec{v}$)

Analysis and Solution
The momentum of the bullet is in the direction of its velocity. So use the scalar form of $\vec{p} = m\vec{v}$ to find the speed.

$$p = mv$$

$$v = \frac{p}{m}$$

$$= \frac{9.00\ \text{kg}\bullet\frac{\text{m}}{\text{s}}}{(75\ \text{g})\left(\frac{1\ \text{kg}}{1000\ \text{g}}\right)}$$

$$= 1.2 \times 10^2\ \text{m/s}$$

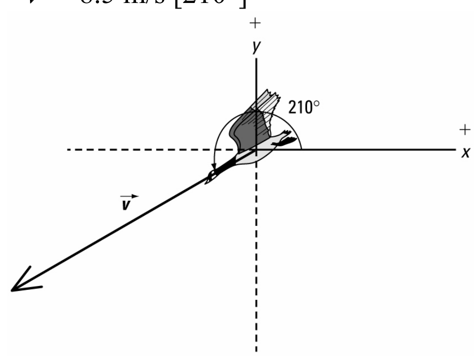
Paraphrase

The velocity of the bullet is 1.2×10^2 m/s [N].

10. (a) ***Given***

$m = 4.6$ kg

$\vec{v} = 8.5$ m/s [210°]



Required

momentum vector diagram

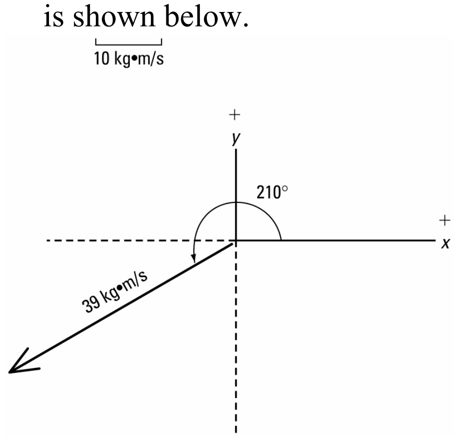
Analysis and Solution

The momentum of the goose is in the direction of its velocity. So use the scalar form of $\vec{p} = m\vec{v}$ to find the magnitude of the momentum.

$$p = mv$$
$$= (4.6\ \text{kg})(8.5\ \text{m/s})$$
$$= 39\ \text{kg}\bullet\text{m/s}$$

Paraphrase

The momentum of the goose is 39 kg•m/s [210°]. The momentum vector diagram is shown below.

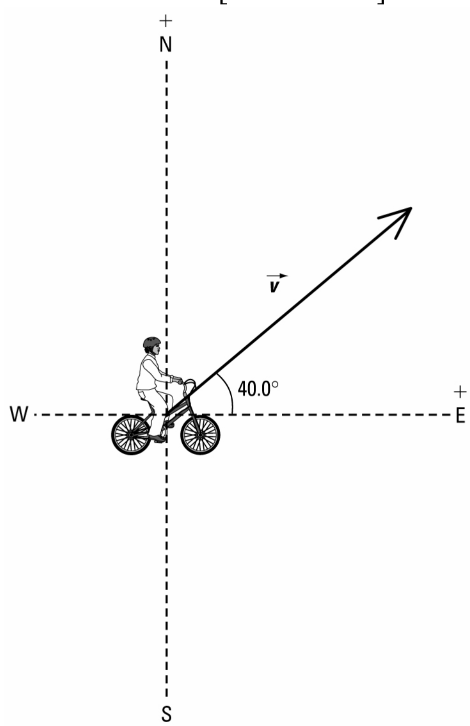


(b) ***Given***

m_b = 10.0 kg

m_r = 54.0 kg

$\vec{v}$ = 4.2 m/s [40.0° N of E]



Required

momentum vector diagram for each mass and for system

Analysis and Solution

Calculate the momentum of each mass.

$$p_r = m_r v = (54.0 \text{ kg})(4.2 \text{ m/s}) = 2.3 \times 10^2 \text{ kg•m/s}$$

and

$$p_b = m_b v = (10.0 \text{ kg})(4.2 \text{ m/s}) = 42 \text{ kg•m/s}$$

The rider and bicycle are a system because they move together as a unit. Find the total mass of the system.

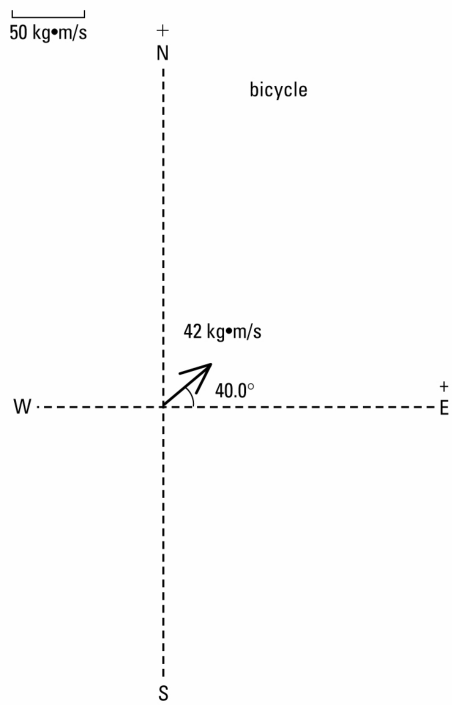
$$m_T = m_r + m_b = 54.0 \text{ kg} + 10.0 \text{ kg} = 64.0 \text{ kg}$$

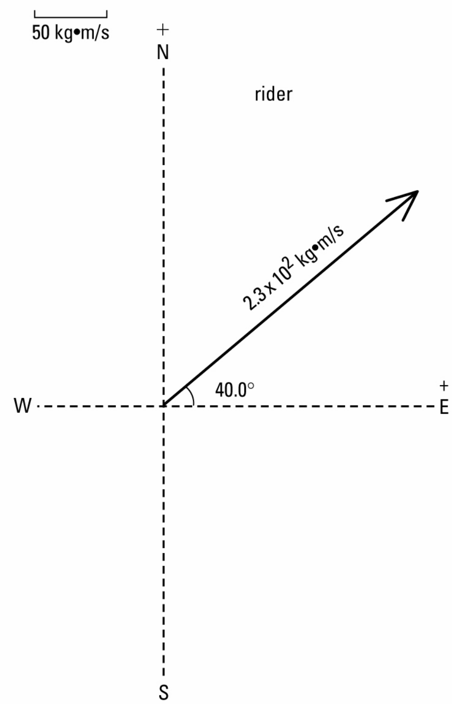
The momentum of the system is in the direction of the velocity of the system.
So use the scalar form of $\vec{p} = m\vec{v}$ to find the magnitude of the momentum.

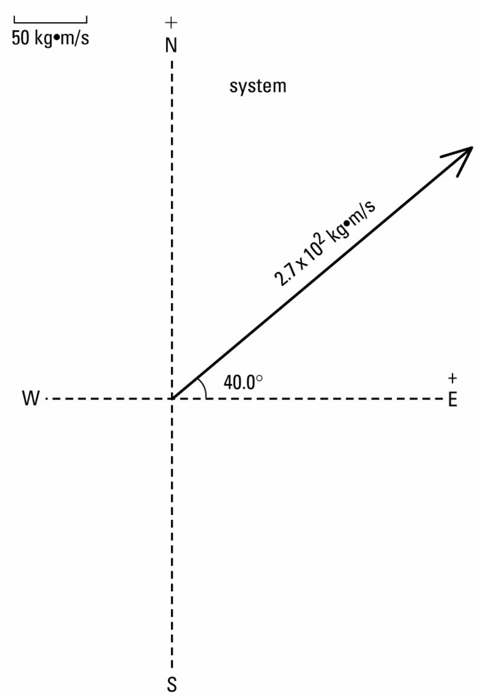
$$p_T = m_T v = (64.0 \text{ kg})(4.2 \text{ m/s}) = 2.7 \times 10^2 \text{ kg•m/s}$$

Paraphrase

The momentum of the rider is 2.3×10^2 kg•m/s [40.0° N of E], the momentum of the bicycle is 42 kg•m/s [40.0° N of E], and the momentum of the system is 2.7×10^2 kg•m/s [40.0° N of E]. The momentum vector diagrams are shown below.



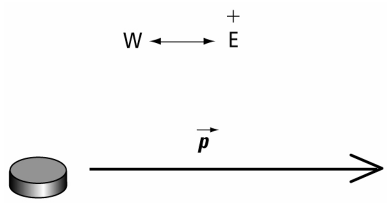




11. ***Given***

$\vec{p}$ = 3.8 kg•m/s [E]

v = 24 m/s



Required

mass of hockey puck (m)

Analysis and Solution

The momentum of the puck is in the direction of its velocity. So use the scalar form of $\vec{p} = m\vec{v}$ to find the mass.

$$p = mv$$

$$m = \frac{p}{v}$$

$$= \frac{3.8\ \text{kg}\bullet\frac{\text{m}}{\text{s}}}{24\ \frac{\text{m}}{\text{s}}}$$

$$= 0.16\ \text{kg}$$

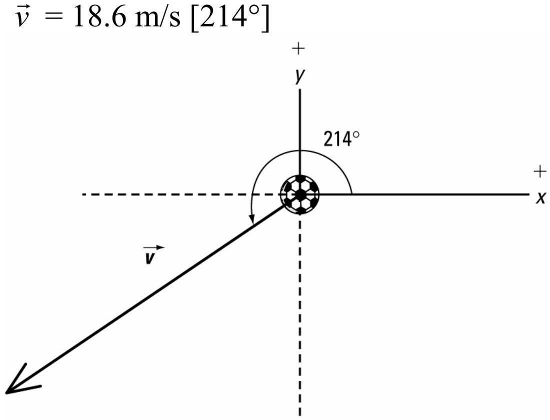
Paraphrase

The mass of the hockey puck is 0.16 kg.

12. ***Given***

m = 425 g

$\vec{v}$ = 18.6 m/s [214°]



Required

momentum vector diagram

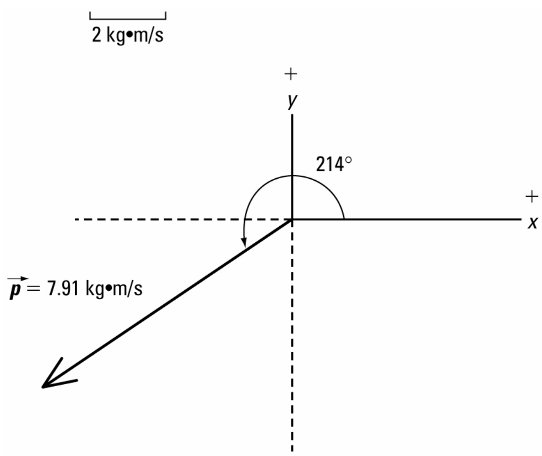
Analysis and Solution

The momentum of the soccer ball is in the direction of its velocity. So use the scalar form of $\vec{p} = m\vec{v}$ to find the magnitude of the momentum.

$$p = mv$$
$$= \left(425\ \cancel{g} \times \frac{1\ kg}{1000\ \cancel{g}}\right)(18.6\ m/s)$$
$$= 7.91\ kg \bullet m/s$$

Paraphrase

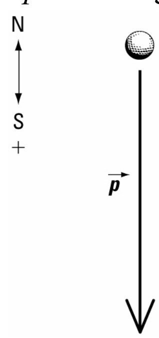
The momentum of the soccer ball is 7.91 kg•m/s [214°]. The momentum vector diagram is shown below.



13. ***Given***

$m = 0.046$ kg

$\vec{p} = 3.45$ kg•m/s [S]



Required

velocity of golf ball ($\vec{v}$)

Analysis and Solution

The momentum of the golf ball is in the direction of its velocity. So use the scalar form of $\vec{p} = m\vec{v}$ to find the speed.

$$p = mv$$
$$v = \frac{p}{m}$$

$$= \frac{3.45\ \cancel{\text{kg}} \bullet \frac{\text{m}}{\text{s}}}{0.046\ \cancel{\text{kg}}}$$

$$= 75 \text{ m/s}$$

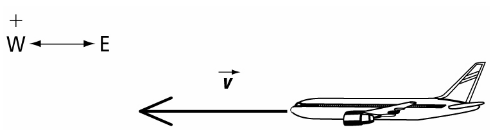
Paraphrase and Verify

The velocity of the golf ball is 75 m/s [S]. The speed is equivalent to 270 km/h, which is approximately the initial speed of a long drive hit by a professional golfer.

14. (a) ***Given***

$\vec{v} = 190$ m/s [W]

$m = 2250$ kg



Required

momentum of jet ($\vec{p}$)

Analysis and Solution

The momentum of the jet is in the direction of its velocity. So use the scalar form of $\vec{p} = m\vec{v}$ to find the magnitude of the momentum.

$$\begin{aligned} p &= mv \\ &= (2250 \text{ kg})(190 \text{ m/s}) \\ &= 4.28 \times 10^5 \text{ kg•m/s} \end{aligned}$$

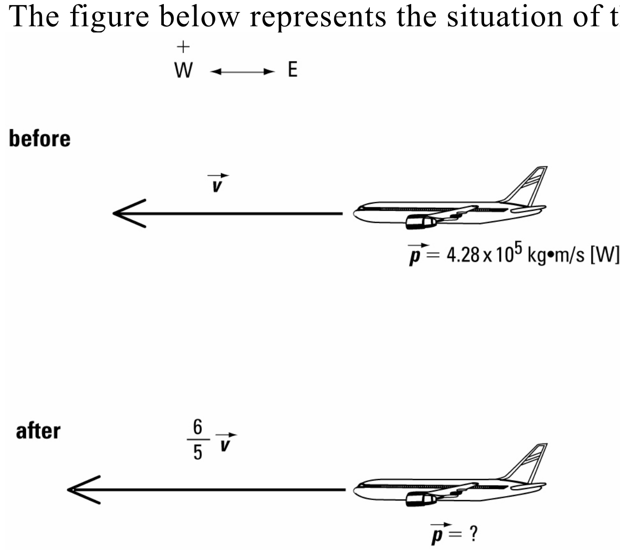
Paraphrase

The momentum of the jet is 4.28×10^5 kg•m/s [W].

(b) ***Analysis and Solution***

From the equation $\vec{p} = m\vec{v}$, $p \propto m$ and $p \propto v$.

The figure below represents the situation of the problem.



$p \propto \frac{3}{4} m$ and $p \propto \frac{6}{5} v$

Calculate the factor change of p.

$$\frac{3}{4} \times \frac{6}{5} = \frac{9}{10}$$

Calculate p.

$$\frac{9}{10}p = \frac{9}{10} \times (4.28 \times 10^5 \text{ kg•m/s})$$

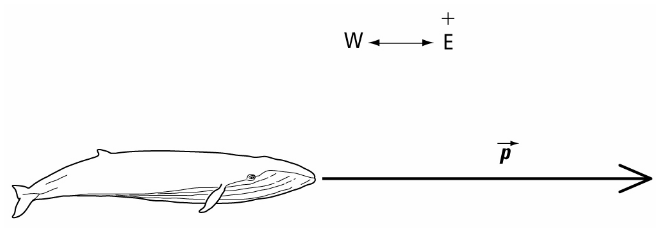
$$= 3.85 \times 10^5 \text{ kg•m/s}$$

The new momentum will be 3.85×10^5 kg•m/s [W].

15. ***Given***

$\vec{v} = 57.0$ km/h [E]

$\vec{p} = 2.15 \times 10^6$ kg•m/s [E]



Required

mass of blue whale (m)

Analysis and Solution

Convert speed to metres per second.

$$v = \frac{57.0 \text{ km}}{1 \text{ h}} \times \frac{1 \text{ h}}{3600 \text{ s}} \times \frac{1000 \text{ m}}{1 \text{ km}}$$

$$= 15.83 \text{ m/s}$$

The momentum of the whale is in the direction of its velocity. So use the scalar form of $\vec{p} = m\vec{v}$ to find the mass.

$$p = mv$$

$$m = \frac{p}{v}$$

$$= \frac{2.15 \times 10^6 \text{ kg•}\frac{\text{m}}{\text{s}}}{15.83 \frac{\text{m}}{\text{s}}}$$

$$= 1.36 \times 10^5 \text{ kg}$$

Paraphrase

The mass of the blue whale is 1.36×10^5 kg.

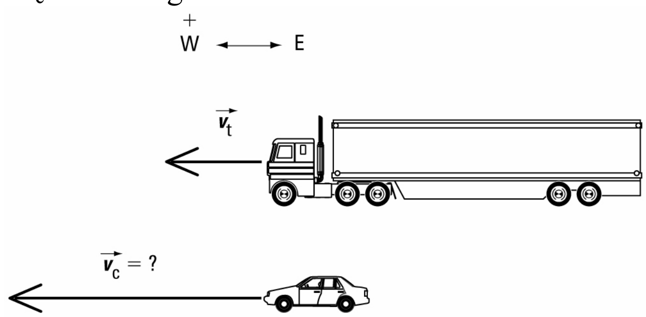
Extensions

16. ***Given***

$m_t = 38\ 000$ kg

$\vec{v}_t = 1.20$ m/s [W]

$m_c = 1400$ kg



Required

velocity of car ($\vec{v}_c$)

Analysis and Solution

The momentum of each vehicle is directed west. So use the scalar form of $\vec{p} = m\vec{v}$ to get an expression for the magnitude of the momentum of each vehicle.

$p_t = m_t v_t$ and $p_c = m_c v_c$

Since the momentum of each vehicle is the same, set both equations equal to each other.

$m_t v_t = m_c v_c$

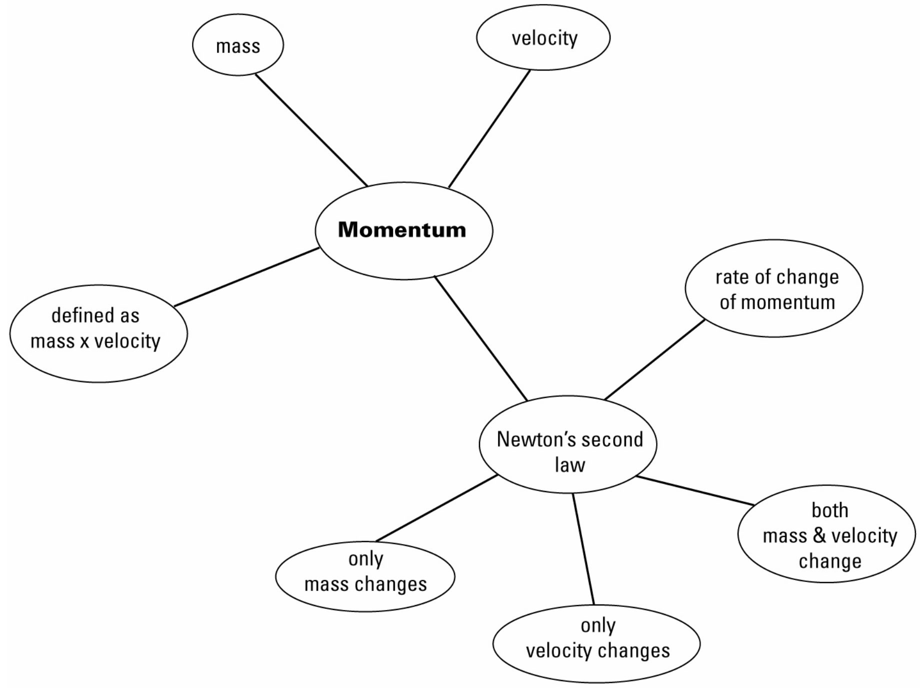
Solve to find the speed of the car.

$$
\begin{aligned}
v_c &= \left(\frac{m_t}{m_c}\right) v_t \\
&= \left(\frac{38\ 000\ \cancel{\text{kg}}}{1400\ \cancel{\text{kg}}}\right)(1.20\text{ m/s}) \\
&= 32.6\text{ m/s}
\end{aligned}
$$

Paraphrase and Verify

The velocity of the car would have to be 32.6 m/s [W] in order to have the same momentum as the truck. Since the car is about 30 times less massive than the truck, you would expect the car to be travelling about 30 times faster than the truck. So the calculated answer is reasonable.

17. For example:



Student Book page 456

Concept Check

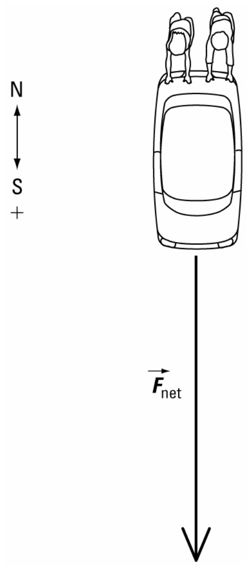
At the first instant of impact, the momentum of each putty ball was the same but its value was not zero. Each ball was still moving downward during the interaction. It was only after the interaction was over that each ball stopped moving and its momentum became zero.

Example 9.3 Practice Problems

1. (a) ***Given***

$\Delta t = 3.64$ s

$\vec{F}_{net} = 200$ N [S]



Required

impulse provided to car

Analysis and Solution

Use the equation of impulse to calculate the impulse provided to the car.

$$\begin{aligned}\text{impulse} &= \vec{F}_{net}\Delta t \\ &= (+200\text{ N})(3.64\text{ s}) \\ &= +728\text{ N}\bullet\text{s} \\ \text{impulse} &= 728\text{ N}\bullet\text{s [S]}\end{aligned}$$

Paraphrase

The impulse provided to the car is 728 N•s [S].

(b) ***Given***

impulse = 728 N•s [S] from part (a)

m = 1100 kg

Required

change in velocity of car ($\Delta\vec{v}$)

Analysis and Solution

Impulse is numerically equal to $m\Delta\vec{v}$.

$$+728\ \text{N}\cdot\text{s} = m\Delta\vec{v}$$

$$\Delta\vec{v} = \frac{+728\ \text{N}\cdot\text{s}}{m}$$

$$= \frac{+728\ \text{N}\cdot\text{s}}{1100\ \text{kg}}$$

$$= \frac{+728\left(\cancel{\text{kg}}\cdot\frac{\text{m}}{\text{s}^{\cancel{2}}}\right)(\cancel{\text{s}})}{1100\ \cancel{\text{kg}}}$$

$$= +0.662\ \text{m/s}$$

$$\Delta\vec{v} = 0.662\ \text{m/s [S]}$$

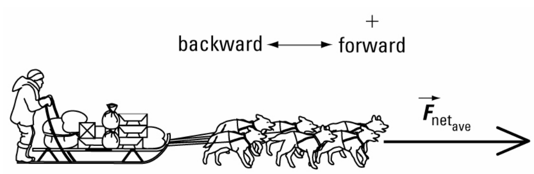
Paraphrase

The change in velocity of the car is 0.662 m/s [S].

2. ***Given***

$m = 400$ kg $\Delta t = 4.20$ s

$\vec{v}_i = 0.200$ m/s [backward] $\vec{v}_f = 1.80$ m/s [forward]



Required

average net force on sled ($\vec{F}_{\text{net}_{\text{ave}}}$)

Analysis and Solution

Use the equation of impulse to calculate the average net force that the dog team exerts on the sled.

$$\vec{F}_{\text{net}_{\text{ave}}}\Delta t = m\Delta\vec{v}$$

$$\vec{F}_{\text{net}_{\text{ave}}} = \frac{m\Delta\vec{v}}{\Delta t}$$

$$= \frac{m(\vec{v}_f - \vec{v}_i)}{\Delta t}$$

$$= \frac{(400\ \text{kg})\{+1.80\ \text{m/s} - (-0.200\ \text{m/s})\}}{4.20\ \text{s}}$$

$$= +190\ \text{kg}\cdot\text{m/s}^2$$

$$\vec{F}_{\text{net}_{\text{ave}}} = 190\ \text{N [forward]}$$

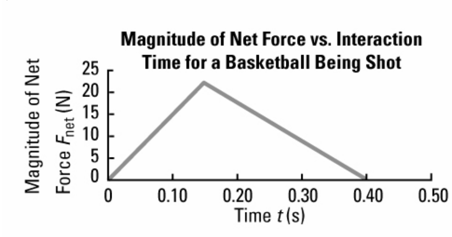
Paraphrase

The average net force on the sled is 190 N [forward].

Student Book page 462

Example 9.4 Practice Problems

1. (a)



(b) and **(c)**

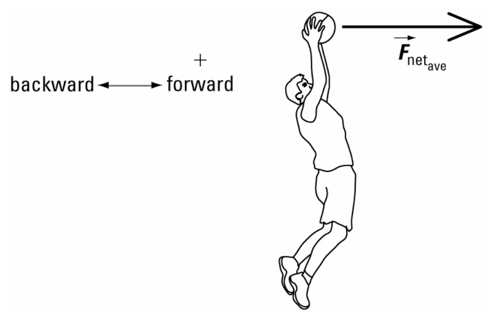
Given

$m = 0.650$ kg

$t_i = 0$ s　　　$t_f = 0.40$ s

$F_{net_i} = 0$ N　　　$F_{net_f} = 0$ N

$F_{net_{max}} = 22$ N



Required

(b) magnitude of impulse provided to basketball

(c) speed of basketball after the interaction (v_f)

(b) ***Analysis and Solution***

The magnitude of the impulse is equal to the area under the net force-time graph. Calculate the time interval.

$$\Delta t = t_f - t_i$$
$$= 0.40 \text{ s} - 0 \text{ s}$$
$$= 0.40 \text{ s}$$

$$\text{magnitude of impulse} = \frac{1}{2}(\Delta t)(F_{net_{max}})$$
$$= \frac{1}{2}(0.40 \text{ s})(22 \text{ N})$$
$$= 4.4 \text{ N•s}$$

Paraphrase

The magnitude of the impulse provided to the basketball is 4.4 N•s.

(c) ***Analysis and Solution***

The impulse and velocity after impact are in the same direction.

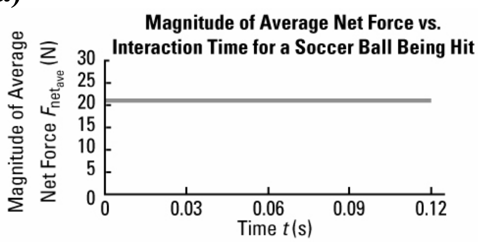
Impulse is numerically equal to $m\Delta\vec{v}$ or $m(\vec{v}_f - \vec{v}_i)$.

$$
\begin{aligned}
+4.4\ \text{N•s} &= m(\vec{v}_f - \vec{v}_i) \\
&= m(\vec{v}_f - 0) \\
&= m\vec{v}_f \\
\vec{v}_f &= \frac{+4.4\ \text{N•s}}{m} \\
&= \frac{+4.4\left(\cancel{\text{kg}}\cdot\frac{\text{m}}{\text{s}^{\cancel{2}}}\right)(\cancel{\text{s}})}{0.650\ \cancel{\text{kg}}} \\
&= +6.8\ \text{m/s} \\
v_f &= 6.8\ \text{m/s}
\end{aligned}
$$

Paraphrase

The final speed of the basketball is 6.8 m/s when it leaves the player's hand.

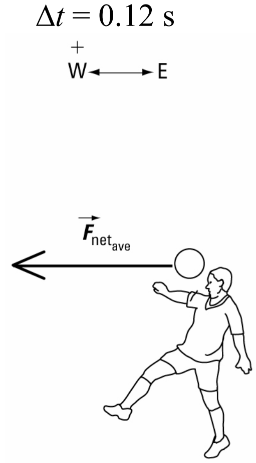
2. (a)



(b) ***Given***

$\vec{F}_{net_{ave}} = 21$ N [W]

$\Delta t = 0.12$ s



Required

impulse provided to soccer ball

Analysis and Solution

The magnitude of the impulse is equal to the area under the net force-time graph.

magnitude of impulse $= (\Delta t)(F_{\text{net}_{\text{max}}})$

$= (0.12\text{ s})(21\text{ N})$

$= 2.5\text{ N•s}$

Impulse is in the direction of the average net force.

impulse $= 2.5\text{ N•s [W]}$

Paraphrase

The impulse provided to the soccer ball is 2.5 N•s [W].

(c) ***Given***

impulse = 2.52 N•s [W] from part (b)

$\vec{v}_i$ = 4.0 m/s [E] $\qquad$ $\vec{v}_f$ = 2.0 m/s [W]

Required

mass of soccer ball (m)

Analysis and Solution

Impulse is numerically equal to $m\Delta\vec{v}$ or $m(\vec{v}_f - \vec{v}_i)$.

$$+2.52\text{ N•s} = m(\vec{v}_f - \vec{v}_i)$$

$$= m\{+2.0\text{ m/s} - (-4.0\text{ m/s})\}$$

$$2.52\text{ N•s} = m(6.0\text{ m/s})$$

$$m = \frac{2.52\text{ N•s}}{6.0\text{ m/s}}$$

$$= \frac{2.52\left(\text{kg•}\frac{\text{m}}{\text{s}^2}\right)(\text{s})}{6.0\ \frac{\text{m}}{\text{s}}}$$

$$= 0.42\text{ kg}$$

Paraphrase

The mass of the soccer ball is 0.42 kg.

Student Book page 467

9.2 Check and Reflect

Knowledge

1. (a) In order to calculate impulse, you need to measure either the net force (or average net force) acting on an object during an interaction and the interaction time, or the mass of the object and its initial velocity at the instant the interaction begins and its final velocity at the instant the interaction ends.

(b) The units of impulse are either newton-seconds (N•s) or kilogram-metres per second (kg•m/s).

2. Impulse is equivalent to the change in momentum in an object.

3. A net force-time graph can be used to determine the impulse provided to an object. To calculate impulse, you need to find the area under the graph.

4. (a) If the time interval is doubled, the magnitude of the impulse will double.

magnitude of impulse $= F_{net}\Delta t$

$= F_{net}\{2(\Delta t)\}$

$= 2(F_{net}\Delta t)$

(b) If the net force is reduced to $\frac{1}{3}$ of its original magnitude, the magnitude of the impulse will be reduced to $\frac{1}{3}$.

magnitude of impulse $= F_{net}\Delta t$

$= \left(\frac{1}{3}F_{net}\right)\Delta t$

$= \left(\frac{1}{3}\right)(F_{net}\Delta t)$

5. Curling stones used in championship games have a maximum mass of 19.96 kg. When a curling stone is moving, it has a lot of inertia because its mass is fairly large. Since a curling stone is a rigid object, the time interval during a collision would be very short. If a stone were to hit your feet, the impact would be very painful since the bones and soft tissue in your feet would have to cause the stone to stop moving in a short time interval.

Applications

6. A karate expert is trained to have very fast reflexes. This allows the expert to provide a very large impulse to the board during a very short time interval. Since the impulse is very large and the time interval of interaction is very short, the net force that the expert applies to the board is very large, resulting in the board being broken.

7. (a) ***Given***

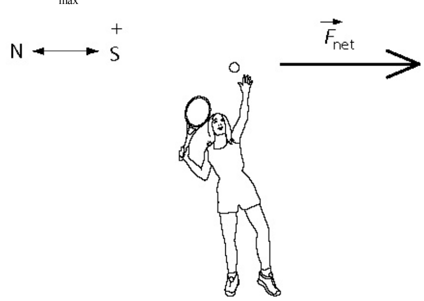
$t_i = 0.5$ ms

$F_{net_i} = 0$ N

$F_{net_{max}} = 900$ N

$t_f = 5.5$ ms

$F_{net_f} = 0$ N



Required

magnitude of impulse provided to tennis ball

Analysis and Solution

The magnitude of the impulse is equal to the area under the net force-time graph. Calculate the time interval.

$$\begin{aligned}\Delta t &= t_f - t_i \\ &= 5.5 \text{ ms} - 0.5 \text{ ms} \\ &= 5.0 \text{ ms or } 5.0 \times 10^{-3} \text{ s}\end{aligned}$$

$$\begin{aligned}\text{magnitude of impulse} &= \frac{1}{2}(\Delta t)(F_{\text{net}_{\text{max}}}) \\ &= \frac{1}{2}(5.0 \times 10^{-3} \text{ s})(900 \text{ N}) \\ &= 2.3 \text{ N•s}\end{aligned}$$

Paraphrase

The magnitude of the impulse provided to the tennis ball is 2.3 N•s.

(b) ***Given***

impulse = 2.25 N•s [forward] from part (a)

m = 48 g

Required

velocity of tennis ball after the interaction ($\vec{v}_f$)

Analysis and Solution

The impulse and velocity after impact are in the same direction.

Impulse is numerically equal to $m\Delta\vec{v}$ or $m(\vec{v}_f - \vec{v}_i)$.

$$\begin{aligned}+2.25 \text{ N•s} &= m(\vec{v}_f - \vec{v}_i) \\ &= m(\vec{v}_f - 0) \\ &= m\vec{v}_f \\ \vec{v}_f &= \frac{+2.25 \text{ N•s}}{m} \\ &= \frac{+2.25\left(\cancel{\text{kg}}\bullet\frac{\text{m}}{\text{s}^{\cancel{2}}}\right)(\cancel{\text{s}})}{(48\ \cancel{\text{g}})\left(\frac{1\ \cancel{\text{kg}}}{1000\ \cancel{\text{g}}}\right)} \\ &= +47 \text{ m/s} \\ \vec{v}_f &= 47 \text{ m/s [S]}\end{aligned}$$

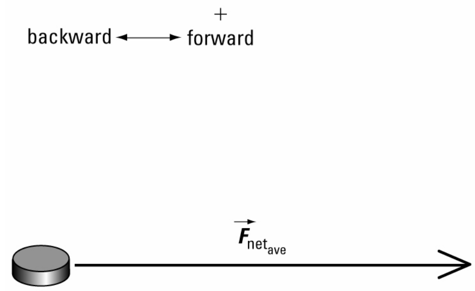
Paraphrase

The velocity of the tennis ball after the interaction is 47 m/s [S].

8. ***Given***

$F_{net_{ave}}$ = 520 N

Δt = 0.012 s



Required

impulse provided to puck

Analysis and Solution

Use the equation of impulse to calculate the impulse provided to the puck.

$$\begin{aligned} \text{impulse} &= \vec{F}_{net_{ave}} \Delta t \\ &= (+520\ \text{N})(0.012\ \text{s}) \\ &= +6.2\ \text{N•s} \\ \text{impulse} &= 6.2\ \text{N•s [forward]} \end{aligned}$$

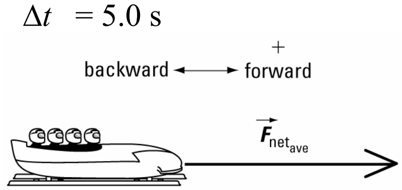
Paraphrase

The impulse provided to the puck is 6.2 N•s [forward].

9. (a) ***Given***

$F_{net_{ave}}$ = 1390 N

Δt = 5.0 s



Required

impulse provided to bobsled

Analysis and Solution

Use the equation of impulse to calculate the impulse provided to the bobsled.

$$\begin{aligned} \text{impulse} &= \vec{F}_{net_{ave}} \Delta t \\ &= (+1390\ \text{N})(5.0\ \text{s}) \\ &= +7.0 \times 10^3\ \text{N•s} \\ \text{impulse} &= 7.0 \times 10^3\ \text{N•s [forward]} \end{aligned}$$

Paraphrase

The impulse provided to the bobsled is 7.0×10^3 N•s [forward].

(b) ***Given***

impulse $= 6.95 \times 10^3$ N•s [forward] from part (a)
$m = 630$ kg

Required

velocity of bobsled after the interaction ($\vec{v}_f$)

Analysis and Solution

The impulse and velocity after impact are in the same direction.
Impulse is numerically equal to $m\Delta\vec{v}$ or $m(\vec{v}_f - \vec{v}_i)$.

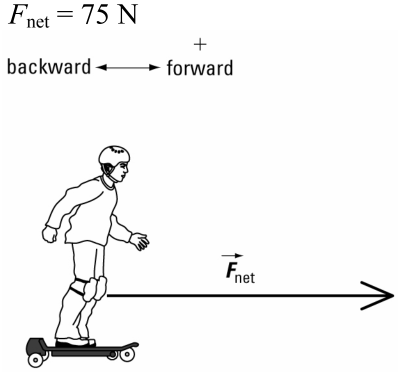
$$+6.95 \times 10^3 \text{ N•s} = m(\vec{v}_f - \vec{v}_i)$$
$$= m(\vec{v}_f - 0)$$
$$= m\vec{v}_f$$
$$\vec{v}_f = \frac{+6.95 \times 10^3 \text{ N•s}}{m}$$
$$= \frac{+6.95 \times 10^3 \left(\cancel{\text{kg}} \bullet \frac{\text{m}}{\text{s}^{\cancel{2}}}\right)(\cancel{\text{s}})}{630 \ \cancel{\text{kg}}}$$
$$= +11 \text{ m/s}$$
$$\vec{v}_f = 11 \text{ m/s [forward]}$$

Paraphrase

The velocity of the bobsled after the interaction is 11 m/s [forward].

10. ***Given***

$m_s = 25$ kg $\quad m_p = 80$ kg
$v_i = 0$ m/s $\quad v_f = 8.5$ m/s
$F_{net} = 75$ N



Required
interaction time (Δt)

Analysis and Solution
The skateboard and person move together as a unit. So calculate the total mass.

$$\begin{aligned} m_T &= m_s + m_p \\ &= 25 \text{ kg} + 80 \text{ kg} \\ &= 105 \text{ kg} \end{aligned}$$

From Newton's third law, if the skateboard exerts a backward net force on the ground, the ground will exert a force of equal magnitude but opposite direction on the skateboard. It is this reaction force acting for the minimum time interval that provides the required impulse to accelerate the skateboard.
Use the equation of impulse to calculate the time interval.

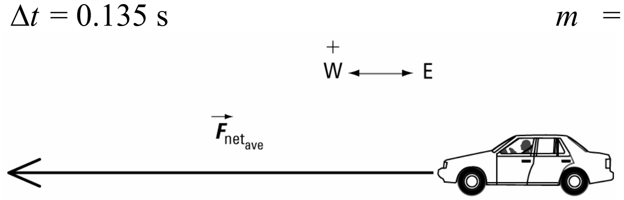
$$\begin{aligned} \vec{F}_{\text{net}} \Delta t &= m\Delta \vec{v} \\ \Delta t &= \frac{m\Delta \vec{v}}{\vec{F}_{\text{net}}} \\ &= \frac{m(\vec{v}_f - \vec{v}_i)}{\vec{F}_{\text{net}}} \\ &= \frac{(105 \text{ kg})(+8.5 \frac{\text{m}}{\text{s}} - 0 \frac{\text{m}}{\text{s}})}{+75 \text{ N}} \\ &= \frac{(105 \cancel{\text{kg}})(+8.5 \cancel{\frac{\text{m}}{\text{s}}})}{+75 \cancel{\text{kg}} \bullet \frac{\cancel{\text{m}}}{\text{s}^{\cancel{2}}}} \\ &= 12 \text{ s} \end{aligned}$$

Paraphrase
It will take 12 s for the skateboard to reach a speed of 8.5 m/s if starting from rest.

11. ***Given***

$\vec{v}_i = 0$ m/s $\vec{v}_f = 14.0$ m/s [W]

$\Delta t = 0.135$ s $m = 5.40$ kg



Required
average net force on neck ($\vec{F}_{\text{net}_{\text{ave}}}$)

Analysis and Solution

From Newton's third law, if the torso exerts a westward average net force on the neck, the neck will exert a force of equal magnitude but opposite direction on the torso. It is this reaction force acting for 0.135 s that provides the impulse during whiplash.

Use the equation of impulse to calculate the average net force that the torso exerts on the neck.

$$\vec{F}_{\text{net}_{\text{ave}}}\Delta t = m\Delta\vec{v}$$

$$\begin{aligned}\vec{F}_{\text{net}_{\text{ave}}} &= \frac{m\Delta\vec{v}}{\Delta t}\\ &= \frac{m(\vec{v}_{\text{f}} - \vec{v}_{\text{i}})}{\Delta t}\\ &= \frac{(5.40\text{ kg})(+14\ \frac{\text{m}}{\text{s}} - 0\ \frac{\text{m}}{\text{s}})}{0.135\text{ s}}\\ &= +560\text{ kg•m/s}^2\\ \vec{F}_{\text{net}_{\text{ave}}} &= 560\text{ N [W]}\end{aligned}$$

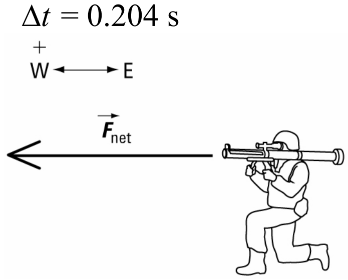
Paraphrase

The average net force that the torso exerts on the neck is 560 N [W].

12. ***Given***

$\vec{F}_{\text{net}} = 2.67$ kN [W]

$\Delta t = 0.204$ s



Required

change in momentum of rocket ($\Delta\vec{p}$)

Analysis and Solution

Impulse is equivalent to change in momentum.

Use the equation of impulse to calculate the change in momentum of the rocket.

$$\begin{aligned}\text{impulse} &= \vec{F}_{\text{net}}\Delta t\\ &= (+2.67 \times 10^3\text{ N})(0.204\text{ s})\\ &= +545\text{ N•s}\\ \text{impulse} &= 545\text{ N•s [W]}\end{aligned}$$

Paraphrase
The change in momentum of the rocket will be 545 N•s [W].

Extensions

13. There are two ways to safely stop a moving curling stone:

- Stop sweeping the ice in front of the stone. Sweeping causes the stone to travel farther because the force of friction acting on the stone decreases. By not sweeping, the friction on the stone increases, which in turn causes the stone to stop sooner.
- Use a rigid object such as the curling broom to slow down the stone. By causing the stone to collide with the broom, the speed of the stone can be decreased more gradually, so the interaction time is increased.

14. For example, a football helmet has cushioning material inside, which compresses when the helmet collides with another object. For a given impulse, the cushioning material increases the interaction time, which results in a decreased net force during impact. The helmet also spreads the net force over the whole head instead of the force being concentrated at a single spot on the head. This tends to decrease the net force during impact at any one point on the head.

Student Book page 473

Concept Check

When a cannon is fired, the expanding gases within the barrel (from the ignited gunpowder) cause the ball to be ejected. From Newton's third law, the expanding gases exert a forward force on the ball, and the ball exerts a force of equal magnitude but opposite direction on the gases. These gases, in turn, exert a force on the cannon, causing the cannon to move backward.

If you analyze the situation in terms of momentum, the cannon and cannon ball form an isolated system. So the law of conservation of momentum applies. Initially, the cannon and ball have a momentum of zero. Once the cannon is fired, the ball has a forward momentum, so the cannon must have a backward momentum of the same magnitude. This backward momentum is large enough to move the cannon away from its original position. That is why ropes were used to tie cannons to the sides of 16th- to 19th-century warships. The ropes restricted the movement of cannons so that the chance of injuring any sailors was minimized and so that the cannons were in position for the next firing.

It is interesting to note that modern battleships do not fire their very large guns simultaneously. The shells are large enough and have enough velocity that, from the conservation of momentum, simultaneous firing has resulted in battleships capsizing.

Student Book page 475

Concept Check

(a) In order for a rocket to orbit Earth, it must exert enough force to counter its weight and reach the appropriate orbital speed. The magnitude of Earth's gravitational field is greater at Earth's surface than in orbit. On the Apollo lunar mission, the first-stage engines were ignited at liftoff and burned for 2.5 min exerting a forward force of 4.5*g*'s. After all fuel was consumed, the first stage was jettisoned at an altitude of 61 km. At this point, the second-stage engines were ignited for about 6 min to reach an altitude of 185 km and a near orbital speed of 2.46×10^4 km/h. Once all the fuel in the second stage was consumed, that stage was jettisoned. The third-stage engine was then ignited and burned for 2.75 min, enough to accelerate the spacecraft to the appropriate orbital speed of 2.82×10^4 km/h. The third-stage engine was then turned off while the spacecraft orbited Earth one to three times. The spacecraft eventually was in the correct position for a lunar trajectory, and at that point the third-stage engine was reignited and burned for slightly more than 5 min. Since the momentum of the rocket is initially zero on the launch pad, the first-stage engines must exert the greatest thrust. Once the rocket is moving at enough velocity, its mass reduced due to the burning of fuel and the jettisoning of stages, and the gravitational field strength is reduced at the location of the rocket, the second-stage engines do not need to supply as much thrust.

(b) A similar reasoning applies to why the third-stage engine supplies the least thrust. The Apollo spacecraft had a near orbital speed of 2.46×10^4 km/h at the moment the third-stage engine was ignited. Since the spacecraft was moving very fast, the magnitude of its momentum was very large. So the third-stage engine did not have to accelerate the rocket as much in order to reach the orbital speed of 2.82×10^4 km/h.

Student Book page 476

Example 9.5 Practice Problems

1. ***Given***

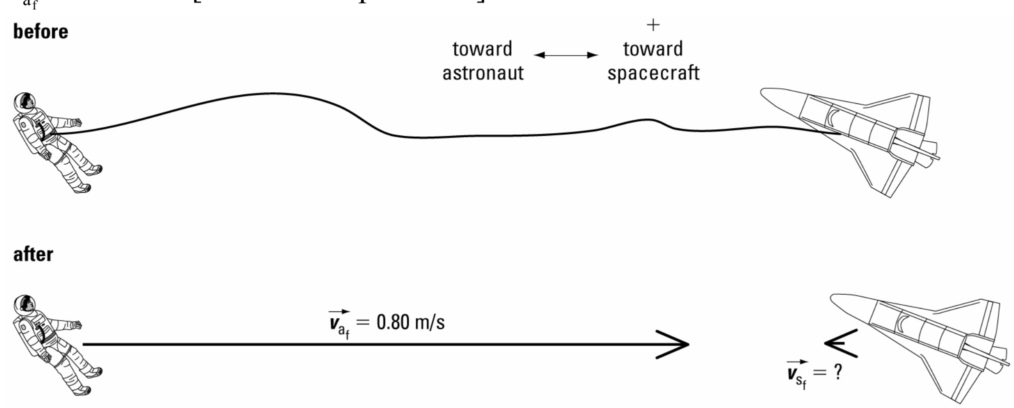
$m_a = 110$ kg

$\vec{v}_{a_i} = 0$ m/s

$\vec{v}_{a_f} = 0.80$ m/s [toward the spacecraft]

$m_s = 4000$ kg

$\vec{v}_{s_i} = 0$ m/s



Required
change in velocity of spacecraft ($\Delta\vec{v}_s$)

Analysis and Solution
Choose the astronaut and spacecraft as an isolated system.
The astronaut and spacecraft each have an initial velocity of zero. So the system has an initial momentum of zero.

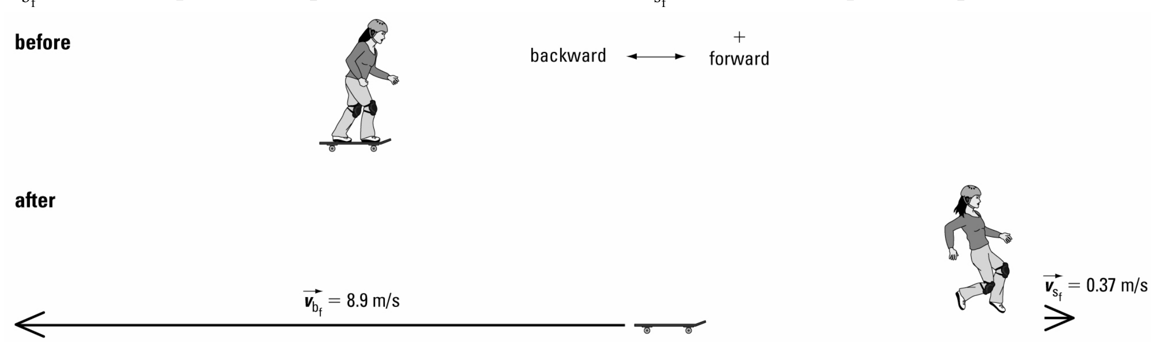
$$\vec{p}_{sys_i} = 0$$

Apply the law of conservation of momentum to the system.

$$\begin{aligned}
\vec{p}_{sys_i} &= \vec{p}_{sys_f} \\
\vec{p}_{sys_i} &= \vec{p}_{a_i} + \vec{p}_{s_i} \\
0 &= m_a\vec{v}_{a_f} + m_s\vec{v}_{s_f} \\
m_s\vec{v}_{s_f} &= -m_a\vec{v}_{a_f} \\
\vec{v}_{s_f} &= -\left(\frac{m_a}{m_s}\right)\vec{v}_{a_f} \\
&= -\left(\frac{110\ \text{kg}}{4000\ \text{kg}}\right)(+0.80\ \text{m/s}) \\
&= -0.022\ \text{m/s} \\
\vec{v}_{s_f} &= 0.022\ \text{m/s [toward the astronaut]} \\
\Delta\vec{v}_s &= 0.022\ \text{m/s [toward the astronaut]}
\end{aligned}$$

Paraphrase and Verify
The change in velocity of the spacecraft will be 0.022 m/s [toward the astronaut] immediately after the astronaut pulls on the cable. The spacecraft is about 36 times more massive than the astronaut. So the change in speed of the spacecraft should be about $\frac{1}{36}$ times the change in speed of the astronaut, which it is.

2. ***Given***
$m_b = 2.3$ kg
$\vec{v}_{b_i} = 0$ m/s
$\vec{v}_{b_f} = 8.9$ m/s [backward]
$\vec{v}_{s_i} = 0$ m/s
$\vec{v}_{s_f} = 0.37$ m/s [forward]



Required
mass of student (m_s)

Analysis and Solution
Choose the student and skateboard as an isolated system.
The student and skateboard each have an initial velocity of zero. So the system has an initial momentum of zero.

$$\vec{p}_{sys_i} = 0$$

Apply the law of conservation of momentum to the system.

$$\begin{aligned}
\vec{p}_{sys_i} &= \vec{p}_{sys_f} \\
\vec{p}_{sys_i} &= \vec{p}_{b_i} + \vec{p}_{s_i} \\
0 &= m_b \vec{v}_{b_f} + m_s \vec{v}_{s_f} \\
m_s \vec{v}_{s_f} &= -m_b \vec{v}_{b_f} \\
m_s(+0.37 \text{ m/s}) &= -(2.3 \text{ kg})(-8.9 \text{ m/s}) \\
m_s(0.37 \text{ m/s}) &= (2.3 \text{ kg})(8.9 \text{ m/s}) \\
m_s &= \left(\frac{8.9 \ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}}{0.37 \ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}}\right)(2.3 \text{ kg}) \\
&= 55 \text{ kg}
\end{aligned}$$

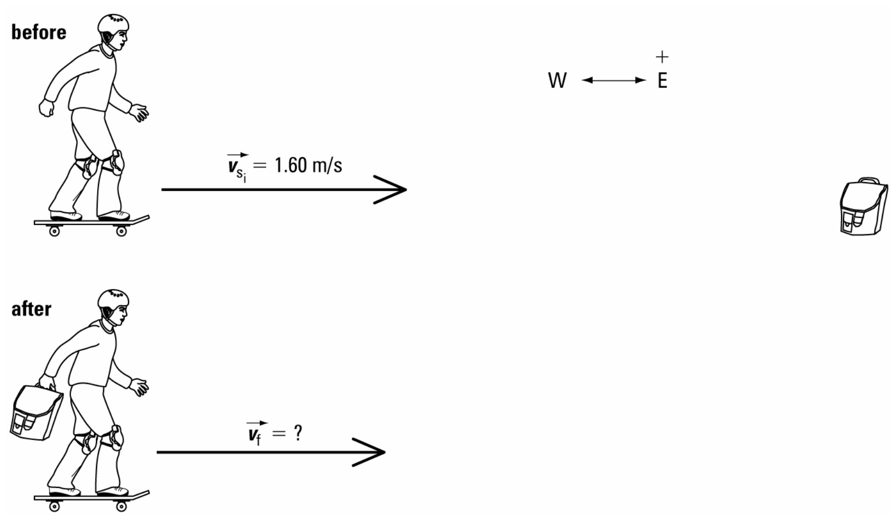
Paraphrase and Verify
The student has a mass of 55 kg. The final speed of the skateboard is about 24 times the final speed of the student. So the mass of the student should be about 24 times the mass of the skateboard, which it is.

Example 9.6 Practice Problems

1. ***Given***

$m_s = 78.2$ kg　　$m_b = 6.4$ kg

$\vec{v}_{s_i} = 1.60$ m/s [E]　　$\vec{v}_{b_i} = 0$ m/s



Required

final velocity of student ($\vec{v}_f$)

Analysis and Solution

Choose the student, skateboard, and backpack as an isolated system.
The student, skateboard, and backpack move together as a unit after the interaction.
The backpack has an initial velocity of zero. So its initial momentum is zero.

$$\vec{p}_{b_i} = 0$$

Apply the law of conservation of momentum to the system.

$$\begin{aligned}
\vec{p}_{sys_i} &= \vec{p}_{sys_f} \\
\vec{p}_{s_i} + \vec{p}_{b_i} &= \vec{p}_{sys_f} \\
m_s\vec{v}_{s_i} + 0 &= (m_s + m_b)\vec{v}_f \\
m_s\vec{v}_{s_i} &= (m_s + m_b)\vec{v}_f \\
\vec{v}_f &= \left(\frac{m_s}{m_s + m_b}\right)\vec{v}_{s_i} \\
&= \left(\frac{78.2 \text{ kg}}{78.2 \text{ kg} + 6.4 \text{ kg}}\right)(+1.60 \text{ m/s}) \\
&= \left(\frac{78.2 \cancel{\text{kg}}}{84.6 \cancel{\text{kg}}}\right)(+1.60 \text{ m/s}) \\
&= +1.5 \text{ m/s}
\end{aligned}$$

$\vec{v}_f = 1.5$ m/s [E]

Paraphrase and Verify

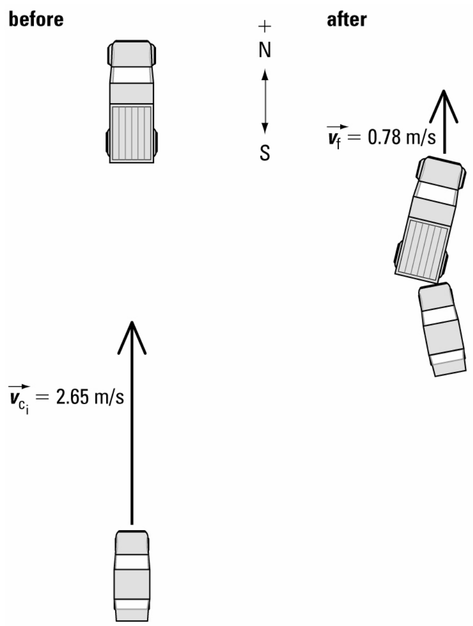
The student will have a velocity of 1.5 m/s [E] immediately after picking up his backpack. Since there is a small increase in mass after the interaction, you would expect that there will be a small decrease in speed, which there is.

2. *Given*

$m_c = 1050$ kg

$\vec{v}_{c_i} = 2.65$ m/s [N] $\qquad \vec{v}_{t_i} = 0$ m/s

$\vec{v}_f = 0.78$ m/s [N]



Required

mass of truck (m_t)

Analysis and Solution

Choose the car and truck as an isolated system.

The car and truck move together as a unit after collision.

The truck has an initial velocity of zero. So its initial momentum is zero.

$$\vec{p}_{t_i} = 0$$

Apply the law of conservation of momentum to the system.

$$\begin{aligned}
\vec{p}_{sys_i} &= \vec{p}_{sys_f} \\
\vec{p}_{c_i} + \vec{p}_{t_i} &= \vec{p}_{sys_f} \\
m_c \vec{v}_{c_i} + 0 &= (m_c + m_t)\vec{v}_f \\
m_c \vec{v}_{c_i} &= m_c \vec{v}_f + m_t \vec{v}_f \\
m_c \vec{v}_{c_i} - m_c \vec{v}_f &= m_t \vec{v}_f \\
m_c(\vec{v}_{c_i} - \vec{v}_f) &= m_t \vec{v}_f
\end{aligned}$$

$$(1050 \text{ kg})(+2.65 \text{ m/s} - 0.78 \text{ m/s}) = m_t(+0.78 \text{ m/s})$$

$$(1050 \text{ kg})(1.87 \text{ m/s}) = m_t(0.78 \text{ m/s})$$

$$m_t = \left(\frac{1.87 \frac{\text{m}}{\text{s}}}{0.78 \frac{\text{m}}{\text{s}}}\right)(1050 \text{ kg})$$

$$= 2.5 \times 10^3 \text{ kg}$$

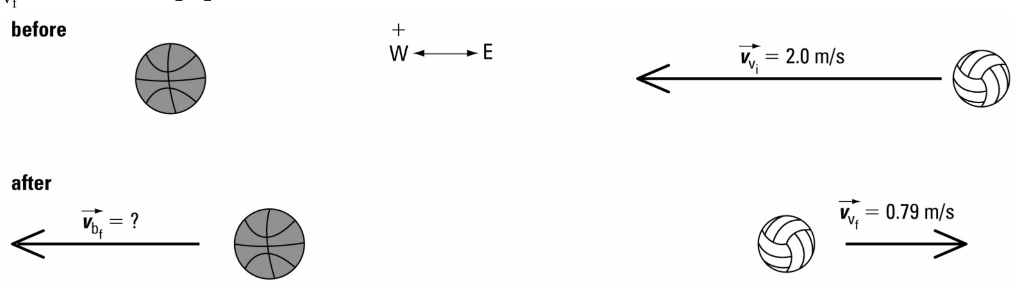
Paraphrase
The truck has a mass of 2.5×10^3 kg.

Student Book page 478

Example 9.7 Practice Problems

1. ***Given***

$m_v = 0.25$ kg
$\vec{v}_{v_i} = 2.0$ m/s [W]
$\vec{v}_{v_f} = 0.79$ m/s [E]
$m_b = 0.58$ kg
$\vec{v}_{b_i} = 0$ m/s



Required
final velocity of basketball ($\vec{v}_{b_f}$)

Analysis and Solution
Choose the volleyball and basketball as an isolated system.
The basketball has an initial velocity of zero. So its initial momentum is zero.

$$\vec{p}_{b_i} = 0$$

Apply the law of conservation of momentum to the system.

$$\vec{p}_{sys_i} = \vec{p}_{sys_f}$$

$$\vec{p}_{v_i} + \vec{p}_{b_i} = \vec{p}_{v_f} + \vec{p}_{b_f}$$

$$m_v \vec{v}_{v_i} + 0 = m_v \vec{v}_{v_f} + m_b \vec{v}_{b_f}$$

$$m_b \vec{v}_{b_f} = m_v \vec{v}_{v_i} - m_v \vec{v}_{v_f}$$

$$\vec{v}_{b_f} = \left(\frac{m_v}{m_b}\right)(\vec{v}_{v_i} - \vec{v}_{v_f})$$

$$= \left(\frac{0.25\ \cancel{kg}}{0.58\ \cancel{kg}}\right)\{+2.0\text{ m/s} - (-0.79\text{ m/s})\}$$

$$= \left(\frac{0.25}{0.58}\right)(2.0\text{ m/s} + 0.79\text{ m/s})$$

$$= +1.2\text{ m/s}$$

$$\vec{v}_{b_f} = 1.2\text{ m/s [W]}$$

Paraphrase

The basketball will have a velocity of 1.2 m/s [W] immediately after impact.

2. ***Given***

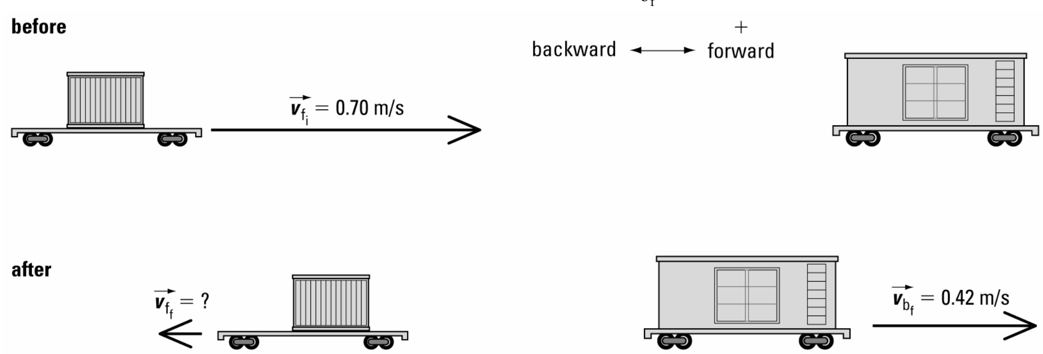
$m_f = 9500$ kg

$\vec{v}_{f_i} = 0.70$ m/s [forward]

$m_b = 18\ 000$ kg

$\vec{v}_{b_i} = 0$ m/s

$\vec{v}_{b_f} = 0.42$ m/s [forward]



Required

final velocity of flatcar ($\vec{v}_{f_f}$)

Analysis and Solution

Choose the flatcar and boxcar as an isolated system.

The boxcar has an initial velocity of zero. So its initial momentum is zero.

$$\vec{p}_{b_i} = 0$$

Apply the law of conservation of momentum to the system.

$$\vec{p}_{sys_i} = \vec{p}_{sys_f}$$

$$\vec{p}_{f_i} + \vec{p}_{b_i} = \vec{p}_{f_f} + \vec{p}_{b_f}$$

$$m_f\vec{v}_{f_i} + 0 = m_f\vec{v}_{f_f} + m_b\vec{v}_{b_f}$$

$$m_f\vec{v}_{f_i} = m_f\vec{v}_{f_f} + m_b\vec{v}_{b_f}$$

$$\vec{v}_{f_i} = \vec{v}_{f_f} + \left(\frac{m_b}{m_f}\right)\vec{v}_{b_f}$$

$$\vec{v}_{f_f} = \vec{v}_{f_i} - \left(\frac{m_b}{m_f}\right)\vec{v}_{b_f}$$

$$= +0.70\text{ m/s} - \left(\frac{18\ 000\ \cancel{\text{kg}}}{9500\ \cancel{\text{kg}}}\right)(+0.42\text{ m/s})$$

$$= -0.096\text{ m/s}$$

$$\vec{v}_{f_f} = 0.096\text{ m/s [backward]}$$

Paraphrase
The flatcar will have a velocity of 0.096 m/s [backward] immediately after collision.

Student Book page 479

Example 9.8 Practice Problems

1. ***Given***

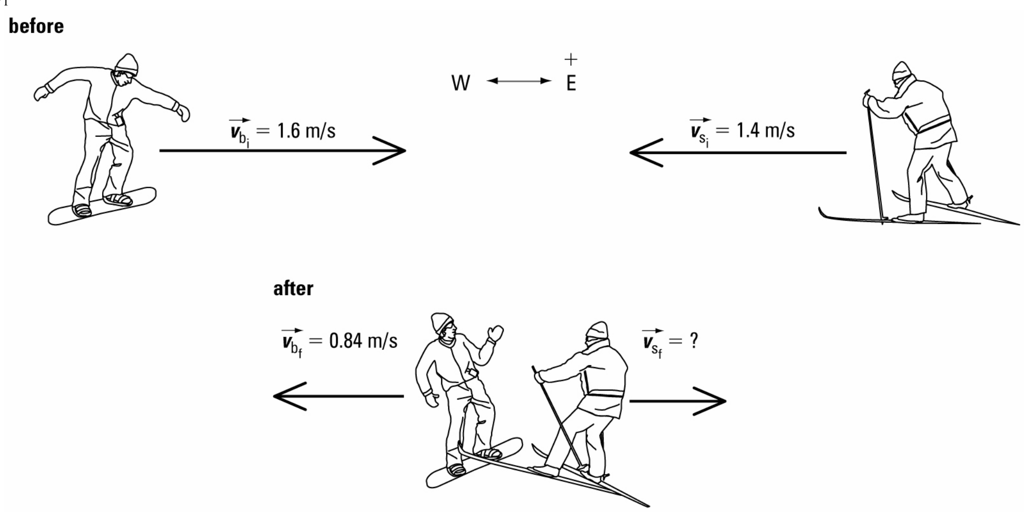
$m_b = 72$ kg

$\vec{v}_{b_i} = 1.6$ m/s [E]

$\vec{v}_{b_f} = 0.84$ m/s [W]

$m_s = 87$ kg

$\vec{v}_{s_i} = 1.4$ m/s [W]



Required
final velocity of skier ($\vec{v}_{s_f}$)

Analysis and Solution
Choose the snowboarder and skier as an isolated system.
Apply the law of conservation of momentum to the system.

$$\vec{p}_{sys_i} = \vec{p}_{sys_f}$$

$$\vec{p}_{b_i} + \vec{p}_{s_i} = \vec{p}_{b_f} + \vec{p}_{s_f}$$

$$m_b\vec{v}_{b_i} + m_s\vec{v}_{s_i} = m_b\vec{v}_{b_f} + m_s\vec{v}_{s_f}$$

$$\left(\frac{m_b}{m_s}\right)\vec{v}_{b_i} + \vec{v}_{s_i} = \left(\frac{m_b}{m_s}\right)\vec{v}_{b_f} + \vec{v}_{s_f}$$

$$\vec{v}_{s_f} = \left(\frac{m_b}{m_s}\right)(\vec{v}_{b_i} - \vec{v}_{b_f}) + \vec{v}_{s_i}$$

$$= \left(\frac{72\ \cancel{kg}}{87\ \cancel{kg}}\right)\{+1.6\text{ m/s} - (-0.84\text{ m/s})\} + (-1.4\text{ m/s})$$

$$= \left(\frac{72}{87}\right)(1.6\text{ m/s} + 0.84\text{ m/s}) - 1.4\text{ m/s}$$

$$= +0.62\text{ m/s}$$

$$\vec{v}_{s_f} = 0.62\text{ m/s [E]}$$

Paraphrase

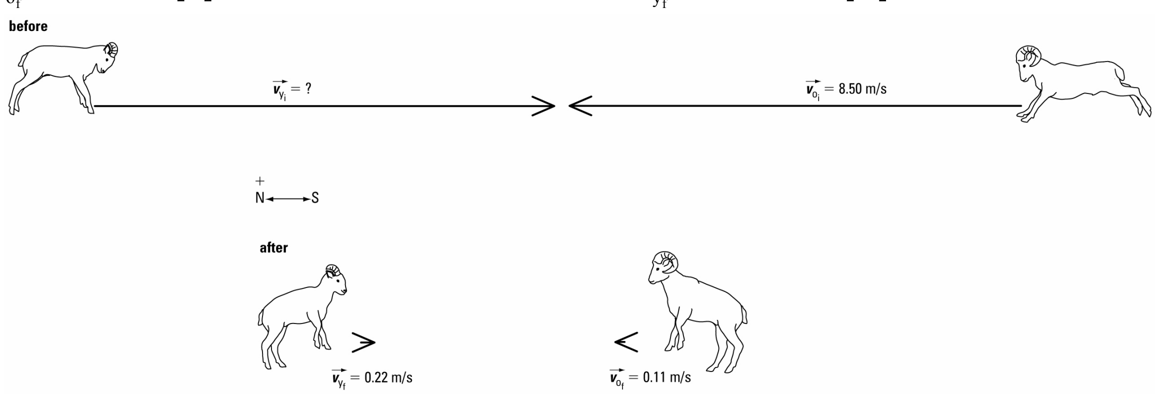
The skier will have a velocity of 0.62 m/s [E] immediately after impact.

2. ***Given***

$m_o = 125$ kg　　$m_y = 122$ kg

$\vec{v}_{o_i} = 8.50$ m/s [N]

$\vec{v}_{o_f} = 0.11$ m/s [S]　　$\vec{v}_{y_f} = 0.22$ m/s [N]



Required

initial velocity of younger ram ($\vec{v}_{y_i}$)

Analysis and Solution

Choose both rams as an isolated system.

Apply the law of conservation of momentum to the system.

$$\vec{p}_{sys_i} = \vec{p}_{sys_f}$$

$$\vec{p}_{o_i} + \vec{p}_{y_i} = \vec{p}_{o_f} + \vec{p}_{y_f}$$

$$m_o\vec{v}_{o_i} + m_y\vec{v}_{y_i} = m_o\vec{v}_{o_f} + m_y\vec{v}_{y_f}$$

$$\left(\frac{m_o}{m_y}\right)\vec{v}_{o_i} + \vec{v}_{y_i} = \left(\frac{m_o}{m_y}\right)\vec{v}_{o_f} + \vec{v}_{y_f}$$

$$\vec{v}_{y_i} = \left(\frac{m_o}{m_y}\right)(\vec{v}_{o_f} - \vec{v}_{o_i}) + \vec{v}_{y_f}$$

$$= \left(\frac{125\ \cancel{kg}}{122\ \cancel{kg}}\right)(-0.11\text{ m/s} - 8.50\text{ m/s}) + 0.22\text{ m/s}$$

$= -8.6 \text{ m/s}$

$\vec{v}_{y_i} = 8.6 \text{ m/s [S]}$

Paraphrase
The younger ram had a velocity of 8.6 m/s [S] immediately before impact.

Student Book page 480

Concept Check

(a) In order for an object to have no momentum, it must be stationary. Such an object could have any form of energy, except kinetic energy at the macroscopic level. For example, the object may have gravitational potential energy, as in the case of a boulder at the edge of a cliff.
(b) If an object has momentum, it must be moving. Any object that is moving has kinetic energy. So it is impossible for an object to have momentum and no energy.

Student Book page 482

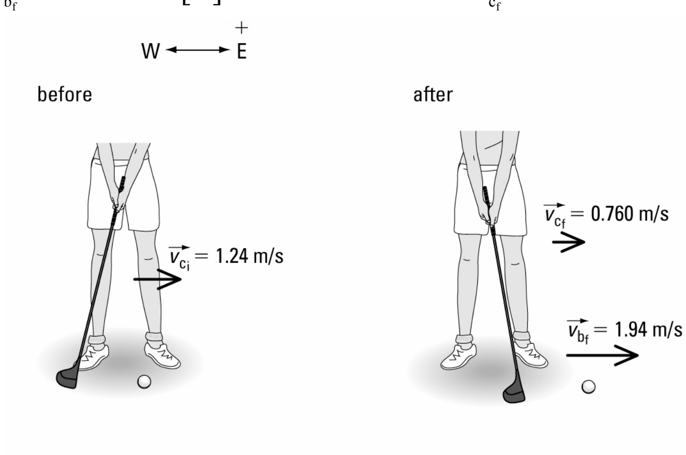
Example 9.9 Practice Problems

1. ***Given***

$m_b = 45.9 \text{ g}$ $\quad m_c = 185 \text{ g}$

$\vec{v}_{b_i} = 0 \text{ m/s}$ $\quad \vec{v}_{c_i} = 1.24 \text{ m/s [E]}$

$\vec{v}_{b_f} = 1.94 \text{ m/s [E]}$ $\quad \vec{v}_{c_f} = 0.760 \text{ m/s [E]}$



Required
determine if the collision is elastic

Analysis and Solution

Convert each mass to kilograms.

$$m_b = 45.9\ \cancel{g} \times \frac{1\ kg}{1000\ \cancel{g}} = 0.0459\ kg \quad \text{and} \quad m_c = 185\ \cancel{g} \times \frac{1\ kg}{1000\ \cancel{g}} = 0.185\ kg$$

Choose the golf ball and golf club as an isolated system.
Calculate the total initial kinetic energy and the total final kinetic energy of the system.

$$E_{k_i} = \frac{1}{2} m_b (v_{b_i})^2 + \frac{1}{2} m_c (v_{c_i})^2$$
$$= 0 + \frac{1}{2}(0.185\ kg)(1.24\ m/s)^2$$
$$= 0.142\ kg \bullet m^2/s^2$$
$$= 0.142\ J$$

$$E_{k_f} = \frac{1}{2} m_b (v_{b_f})^2 + \frac{1}{2} m_c (v_{c_f})^2$$
$$= \frac{1}{2}(0.0459\ kg)(1.94\ m/s)^2 + \frac{1}{2}(0.185\ kg)(0.760\ m/s)^2$$
$$= 0.140\ kg \bullet m^2/s^2$$
$$= 0.140\ J$$

Since $E_{k_i} \neq E_{k_f}$, the collision is not elastic.

Paraphrase

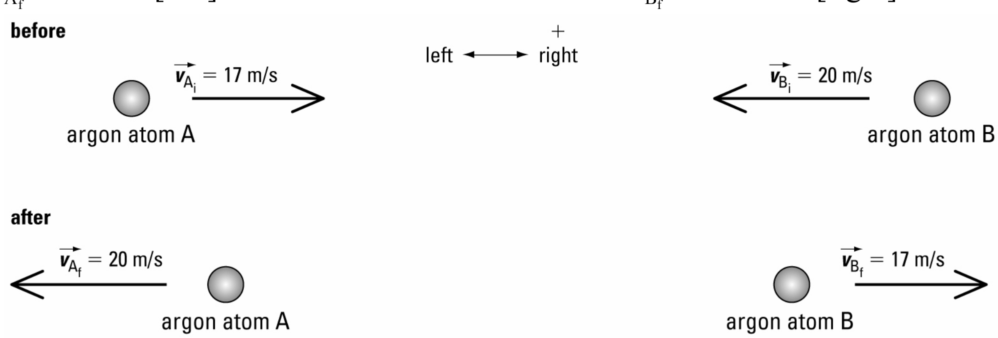
The collision between the golf club face and the golf ball is not elastic.

2. ***Given***

$m_A = 6.63 \times 10^{-23}$ kg $\quad$ $m_B = 6.63 \times 10^{-23}$ kg

$\vec{v}_{A_i} = 17$ m/s [right] $\quad$ $\vec{v}_{B_i} = 20$ m/s [left]

$\vec{v}_{A_f} = 20$ m/s [left] $\quad$ $\vec{v}_{B_f} = 17$ m/s [right]



Required

determine if the collision is elastic

Analysis and Solution

Choose both argon atoms as an isolated system.
Calculate the total initial kinetic energy and the total final kinetic energy of the system.

$$E_{k_i} = \frac{1}{2} m_A (v_{A_i})^2 + \frac{1}{2} m_B (v_{B_i})^2$$
$$= \frac{1}{2}(6.63 \times 10^{-23}\ \text{kg})(17\ \text{m/s})^2 + \frac{1}{2}(6.63 \times 10^{-23}\ \text{kg})(20\ \text{m/s})^2$$

$$E_{k_f} = \frac{1}{2} m_A (v_{A_f})^2 + \frac{1}{2} m_B (v_{B_f})^2$$
$$= \frac{1}{2}(6.63 \times 10^{-23}\ \text{kg})(20\ \text{m/s})^2 + \frac{1}{2}(6.63 \times 10^{-23}\ \text{kg})(17\ \text{m/s})^2$$

By inspection, the numbers in the expression for E_{k_i} are the same as those in the expression for E_{k_f}. So $E_{k_i} = E_{k_f}$, and the collision is elastic.

Paraphrase
The collision between both argon atoms is elastic.

Student Book page 484

Example 9.10 Practice Problems

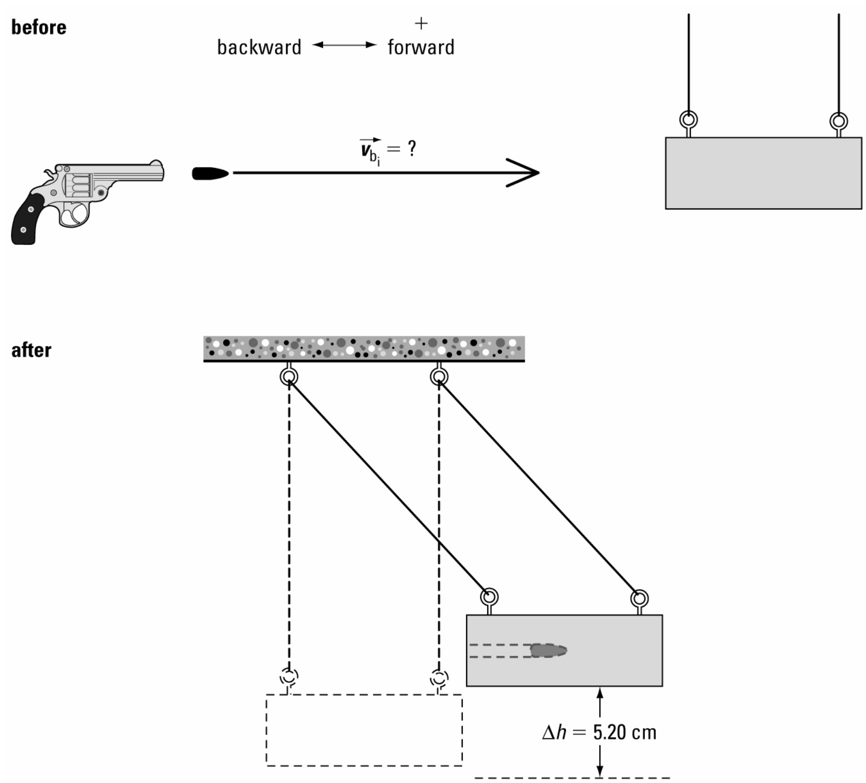
1. ***Given***

$m_b = 2.59$ g

$\Delta h = 5.20$ cm

$m_p = 1.00$ kg

$\vec{v}_{p_i} = 0$ m/s



Required
initial speed of bullet (v_{b_i})

Analysis and Solution
Convert the mass of the bullet to kilograms.

$$m_b = 2.59\ \not{g} \times \frac{1\ \text{kg}}{1000\ \not{g}}$$

$$= 0.00259\ \text{kg}$$

Choose the pendulum and the bullet as an isolated system.
Since the pendulum is stationary before impact, its initial velocity is zero. So its initial momentum is zero.

$$\vec{p}_{p_i} = 0$$

Immediately after collision, the bullet and pendulum move together as a unit. The kinetic energy of the pendulum-bullet system just after impact is converted to gravitational potential energy.

$$E_k = E_p$$

Apply the law of conservation of energy to find the speed of the pendulum-bullet system just after impact.

$$E_k = E_p$$

$$\frac{1}{2}\cancel{(m_b + m_p)}(v_f)^2 = \cancel{(m_b + m_p)}g(\Delta h)$$

$$(v_f)^2 = 2g(\Delta h)$$

$$v_f = \sqrt{2g(\Delta h)}$$

$$= \sqrt{2\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\left(5.20\ \cancel{\text{cm}} \times \frac{1\ \text{m}}{100\ \cancel{\text{cm}}}\right)}$$

$$= 1.010\ \text{m/s}$$

Apply the law of conservation of momentum to the system to find the initial velocity of the bullet.

$$\vec{p}_{sys_i} = \vec{p}_{sys_f}$$

$$\vec{p}_{b_i} + \vec{p}_{p_i} = \vec{p}_{b_f} + \vec{p}_{p_f}$$

$$m_b\vec{v}_{b_i} + 0 = (m_b + m_p)\vec{v}_f$$

$$m_b\vec{v}_{b_i} = (m_b + m_p)\vec{v}_f$$

$$\vec{v}_{b_i} = \left(\frac{m_b + m_p}{m_b}\right)\vec{v}_f$$

$$= \left(\frac{0.00259\ \text{kg} + 1.00\ \text{kg}}{0.00259\ \text{kg}}\right)(+1.010\ \text{m/s})$$

$$= \left(\frac{1.00259\ \cancel{\text{kg}}}{0.00259\ \cancel{\text{kg}}}\right)(+1.010\ \text{m/s})$$

$$= +391 \text{ m/s}$$

$$\vec{v}_{b_i} = 391 \text{ m/s [forward]}$$

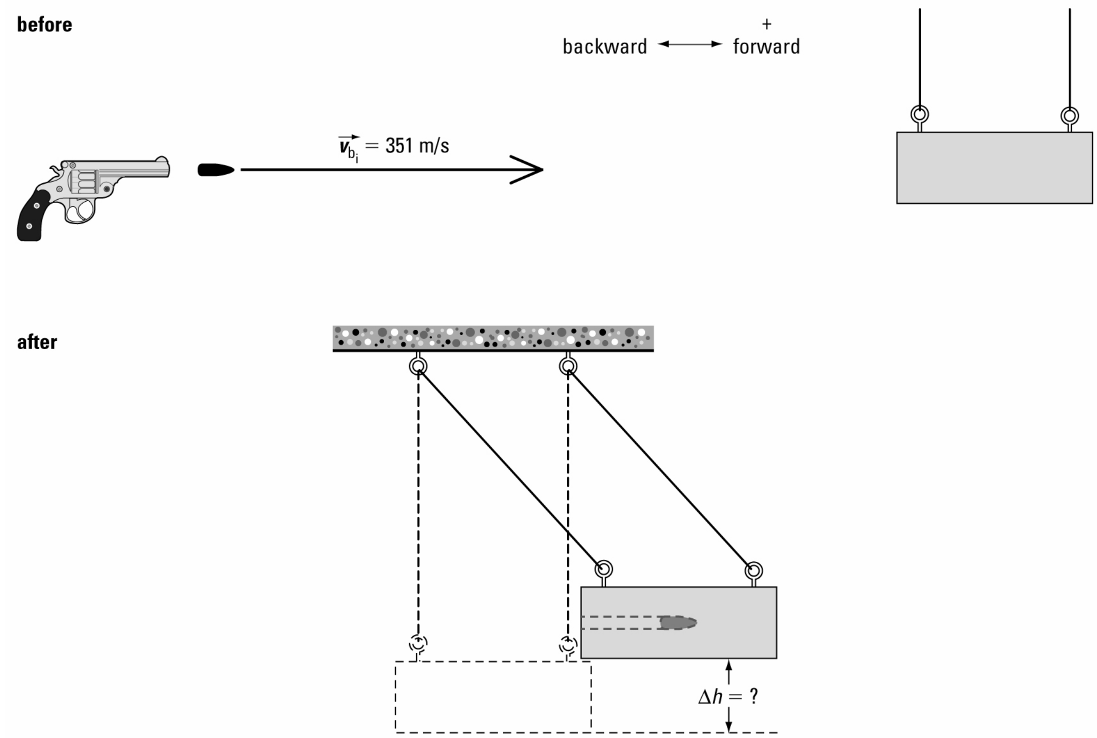
Paraphrase

The initial speed of the bullet immediately before impact was 391 m/s.

2. ***Given***

$m_b = 7.75$ g $m_p = 2.5$ kg

$\vec{v}_{b_i} = 351$ m/s [forward] $\vec{v}_{p_i} = 0$ m/s



Required

height of pendulum swing (Δh)

Analysis and Solution

Convert the mass of the bullet to kilograms.

$$m_b = 7.75 \not{g} \times \frac{1 \text{ kg}}{1000 \not{g}}$$

$$= 0.00775 \text{ kg}$$

Choose the pendulum and the bullet as an isolated system.

Since the pendulum is stationary before impact, its initial velocity is zero. So its initial momentum is zero.

$$\vec{p}_{p_i} = 0$$

Immediately after collision, the bullet and pendulum move together as a unit. Apply the law of conservation of momentum to find the velocity of the pendulum-bullet system just after impact.

$$\vec{p}_{sys_i} = \vec{p}_{sys_f}$$

$$\vec{p}_{b_i} + \vec{p}_{p_i} = \vec{p}_{b_f} + \vec{p}_{p_f}$$

$$m_b \vec{v}_{b_i} + 0 = (m_b + m_p)\vec{v}_f$$

$$m_b \vec{v}_{b_i} = (m_b + m_p)\vec{v}_f$$

$$\vec{v}_f = \left(\frac{m_b}{m_b + m_p}\right)\vec{v}_{b_i}$$

$$= \left(\frac{0.00775 \text{ kg}}{0.00775 \text{ kg} + 2.5 \text{ kg}}\right)(+351 \text{ m/s})$$

$$= \left(\frac{0.00775 \cancel{\text{kg}}}{2.50775 \cancel{\text{kg}}}\right)(+351 \text{ m/s})$$

$$= +1.08 \text{ m/s}$$

The kinetic energy of the pendulum-bullet system just after impact is converted to gravitational potential energy.

$$E_k = E_p$$

$$\frac{1}{2}\cancel{(m_b + m_p)}(v_f)^2 = \cancel{(m_b + m_p)}g(\Delta h)$$

$$(v_f)^2 = 2g(\Delta h)$$

$$\Delta h = \frac{(v_f)^2}{2g}$$

$$= \frac{\left(1.08 \dfrac{\text{m}}{\text{s}}\right)^2}{2\left(9.81 \dfrac{\text{m}}{\text{s}^2}\right)}$$

$$= 0.060 \text{ m}$$

$$= 6.0 \text{ cm}$$

Paraphrase

The pendulum will swing to a height of 6.0 cm immediately after the bullet strikes it.

Student Book page 485

Example 9.11 Practice Problems

1. ***Given***

$m_b = 0.200$ kg

$m_d = 0.012$ kg

$\vec{v}_{b_i} = 0$ m/s

$\vec{v}_{d_i} = 13.78$ m/s [forward] from Example 9.6

$\vec{v}_f = 0.78$ m/s [forward]

Required
amount of kinetic energy lost immediately after impact

Analysis and Solution
The collision between the bullet and the block-glider unit is inelastic, because the bullet becomes embedded in the block.
Calculate the total initial kinetic energy and the total final kinetic energy of the system.

$$\begin{aligned} E_{k_i} &= \frac{1}{2} m_b \left(v_{b_i}\right)^2 + \frac{1}{2} m_d \left(v_{d_i}\right)^2 \\ &= 0 + \frac{1}{2}(0.012 \text{ kg})(13.78 \text{ m/s})^2 \\ &= 1.139 \text{ kg•m}^2/\text{s}^2 \\ &= 1.139 \text{ J} \end{aligned}$$

$$\begin{aligned} E_{k_f} &= \frac{1}{2}(m_b + m_d)\left(v_f\right)^2 \\ &= \frac{1}{2}(0.200 \text{ kg} + 0.012 \text{ kg})(0.78 \text{ m/s})^2 \\ &= 0.064\ 49 \text{ kg•m}^2/\text{s}^2 \\ &= 0.064\ 49 \text{ J} \end{aligned}$$

Find the difference between these energy values.

$$\begin{aligned} \Delta E_k &= E_{k_i} - E_{k_f} \\ &= 1.139 \text{ J} - 0.064\ 49 \text{ J} \\ &= 1.1 \text{ J} \end{aligned}$$

Paraphrase
Immediately after the dart hit the block, 1.1 J of kinetic energy was lost.

2. (a) ***Given***

$m_S = 110$ kg

$\vec{v}_{S_i} = 1.80$ m/s [N]

$\vec{v}_{S_f} = 0.250$ m/s [S]

$m_E = 140$ kg

$\vec{v}_{E_i} = 1.50$ m/s [S]

$\vec{v}_{E_f} = 0.1107$ m/s [N] from Example 9.8

Required
determine if the collision is elastic

Analysis and Solution
Calculate the total initial kinetic energy and the total final kinetic energy of the system.

$$\begin{aligned} E_{k_i} &= \frac{1}{2} m_S \left(v_{S_i}\right)^2 + \frac{1}{2} m_E \left(v_{E_i}\right)^2 \\ &= \frac{1}{2}(110 \text{ kg})(1.80 \text{ m/s})^2 + \frac{1}{2}(140 \text{ kg})(1.50 \text{ m/s})^2 \\ &= 335.7 \text{ kg•m}^2/\text{s}^2 \\ &= 335.7 \text{ J} \end{aligned}$$

$$\begin{aligned} E_{k_f} &= \frac{1}{2} m_S \left(v_{S_f}\right)^2 + \frac{1}{2} m_E \left(v_{E_f}\right)^2 \\ &= \frac{1}{2}(110 \text{ kg})(0.250 \text{ m/s})^2 + \frac{1}{2}(140 \text{ kg})(0.1107 \text{ m/s})^2 \\ &= 4.295 \text{ kg•m}^2/\text{s}^2 \end{aligned}$$

$= 4.295 \text{ J}$

Since $E_{k_i} \neq E_{k_f}$, the collision is inelastic.

Paraphrase

The collision between both football players is inelastic.

(b) ***Analysis and Solution***

Use the values of E_{k_i} and E_{k_f} from part (a) to determine the percent of kinetic energy lost.

$$\begin{aligned}\% E_k \text{ lost} &= \frac{E_{k_i} - E_{k_f}}{E_{k_i}} \times 100\% \\ &= \frac{335.7 \text{ J} - 4.295 \text{ J}}{335.7 \text{ J}} \times 100\% \\ &= 98.7\%\end{aligned}$$

Immediately after impact, 98.7% of the kinetic energy of the system is lost.

Student Book page 486

9.3 Check and Reflect

Knowledge

1. The law of conservation of momentum states that if no external force acts on the objects of a system, the initial momentum of the system is the same as the final momentum of the system.

2. (a) In the context of momentum, an isolated system is a group of objects that does not experience an external force.

(b) If you do not choose an isolated system when solving a momentum problem, the law of conservation of momentum is not valid. This conservation law only applies to isolated systems.

3. An elastic collision is a type of interaction in which the total kinetic energy of the system is conserved. So the total kinetic energy remains constant before and after collision. Two argon atoms colliding with each other is an example of an elastic collision.

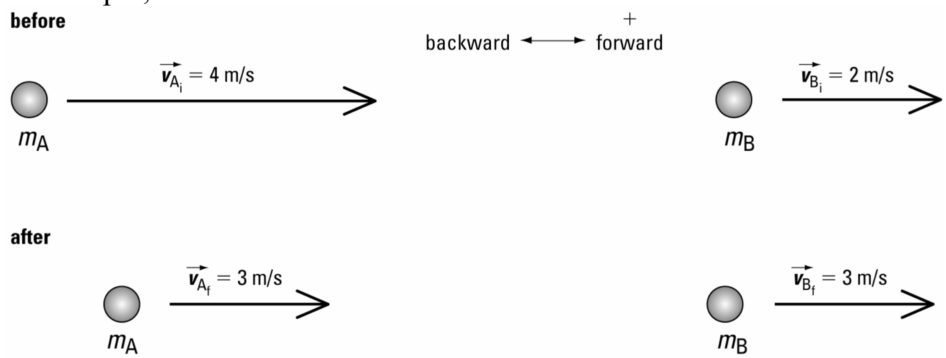
An inelastic collision is a type of interaction in which the total kinetic energy of the system is *not* conserved. In this case, the total initial kinetic energy is *greater* than the total final kinetic energy. When a ball of putty hits a wall and sticks, the collision is inelastic.

4. (a) Evidence that suggests an elastic collision are objects being rigid and bouncing apart after collision without any sound, light, heat, or deformation.

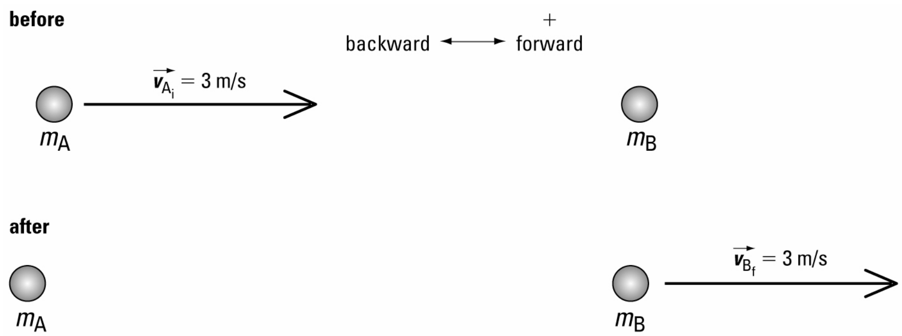
(b) Evidence that suggests an inelastic collision are deformation of one or more of the colliding objects; the colliding objects becoming one unit after collision; and the presence of sound, light, or heat.

Applications

5. For example, first situation:



Second situation:



6. ***Given***

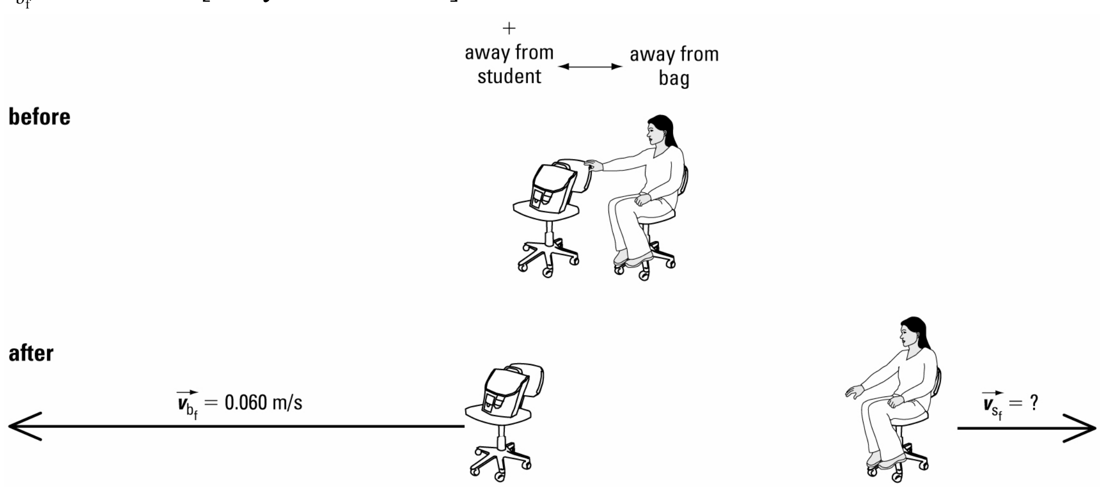
m_b= 20 kg

$\vec{v}_{b_i}$ = 0 m/s

$\vec{v}_{b_f}$ = 0.060 m/s [away from student]

m_s = 65 kg

$\vec{v}_{s_i}$ = 0 m/s



Required

student's final velocity ($\vec{v}_{s_f}$)

Analysis and Solution

Choose the student, her homework bag, and both chairs as an isolated system.

The student, bag, and both chairs each have an initial velocity of zero. So the system has an initial momentum of zero.

$$\vec{p}_{\text{sys}_\text{i}} = 0$$

Apply the law of conservation of momentum to the system.

$$\vec{p}_{\text{sys}_\text{i}} = \vec{p}_{\text{sys}_\text{f}}$$

$$\vec{p}_{\text{sys}_\text{i}} = \vec{p}_{\text{b}_\text{i}} + \vec{p}_{\text{s}_\text{i}}$$

$$0 = m_\text{b}\vec{v}_{\text{b}_\text{f}} + m_\text{s}\vec{v}_{\text{s}_\text{f}}$$

$$m_\text{s}\vec{v}_{\text{s}_\text{f}} = -m_\text{b}\vec{v}_{\text{b}_\text{f}}$$

$$\vec{v}_{\text{s}_\text{f}} = -\left(\frac{m_\text{b}}{m_\text{s}}\right)\vec{v}_{\text{b}_\text{f}}$$

$$= -\left(\frac{20\ \cancel{\text{kg}}}{65\ \cancel{\text{kg}}}\right)(+0.060\text{ m/s})$$

$$= -0.018\text{ m/s}$$

$$\vec{v}_{\text{s}_\text{f}} = 0.018\text{ m/s [away from the bag]}$$

Paraphrase and Verify

The student will have a velocity of 0.018 m/s [away from the bag] immediately after pushing the chair. The student is about 3 times more massive than her bag. So her final speed should be about $\frac{1}{3}$ times the final speed of the bag, which it is.

7. ***Given***

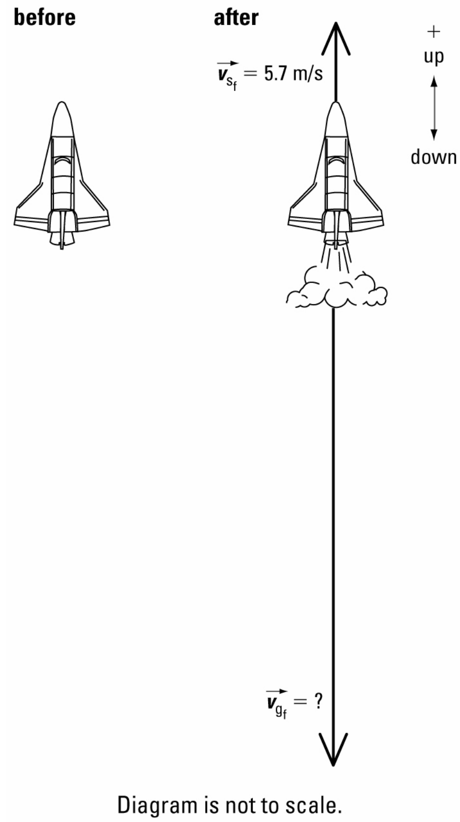
$m_\text{T} = 2.04 \times 10^6$ kg

$\vec{v}_{\text{T}_\text{i}} = 0$ m/s

$\vec{v}_{\text{s}_\text{f}} = 5.7$ m/s [up]

$m_\text{g} = 3.7 \times 10^3$ kg

$\vec{v}_{\text{g}_\text{i}} = 0$ m/s



Required
final velocity of exhaust gas ($\vec{v}_{g_f}$)

Analysis and Solution
Choose the space shuttle and the exhaust gas as an isolated system.
The space shuttle and the exhaust gas each have an initial velocity of zero. So the system has an initial momentum of zero.

$$\vec{p}_{sys_i} = 0$$

After 1 s of liftoff, the total mass of the shuttle changes. Find the new mass of the shuttle.

$$\begin{aligned} m_s &= m_T - m_g \\ &= 2.04 \times 10^6 \text{ kg} - 3.7 \times 10^3 \text{ kg} \\ &= 2.036 \times 10^6 \text{ kg} \end{aligned}$$

Apply the law of conservation of momentum to the system.

$$\begin{aligned} \vec{p}_{sys_i} &= \vec{p}_{sys_f} \\ \vec{p}_{sys_i} &= \vec{p}_{s_i} + \vec{p}_{g_i} \\ 0 &= m_s \vec{v}_{s_f} + m_g \vec{v}_{g_f} \\ m_g \vec{v}_{g_f} &= -m_s \vec{v}_{s_f} \end{aligned}$$

$$\vec{v}_{g_f} = -\left(\frac{m_s}{m_g}\right)\vec{v}_{s_f}$$

$$= -\left(\frac{2.036 \times 10^6\ \text{kg}}{3.7 \times 10^3\ \text{kg}}\right)(+5.7\ \text{m/s})$$

$$= -3.1 \times 10^3\ \text{m/s}$$

$$\vec{v}_{g_f} = 3.1 \times 10^3\ \text{m/s [down]}$$

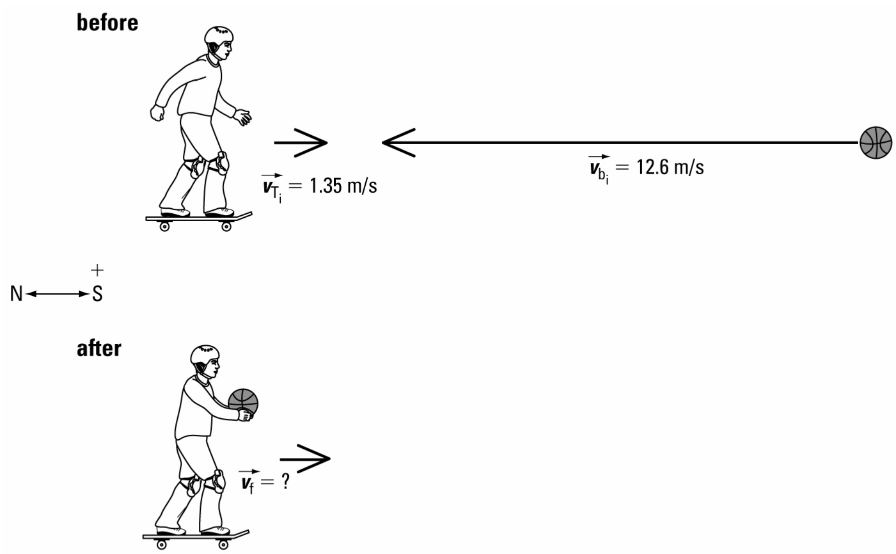
Paraphrase

The exhaust gas will have a velocity of 3.1×10^3 m/s [down] immediately after 1 s of liftoff.

8. ***Given***

$m_s = 60.0$ kg $m_{sk} = 4.2$ kg $m_b = 0.585$ kg

$\vec{v}_{T_i} = 1.35$ m/s [S] $\vec{v}_{b_i} = 12.6$ m/s [N]



Required

final velocity of student-skateboard-basketball combination ($\vec{v}_f$)

Analysis and Solution

The student and the skateboard move at the same velocity. So find the total mass.

$$m_T = m_s + m_{sk}$$

$$m_T = 60.0\ \text{kg} + 4.2\ \text{kg}$$

$$= 64.2\ \text{kg}$$

Choose the student, skateboard, and basketball as an isolated system.

The student, skateboard, and basketball move together as a unit after the interaction.

Apply the law of conservation of momentum to the system.

$$\vec{p}_{sys_i} = \vec{p}_{sys_f}$$

$$\vec{p}_{T_i} + \vec{p}_{b_i} = \vec{p}_{sys_f}$$

$$m_T\vec{v}_{T_i} + m_b\vec{v}_{b_i} = (m_T + m_b)\vec{v}_f$$

$$\vec{v}_f = \left(\frac{1}{m_T + m_b}\right)(m_T \vec{v}_{T_i} + m_b \vec{v}_{b_i})$$

$$= \left(\frac{1}{64.2 \text{ kg} + 0.585 \text{ kg}}\right)\{(64.2 \text{ kg})(+1.35 \text{ m/s}) + (0.585 \text{ kg})(-12.6 \text{ m/s})\}$$

$$= +1.2 \text{ m/s}$$

$$\vec{v}_f = 1.2 \text{ m/s [S]}$$

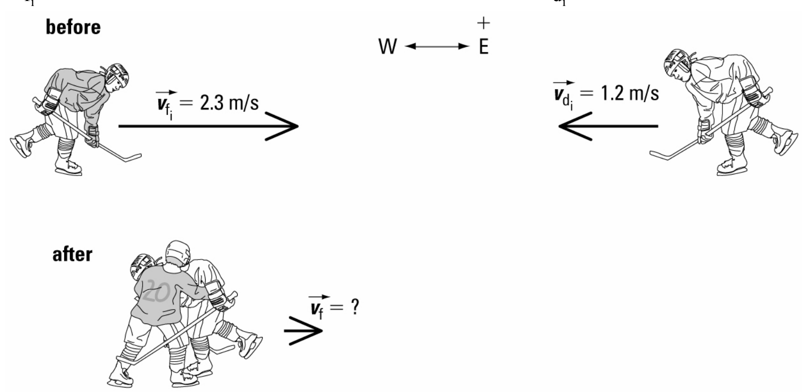
Paraphrase

The student-skateboard-basketball combination will have a velocity of 1.2 m/s [S] immediately after the catch.

9. ***Given***

$m_f = 95$ kg $\qquad m_d = 104$ kg

$\vec{v}_{f_i} = 2.3$ m/s [E] $\qquad \vec{v}_{d_i} = 1.2$ m/s [W]



Required

final velocity of tangle of players ($\vec{v}_f$)

Analysis and Solution

Choose the forward and defenceman as an isolated system.

Both hockey players move together as a unit after the interaction.

Apply the law of conservation of momentum to the system.

$$\vec{p}_{sys_i} = \vec{p}_{sys_f}$$

$$\vec{p}_{f_i} + \vec{p}_{d_i} = \vec{p}_{sys_f}$$

$$m_f \vec{v}_{f_i} + m_d \vec{v}_{d_i} = (m_f + m_d)\vec{v}_f$$

$$\vec{v}_f = \left(\frac{1}{m_f + m_d}\right)(m_f \vec{v}_{f_i} + m_d \vec{v}_{d_i})$$

$$= \left(\frac{1}{95 \text{ kg} + 104 \text{ kg}}\right)\{(95 \text{ kg})(+2.3 \text{ m/s}) + (104 \text{ kg})(-1.2 \text{ m/s})\}$$

$$= +0.47 \text{ m/s}$$

$$\vec{v}_f = 0.47 \text{ m/s [E]}$$

Paraphrase
The tangle of players will have a velocity of 0.47 m/s [E] immediately after impact.

10. (a) ***Given***

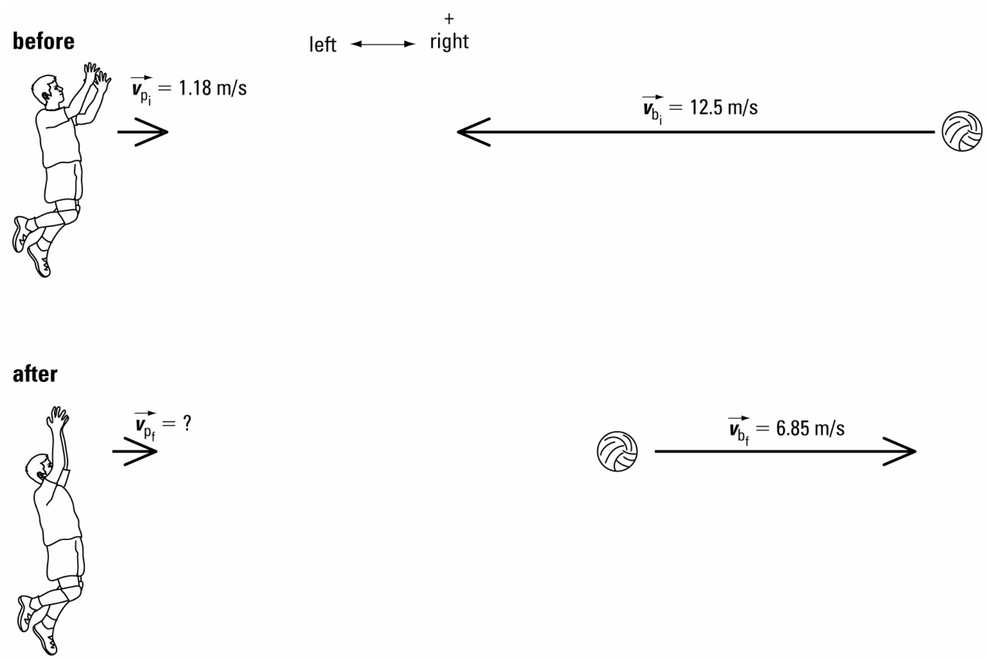
$m_p = 75.6$ kg

$\vec{v}_{p_i} = 1.18$ m/s [right]

$m_b = 0.275$ kg

$\vec{v}_{b_i} = 12.5$ m/s [left]

$\vec{v}_{b_f} = 6.85$ m/s [right]



Required
final velocity of player ($\vec{v}_{p_f}$)

Analysis and Solution
Choose the player and the volleyball as an isolated system.
Apply the law of conservation of momentum to the system.

$$\vec{p}_{sys_i} = \vec{p}_{sys_f}$$

$$\vec{p}_{p_i} + \vec{p}_{b_i} = \vec{p}_{p_f} + \vec{p}_{b_f}$$

$$m_p \vec{v}_{p_i} + m_b \vec{v}_{b_i} = m_p \vec{v}_{p_f} + m_b \vec{v}_{b_f}$$

$$\vec{v}_{p_i} + \left(\frac{m_b}{m_p}\right) \vec{v}_{b_i} = \vec{v}_{p_f} + \left(\frac{m_b}{m_p}\right) \vec{v}_{b_f}$$

$$\vec{v}_{p_f} = \left(\frac{m_b}{m_p}\right)(\vec{v}_{b_i} - \vec{v}_{b_f}) + \vec{v}_{p_i}$$

$$= \left(\frac{0.275 \text{ kg}}{75.6 \text{ kg}}\right)(-12.5 \text{ m/s} - 6.85 \text{ m/s}) + 1.18 \text{ m/s}$$

$$= +1.11 \text{ m/s}$$

$$\vec{v}_{p_f} = 1.11 \text{ m/s [right]}$$

Paraphrase
The player will have a velocity of 1.11 m/s [right] immediately after the block.

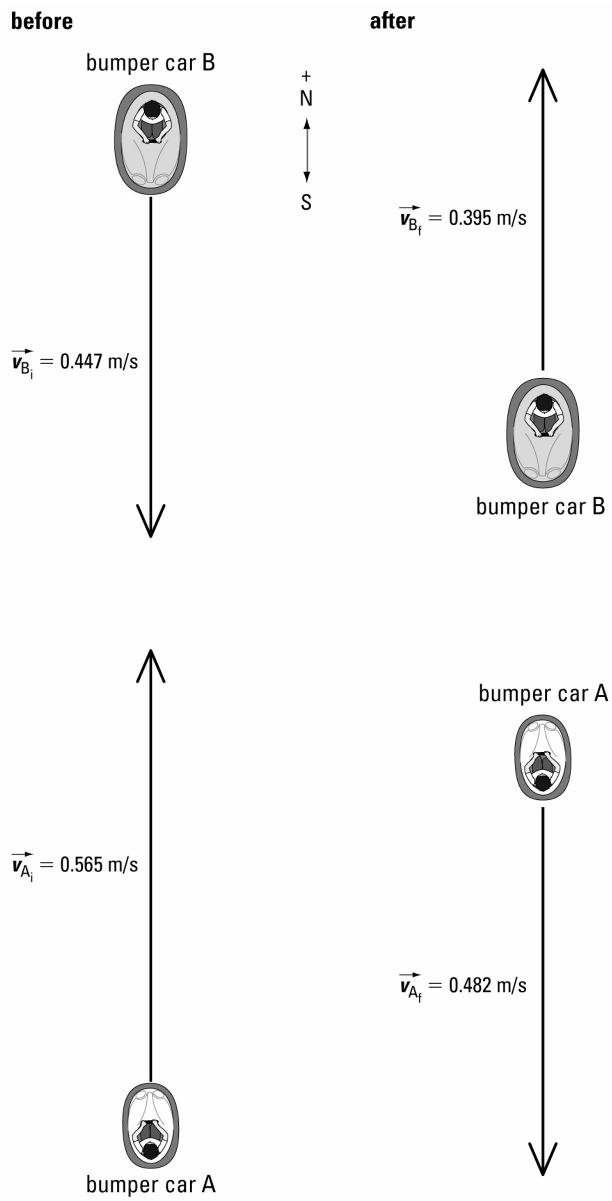
(b) ***Analysis and Solution***
To determine if the collision is elastic, calculate the total initial kinetic energy and the total final kinetic energy of the system.

$$\begin{aligned} E_{k_i} &= \frac{1}{2} m_p \left(v_{p_i}\right)^2 + \frac{1}{2} m_b \left(v_{b_i}\right)^2 \\ &= \frac{1}{2}(75.6 \text{ kg})(1.18 \text{ m/s})^2 + \frac{1}{2}(0.275 \text{ kg})(12.5 \text{ m/s})^2 \\ &= 74.12 \text{ kg}\bullet\text{m}^2/\text{s}^2 \\ &= 74.12 \text{ J} \end{aligned}$$

$$\begin{aligned} E_{k_f} &= \frac{1}{2} m_p \left(v_{p_f}\right)^2 + \frac{1}{2} m_b \left(v_{b_f}\right)^2 \\ &= \frac{1}{2}(75.6 \text{ kg})(1.11 \text{ m/s})^2 + \frac{1}{2}(0.275 \text{ kg})(6.85 \text{ m/s})^2 \\ &= 52.99 \text{ kg}\bullet\text{m}^2/\text{s}^2 \\ &= 52.99 \text{ J} \end{aligned}$$

Since $E_{k_i} \neq E_{k_f}$, the collision between the player and the ball is inelastic.

11. (a) ***Given***
$m_A = 220$ kg
$\vec{v}_{A_i} = 0.565$ m/s [N] $\vec{v}_{B_i} = 0.447$ m/s [S]
$\vec{v}_{A_f} = 0.482$ m/s [S] $\vec{v}_{B_f} = 0.395$ m/s [N]



Required
mass of bumper car B (m_B)

Analysis and Solution
Choose both bumper cars as an isolated system.
Apply the law of conservation of momentum to the system.

$$\vec{p}_{sys_i} = \vec{p}_{sys_f}$$

$$\vec{p}_{A_i} + \vec{p}_{B_i} = \vec{p}_{A_f} + \vec{p}_{B_f}$$

$$m_A \vec{v}_{A_i} + m_B \vec{v}_{B_i} = m_A \vec{v}_{A_f} + m_B \vec{v}_{B_f}$$

$$m_A(\vec{v}_{A_i} - \vec{v}_{A_f}) = m_B(\vec{v}_{B_f} - \vec{v}_{B_i})$$

$$(220 \text{ kg})\{+0.565 \text{ m/s} - (-0.482 \text{ m/s})\} = m_B\{+0.395 \text{ m/s} - (-0.447 \text{ m/s})\}$$

$$(220 \text{ kg})(0.565 \text{ m/s} + 0.482 \text{ m/s}) = m_B(0.395 \text{ m/s} + 0.447 \text{ m/s})$$

$$(220\text{ kg})(1.047\text{ m/s}) = m_B(0.842\text{ m/s})$$

$$m_B = \left(\frac{1.047\ \frac{\cancel{m}}{\cancel{s}}}{0.842\ \frac{\cancel{m}}{\cancel{s}}}\right)(220\text{ kg})$$

$$= 274\text{ kg}$$

Paraphrase

The mass of bumper car B is 274 kg.

(b) ***Analysis and Solution***

To determine if the collision is elastic, calculate the total initial kinetic energy and the total final kinetic energy of the system.

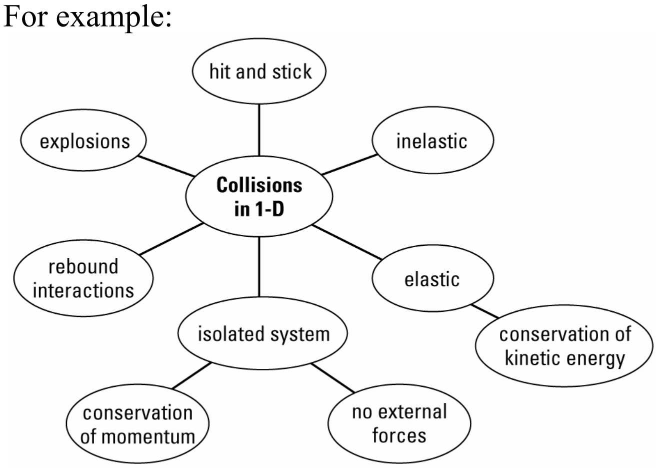
$$\begin{aligned} E_{k_i} &= \frac{1}{2}m_A\left(v_{A_i}\right)^2 + \frac{1}{2}m_B\left(v_{B_i}\right)^2 \\ &= \frac{1}{2}(220\text{ kg})(0.565\text{ m/s})^2 + \frac{1}{2}(273.56\text{ kg})(0.447\text{ m/s})^2 \\ &= 62.44\text{ kg•m}^2/\text{s}^2 \\ &= 62.44\text{ J} \end{aligned}$$

$$\begin{aligned} E_{k_f} &= \frac{1}{2}m_A\left(v_{A_f}\right)^2 + \frac{1}{2}m_B\left(v_{B_f}\right)^2 \\ &= \frac{1}{2}(220\text{ kg})(0.482\text{ m/s})^2 + \frac{1}{2}(273.56\text{ kg})(0.395\text{ m/s})^2 \\ &= 46.90\text{ kg•m}^2/\text{s}^2 \\ &= 46.90\text{ J} \end{aligned}$$

Since $E_{k_i} \neq E_{k_f}$, the collision between both bumper cars is inelastic.

Extensions

12. For example:



Student Book page 490

Concept Check

(a) If a non-zero net force acts on the object, the magnitude of its momentum could decrease, increase, or remain constant.

***p* decreasing and increasing**: If the net force opposes the motion of the object, the object will slow down and its direction of motion may or may not change. For example, when a baseball bat strikes a baseball, the bat exerts a net force on the ball. So for part of the interaction, the magnitude of the momentum of the ball decreases from its initial value (p_i) to zero. At the instant the ball begins to be redirected away from the bat, the magnitude of the momentum of the ball increases from zero to a new value, possibly greater than p_i.

***p* increasing or constant**: If the net force acts in the same direction as the motion of the object, the speed of the object will either remain the same or increase. For example, when a student starts pushing on a skateboard from a stopped position, the net force on the skateboard acts in the same direction as the motion of the skateboard. The speed of the skateboard increases and the magnitude of its momentum also increases. If a small child pushes on the student, the push might not be great enough to change the speed of the skateboard. So the push will not change the momentum of the skateboard.

(b) The momentum of an object can change even though its speed remains the same, if either the mass of the object changes or the motion of the object changes direction. For example, when a billiard ball moving at constant speed rebounds elastically with another object, the speed of the billiard ball immediately after collision will be the same as its speed just before impact. However, the direction of its momentum will be opposite to what it was initially, so its momentum has changed.

Student book page 491

Example 9.12 Practice Problems

1. ***Given***

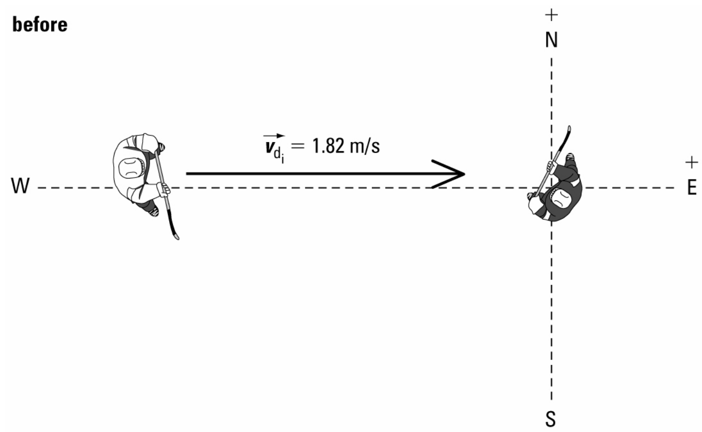
$m_c = 97.0$ kg

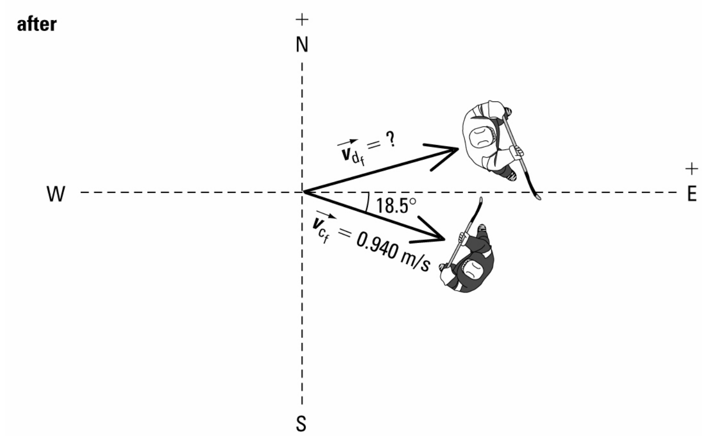
$\vec{v}_{c_i} = 0$ m/s

$\vec{v}_{c_f} = 0.940$ m/s [18.5º S of E]

$m_d = 104$ kg

$\vec{v}_{d_i} = 1.82$ m/s [E]



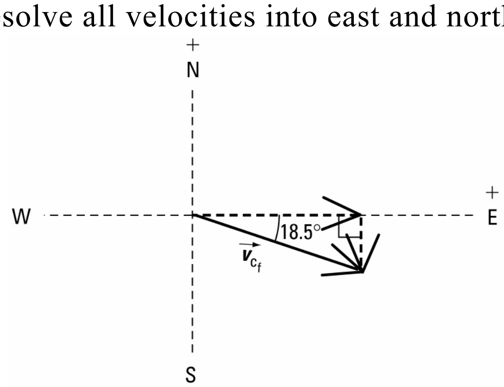


Required
final velocity of defenceman ($\vec{v}_{d_f}$)

Analysis and Solution
Choose the centre and the defenceman as an isolated system.
The centre has an initial velocity of zero. So his initial momentum is zero.

$$\vec{p}_{c_i} = 0$$

Resolve all velocities into east and north components.



Vector	East component	North component
$\vec{v}_{d_i}$	1.82 m/s	0
$\vec{v}_{c_f}$	(0.940 m/s)(cos 18.5°)	–(0.940 m/s)(sin 18.5°)

Apply the law of conservation of momentum to the system in the east and north directions.

E direction

$$p_{sys_{iE}} = p_{sys_{fE}}$$

$$p_{c_{iE}} + p_{d_{iE}} = p_{c_{fE}} + p_{d_{fE}}$$

$$0 + m_d v_{d_{iE}} = m_c v_{c_{fE}} + m_d v_{d_{fE}}$$

$$v_{d_{iE}} = \left(\frac{m_c}{m_d}\right) v_{c_{fE}} + v_{d_{fE}}$$

$$v_{d_{fE}} = v_{d_{iE}} - \left(\frac{m_c}{m_d}\right) v_{c_{fE}}$$

$$= 1.82 \text{ m/s} - \left(\frac{97.0 \text{ kg}}{104 \text{ kg}}\right)\{(0.940 \text{ m/s})(\cos 18.5°)\}$$

$$= 0.9886 \text{ m/s}$$

N direction

$$p_{sys_{iN}} = p_{sys_{fN}}$$

$$p_{c_{iN}} + p_{d_{iN}} = p_{c_{fN}} + p_{d_{fN}}$$

$$0 + 0 = m_c v_{c_{fN}} + m_d v_{d_{fN}}$$

$$m_d v_{d_{fN}} = -m_c v_{c_{fN}}$$

$$v_{d_{fN}} = -\left(\frac{m_c}{m_d}\right) v_{c_{fN}}$$

$$= -\left(\frac{97.0 \text{ kg}}{104 \text{ kg}}\right)\{-(0.940 \text{ m/s})(\sin 18.5°)\}$$

$$= 0.2782 \text{ m/s}$$

Use the Pythagorean theorem to find the magnitude of $\vec{v}_{d_f}$.

$$v_{d_f} = \sqrt{\left(v_{d_{fE}}\right)^2 + \left(v_{d_{fN}}\right)^2}$$

$$= \sqrt{(0.9886 \text{ m/s})^2 + (0.2782 \text{ m/s})^2}$$

$$= 1.03 \text{ m/s}$$

Use the tangent function to find the direction of $\vec{v}_{d_f}$.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

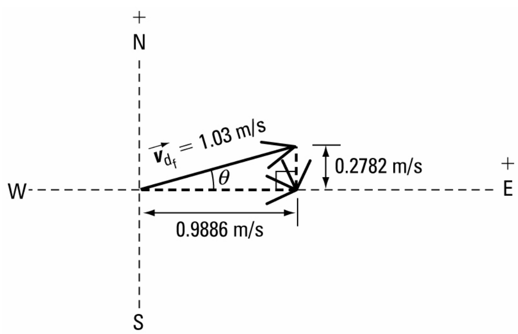
$$= \frac{0.2782\ \cancel{\frac{\text{m}}{\text{s}}}}{0.9886\ \cancel{\frac{\text{m}}{\text{s}}}}$$

$$= 0.2814$$

$$\theta = \tan^{-1}(0.2814)$$

$$= 15.7°$$

From the figure below, this angle is between $\vec{v}_{d_f}$ and the east direction.



$\vec{v}_{d_f}$ = 1.03 m/s [15.7° N of E]

Paraphrase

The velocity of the defenceman is 1.03 m/s [15.7° N of E] immediately after the check.

2. ***Given***

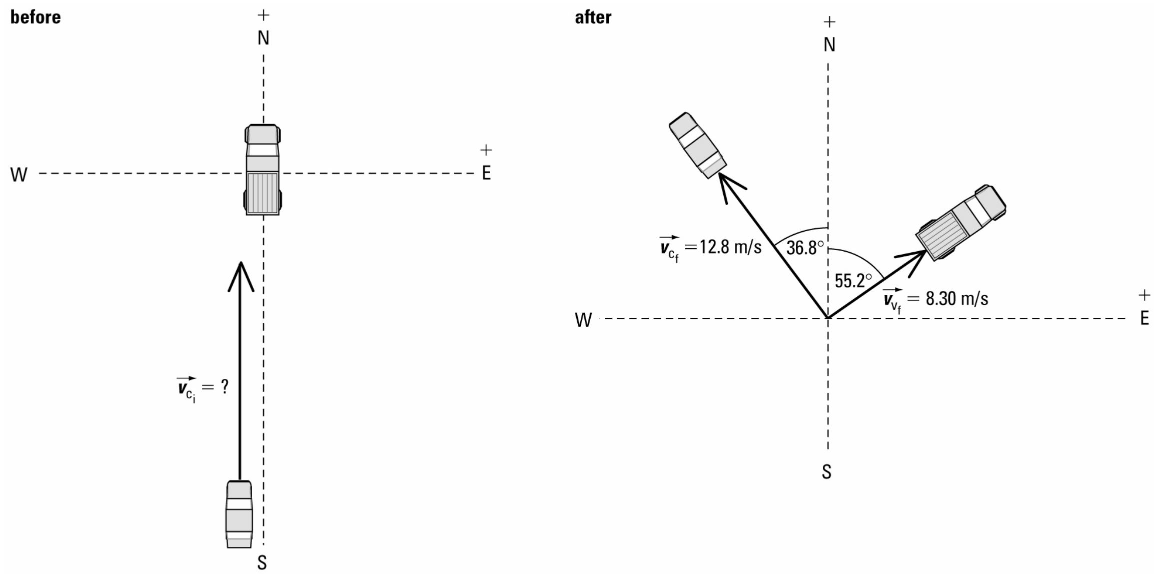
m_c = 1200 kg

m_v = 1350 kg

$\vec{v}_{v_i}$ = 0 m/s

$\vec{v}_{c_f}$ = 12.8 m/s [36.8º W of N]

$\vec{v}_{v_f}$ = 8.30 m/s [55.2º E of N]

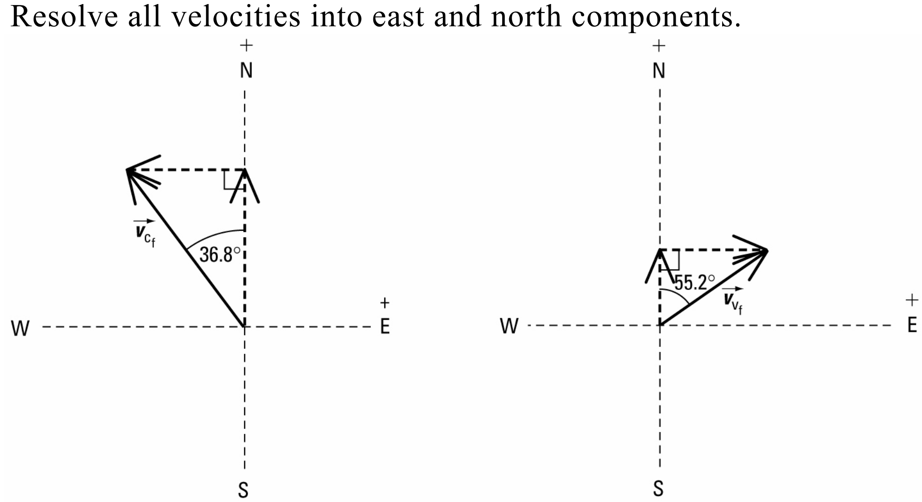


Required
initial velocity of car ($\vec{v}_{c_i}$)

Analysis and Solution
Choose the car and the other vehicle as an isolated system.
The 1350-kg vehicle has an initial velocity of zero. So its initial momentum is zero.

$$\vec{p}_{v_i} = 0$$

Resolve all velocities into east and north components.



Vector	East component	North component
$\vec{v}_{c_f}$	–(12.8 m/s)(sin 36.8°)	(12.8 m/s)(cos 36.8°)
$\vec{v}_{v_f}$	(8.30 m/s)(sin 55.2°)	(8.30 m/s)(cos 55.2°)

Apply the law of conservation of momentum to the system in the east and north directions.

E direction

$$p_{\text{sys}_{\text{i}E}} = p_{\text{sys}_{\text{f}E}}$$

$$p_{\text{c}_{\text{i}E}} + p_{\text{v}_{\text{i}E}} = p_{\text{c}_{\text{f}E}} + p_{\text{v}_{\text{f}E}}$$

$$m_{\text{c}} v_{\text{c}_{\text{i}E}} + 0 = m_{\text{c}} v_{\text{c}_{\text{f}E}} + m_{\text{v}} v_{\text{v}_{\text{f}E}}$$

$$v_{\text{c}_{\text{i}E}} = v_{\text{c}_{\text{f}E}} + \left(\frac{m_{\text{v}}}{m_{\text{c}}}\right) v_{\text{v}_{\text{f}E}}$$

$$= -(12.8 \text{ m/s})(\sin 36.8^{\circ}) + \left(\frac{1350 \cancel{\text{ kg}}}{1200 \cancel{\text{ kg}}}\right)\{(8.30 \text{ m/s})(\sin 55.2^{\circ})\}$$

$$= -2.132 \times 10^{-5} \text{ m/s}$$

N direction

$$p_{\text{sys}_{\text{i}N}} = p_{\text{sys}_{\text{f}N}}$$

$$p_{\text{c}_{\text{i}N}} + p_{\text{v}_{\text{i}N}} = p_{\text{c}_{\text{f}N}} + p_{\text{v}_{\text{f}N}}$$

$$m_{\text{c}} v_{\text{c}_{\text{i}N}} + 0 = m_{\text{c}} v_{\text{c}_{\text{f}N}} + m_{\text{v}} v_{\text{v}_{\text{f}N}}$$

$$v_{\text{c}_{\text{i}N}} = v_{\text{c}_{\text{f}N}} + \left(\frac{m_{\text{v}}}{m_{\text{c}}}\right) v_{\text{v}_{\text{f}N}}$$

$$= (12.8 \text{ m/s})(\cos 36.8^{\circ}) + \left(\frac{1350 \cancel{\text{ kg}}}{1200 \cancel{\text{ kg}}}\right)\{(8.30 \text{ m/s})(\cos 55.2^{\circ})\}$$

$$= 15.58 \text{ m/s}$$

Use the Pythagorean theorem to find the magnitude of $\vec{v}_{\text{c}_{\text{i}}}$.

$$v_{\text{c}_{\text{i}}} = \sqrt{\left(v_{\text{c}_{\text{i}E}}\right)^2 + \left(v_{\text{c}_{\text{i}N}}\right)^2}$$

$$= \sqrt{(-2.132 \times 10^{-5} \text{ m/s})^2 + (15.58 \text{ m/s})^2}$$

$$= 15.6 \text{ m/s}$$

Use the tangent function to find the direction of $\vec{v}_{\text{c}_{\text{i}}}$.

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{15.58 \cancel{\frac{\text{m}}{\text{s}}}}{2.132 \times 10^{-5} \cancel{\frac{\text{m}}{\text{s}}}}$$

$$= 730\ 694$$

$$\theta = \tan^{-1}(730\ 694)$$

$$= 90.0^{\circ}$$

$\vec{v}_{c_i} = 15.6 \text{ m/s [N]}$

Paraphrase
The velocity of the car was 15.6 m/s [N] just before collision.

Student Book page 492

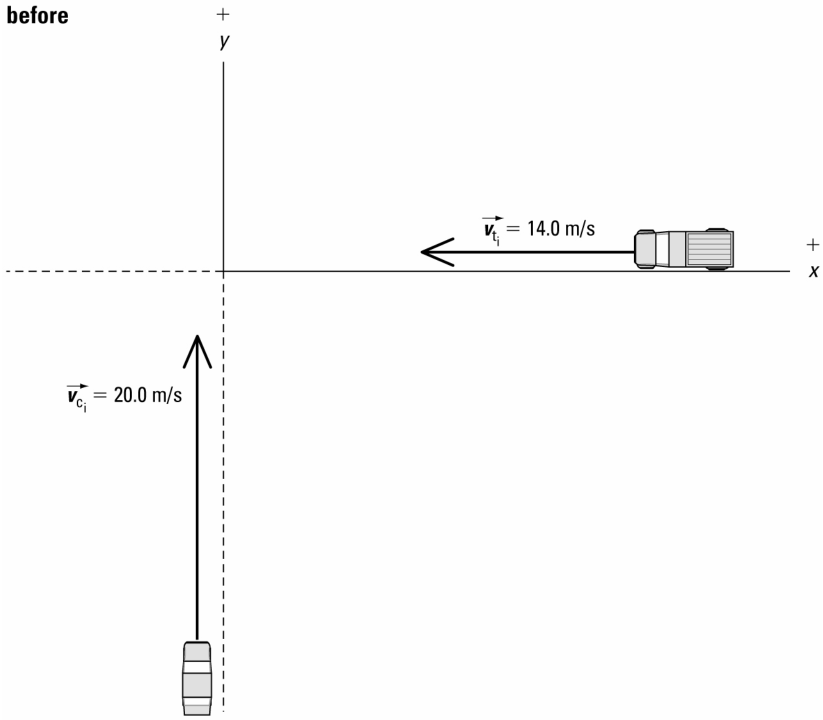
Example 9.13 Practice Problems

1. ***Given***

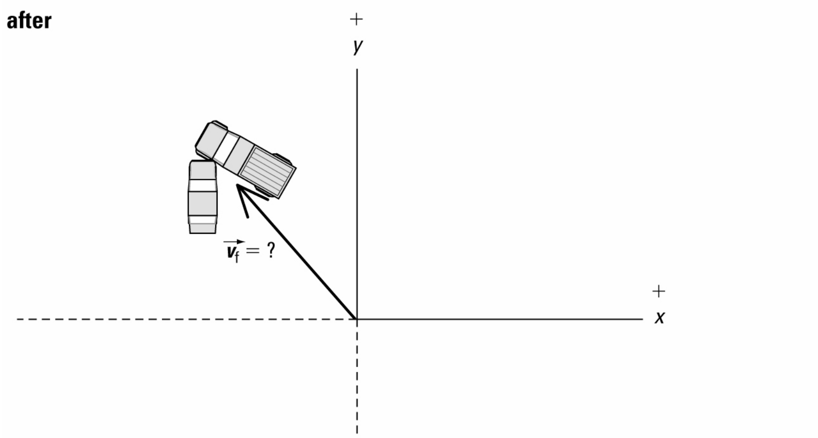
$m_c = 2000 \text{ kg}$ $m_t = 2500 \text{ kg}$

$\vec{v}_{c_i} = 20.0 \text{ m/s [90.0º]}$ $\vec{v}_{t_i} = 14.0 \text{ m/s [180º]}$

before



after



Required
final velocity of centre of mass of both vehicles ($\vec{v}_f$)

Analysis and Solution
Choose the car and the truck as an isolated system.
Both vehicles stick together after collision. So they will have the same final velocity.
Resolve all velocities into x and y components.

Vector	x component	y component
$\vec{v}_{c_i}$	0	20.0 m/s
$\vec{v}_{t_i}$	–(14.0 m/s)	0

Apply the law of conservation of momentum to the system in the x and y directions.

x direction

$$\begin{aligned}
p_{sys_{ix}} &= p_{sys_{fx}} \\
p_{c_{ix}} + p_{t_{ix}} &= p_{sys_{fx}} \\
0 + m_t v_{t_{ix}} &= (m_c + m_t)\, v_{f_x} \\
v_{f_x} &= \left(\frac{m_t}{m_c + m_t}\right) v_{t_{ix}} \\
&= \left(\frac{2500\ \text{kg}}{2000\ \text{kg} + 2500\ \text{kg}}\right) \{-(14.0\ \text{m/s})\} \\
&= \left(\frac{2500\ \cancel{\text{kg}}}{4500\ \cancel{\text{kg}}}\right) \{-(14.0\ \text{m/s})\} \\
&= -7.778\ \text{m/s}
\end{aligned}$$

y direction

$$\begin{aligned}
p_{sys_{iy}} &= p_{sys_{fy}} \\
p_{c_{iy}} + p_{t_{iy}} &= p_{sys_{fy}} \\
m_c v_{c_{iy}} + 0 &= (m_c + m_t)\, v_{f_y} \\
v_{f_y} &= \left(\frac{m_c}{m_c + m_t}\right) v_{c_{iy}} \\
&= \left(\frac{2000\ \text{kg}}{2000\ \text{kg} + 2500\ \text{kg}}\right) (20.0\ \text{m/s}) \\
&= \left(\frac{2000\ \cancel{\text{kg}}}{4500\ \cancel{\text{kg}}}\right) (20.0\ \text{m/s}) \\
&= 8.889\ \text{m/s}
\end{aligned}$$

Use the Pythagorean theorem to find the magnitude of $\vec{v}_{\mathrm{f}}$.

$$v_{\mathrm{f}} = \sqrt{\left(v_{\mathrm{f}_x}\right)^2 + \left(v_{\mathrm{f}_y}\right)^2}$$

$$= \sqrt{(-7.778 \text{ m/s})^2 + (8.889 \text{ m/s})^2}$$

$$= 11.8 \text{ m/s}$$

Use the tangent function to find the direction of $\vec{v}_{\mathrm{f}}$.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{8.889 \ \frac{\text{m}}{\text{s}}}{7.778 \ \frac{\text{m}}{\text{s}}}$$

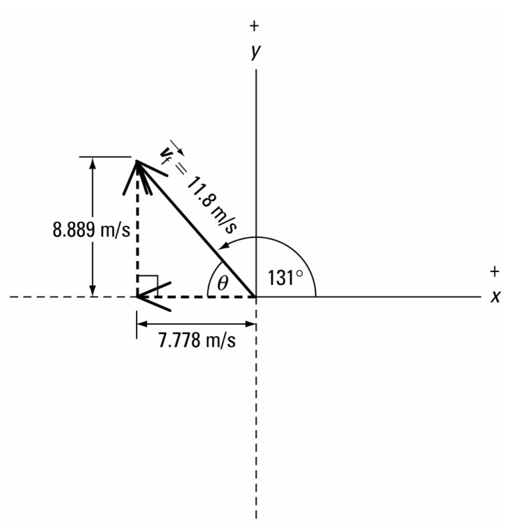
$$= 1.143$$

$$\theta = \tan^{-1}(1.143)$$

$$= 48.8°$$

From the figure below, this angle is between $\vec{v}_{\mathrm{f}}$ and the negative x-axis. So the direction of $\vec{v}_{\mathrm{f}}$ measured *counterclockwise* from the positive x-axis is

180° – 48.8° = 131°.



$$\vec{v}_{\mathrm{f}} = 11.8 \text{ m/s } [131°]$$

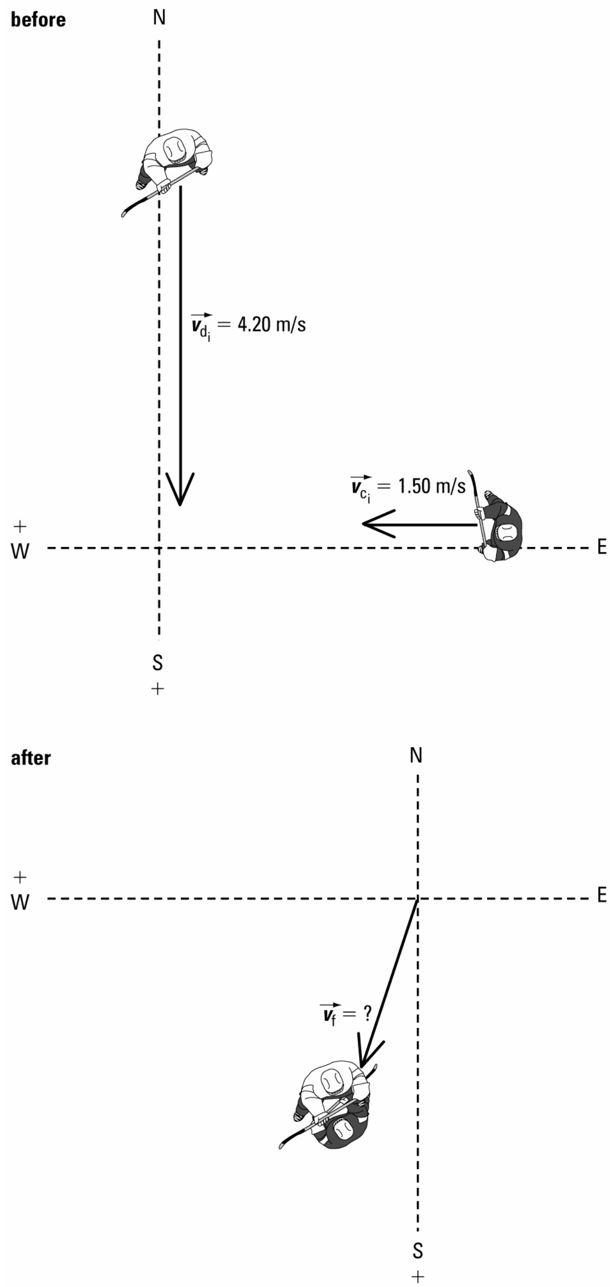
Paraphrase

The velocity of the centre of mass of both vehicles will be 11.8 m/s [131°] immediately after collision.

2. *Given*

$m_c = 100$ kg
$m_d = 108$ kg

$\vec{v}_{c_i} = 1.50$ m/s [W]
$\vec{v}_{d_i} = 4.20$ m/s [S]



Required

final velocity of centre of mass of both players ($\vec{v}_f$)

Analysis and Solution

Choose the centre and the defenceman as an isolated system.

Both players move together after collision. So they will have the same final velocity.

Resolve all velocities into west and south components.

Vector	West component	South component
$\vec{v}_{c_i}$	1.50 m/s	0
$\vec{v}_{d_i}$	0	4.20 m/s

Apply the law of conservation of momentum to the system in the west and south directions.

W direction

$$\begin{aligned} p_{sys_{iW}} &= p_{sys_{fW}} \\ p_{c_{iW}} + p_{d_{iW}} &= p_{sys_{fW}} \\ m_c v_{c_{iW}} + 0 &= (m_c + m_d) v_{fW} \\ v_{fW} &= \left(\frac{m_c}{m_c + m_d}\right) v_{c_{iW}} \\ &= \left(\frac{100 \text{ kg}}{100 \text{ kg} + 108 \text{ kg}}\right)(1.50 \text{ m/s}) \\ &= \left(\frac{100 \not{\text{kg}}}{208 \not{\text{kg}}}\right)(1.50 \text{ m/s}) \\ &= 0.7212 \text{ m/s} \end{aligned}$$

S direction

$$\begin{aligned} p_{sys_{iS}} &= p_{sys_{fS}} \\ p_{c_{iS}} + p_{d_{iS}} &= p_{sys_{fS}} \\ 0 + m_d v_{d_{iS}} &= (m_c + m_d) v_{fS} \\ v_{fS} &= \left(\frac{m_d}{m_c + m_d}\right) v_{d_{iS}} \\ &= \left(\frac{108 \text{ kg}}{100 \text{ kg} + 108 \text{ kg}}\right)(4.20 \text{ m/s}) \\ &= \left(\frac{108 \not{\text{kg}}}{208 \not{\text{kg}}}\right)(4.20 \text{ m/s}) \\ &= 2.181 \text{ m/s} \end{aligned}$$

Use the Pythagorean theorem to find the magnitude of $\vec{v}_f$.

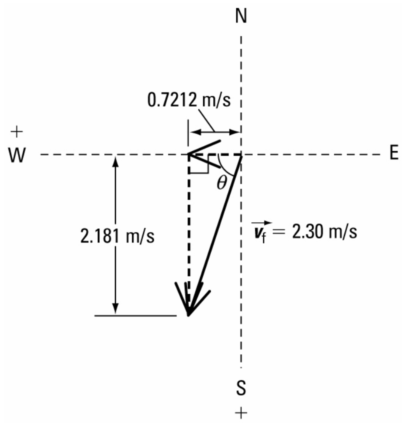
$$\begin{aligned} v_f &= \sqrt{(v_{fW})^2 + (v_{fS})^2} \\ &= \sqrt{(0.7212 \text{ m/s})^2 + (2.181 \text{ m/s})^2} \\ &= 2.30 \text{ m/s} \end{aligned}$$

Use the tangent function to find the direction of $\vec{v}_f$.

$$\begin{aligned} \tan\theta &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{2.181 \not{\frac{\text{m}}{\text{s}}}}{0.7212 \not{\frac{\text{m}}{\text{s}}}} \\ &= 3.024 \end{aligned}$$

$\theta = \tan^{-1}(3.024)$
$= 71.7°$

From the figure below, this angle is between $\vec{v}_f$ and the west direction.



$\vec{v}_f = 2.30$ m/s [71.7° S of W]

Paraphrase
The velocity of the centre of mass of both players will be 2.30 m/s [71.7° S of W] immediately after the check.

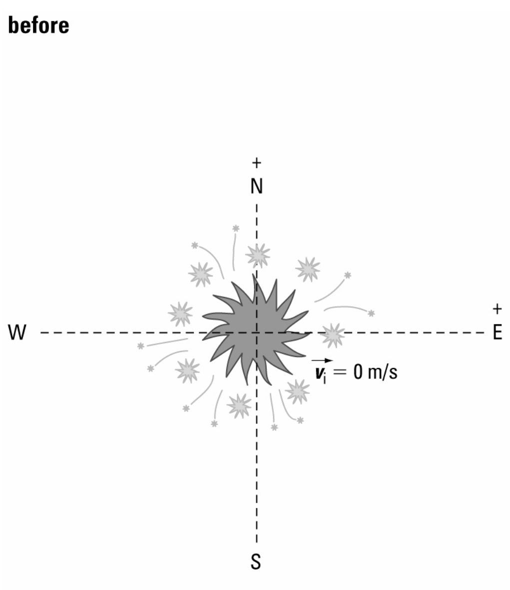
Student Book page 494

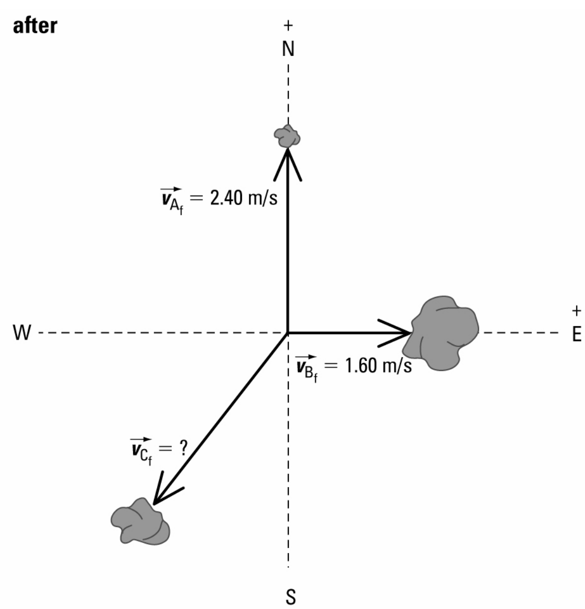
Example 9.14 Practice Problems

1. ***Given***

$m_T = 0.058$ kg $m_A = 0.018$ kg $m_B = 0.021$ kg

$\vec{v}_{T_i} = 0$ m/s $\vec{v}_{A_f} = 2.40$ m/s [N] $\vec{v}_{B_f} = 1.60$ m/s [E]





Required
final velocity of fragment C ($\vec{v}_{C_f}$)

Analysis and Solution
Choose the original firecracker and the three fragments as an isolated system. Since no mass is lost, find the mass of fragment C.

$$\begin{aligned} m_C &= m_T - (m_A + m_B) \\ &= 0.058 \text{ kg} - (0.018 \text{ kg} + 0.021 \text{ kg}) \\ &= 0.019 \text{ kg} \end{aligned}$$

The firecracker has an initial velocity of zero. So the system has an initial momentum of zero.

$$\vec{p}_{sys_i} = 0$$

The momentum of each fragment is in the same direction as its velocity. Calculate the momentum of fragments A and B.

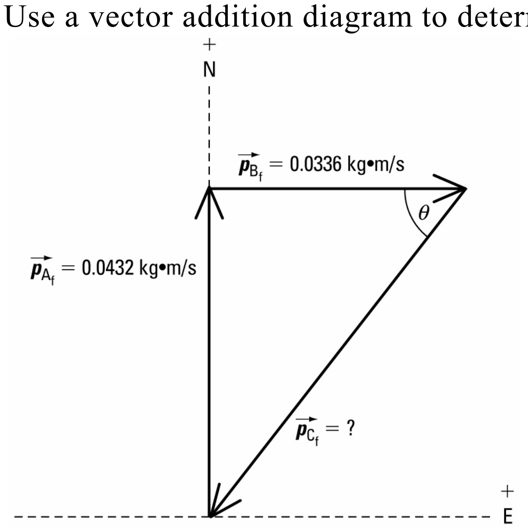
$$\begin{aligned} p_{A_f} &= m_A v_{A_f} \\ &= (0.018 \text{ kg})(2.40 \text{ m/s}) \\ &= 0.0432 \text{ kg•m/s} \\ \vec{p}_{A_f} &= 0.0432 \text{ kg•m/s [N]} \end{aligned}$$

$$\begin{aligned} p_{B_f} &= m_B v_{B_f} \\ &= (0.021 \text{ kg})(1.60 \text{ m/s}) \\ &= 0.0336 \text{ kg•m/s} \\ \vec{p}_{B_f} &= 0.0336 \text{ kg•m/s [E]} \end{aligned}$$

Apply the law of conservation of momentum to the system.

$$\begin{aligned} \vec{p}_{sys_i} &= \vec{p}_{sys_f} \\ 0 &= \vec{p}_{A_f} + \vec{p}_{B_f} + \vec{p}_{C_f} \end{aligned}$$

Use a vector addition diagram to determine the momentum of fragment C.



From the figure above, careful measurements give p_{C_f} = 0.0547 kg•m/s and θ = 52.1° S of W.

Divide the momentum of fragment C by its mass to find the velocity.

$$p_{C_f} = m_c v_{C_f}$$

$$v_{C_f} = \frac{p_{C_f}}{m_C}$$

$$= \frac{0.0547\ \cancel{kg} \bullet \frac{m}{s}}{0.019\ \cancel{kg}}$$

$$= 2.9\ m/s$$

$$\vec{v}_{C_f} = 2.9\ m/s\ [52.1°\ S\ of\ W]$$

Paraphrase

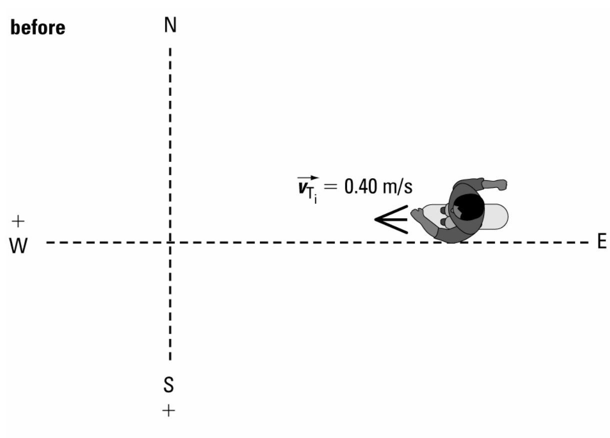
The velocity of the third fragment is 2.9 m/s [52.1º S of W] immediately after the explosion.

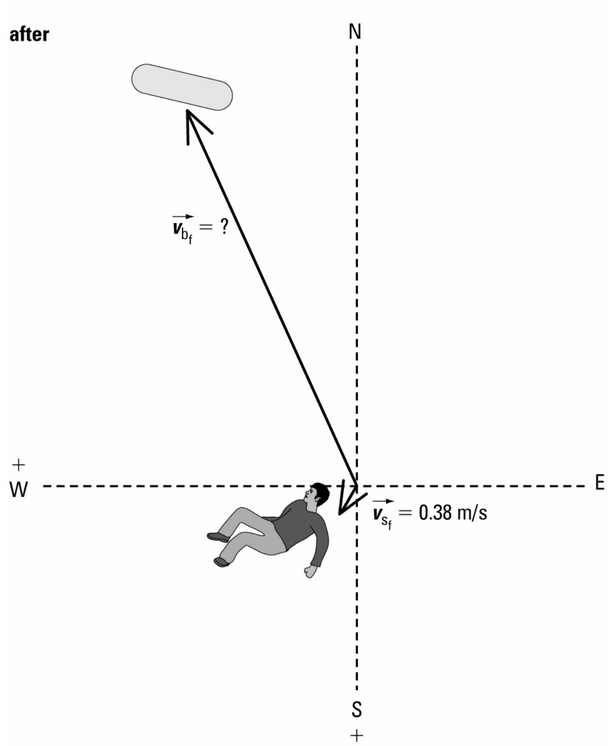
2. ***Given***

$m_s = 65.2$ kg $m_b = 2.50$ kg

$\vec{v}_{T_i} = 0.40$ m/s [W]

$\vec{v}_{s_f} = 0.38$ m/s [30.0° S of W]



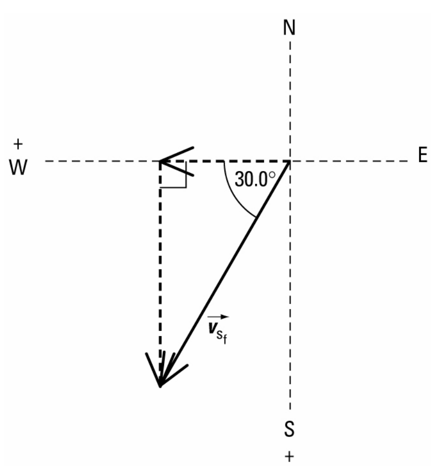


Required
final velocity of skateboard ($\vec{v}_{b_f}$)

Analysis and Solution
Choose the student and skateboard as an isolated system.
The student and skateboard have the same initial velocity. So calculate the total mass.

$$\begin{aligned} m_T &= m_s + m_b \\ &= 65.2 \text{ kg} + 2.50 \text{ kg} \\ &= 67.70 \text{ kg} \end{aligned}$$

Resolve all velocities into west and south components.



Vector	West component	South component
$\vec{v}_{T_i}$	0.40 m/s	0
$\vec{v}_{s_f}$	(0.38 m/s)(cos 30.0°)	(0.38 m/s)(sin 30.0°)

Apply the law of conservation of momentum to the system in the west and south directions.

W direction

$$
\begin{aligned}
p_{\text{sys}_{\text{i}W}} &= p_{\text{sys}_{\text{f}W}} \\
p_{\text{sys}_{\text{i}W}} &= p_{\text{s}_{\text{f}W}} + p_{\text{b}_{\text{f}W}} \\
m_{\text{T}} v_{\text{T}_{\text{i}W}} &= m_{\text{s}} v_{\text{s}_{\text{f}W}} + m_{\text{b}} v_{\text{b}_{\text{f}W}} \\
\left(\frac{m_{\text{T}}}{m_{\text{b}}}\right) v_{\text{T}_{\text{i}W}} &= \left(\frac{m_{\text{s}}}{m_{\text{b}}}\right) v_{\text{s}_{\text{f}W}} + v_{\text{b}_{\text{f}W}} \\
v_{\text{b}_{\text{f}W}} &= \left(\frac{m_{\text{T}}}{m_{\text{b}}}\right) v_{\text{T}_{\text{i}W}} - \left(\frac{m_{\text{s}}}{m_{\text{b}}}\right) v_{\text{s}_{\text{f}W}} \\
&= \left(\frac{67.70\ \cancel{\text{kg}}}{2.50\ \cancel{\text{kg}}}\right)(0.40\ \text{m/s}) - \left(\frac{65.2\ \cancel{\text{kg}}}{2.50\ \cancel{\text{kg}}}\right)\{(0.38\ \text{m/s})(\cos 30.0°)\} \\
&= 2.25\ \text{m/s}
\end{aligned}
$$

S direction

$$
\begin{aligned}
p_{\text{sys}_{\text{i}S}} &= p_{\text{sys}_{\text{f}S}} \\
p_{\text{sys}_{\text{i}S}} &= p_{\text{s}_{\text{f}S}} + p_{\text{b}_{\text{f}S}} \\
0 &= m_{\text{s}} v_{\text{s}_{\text{f}S}} + m_{\text{b}} v_{\text{b}_{\text{f}S}} \\
m_{\text{b}} v_{\text{b}_{\text{f}S}} &= -m_{\text{s}} v_{\text{s}_{\text{f}S}} \\
v_{\text{b}_{\text{f}S}} &= -\left(\frac{m_{\text{s}}}{m_{\text{b}}}\right) v_{\text{s}_{\text{f}S}} \\
&= -\left(\frac{65.2\ \cancel{\text{kg}}}{2.50\ \cancel{\text{kg}}}\right)\{(0.38\ \text{m/s})(\sin 30.0°)\} \\
&= -4.96
\end{aligned}
$$

Use the Pythagorean theorem to find the magnitude of $\vec{v}_{\text{b}_\text{f}}$.

$$
\begin{aligned}
v_{\text{b}_\text{f}} &= \sqrt{\left(v_{\text{b}_{\text{f}W}}\right)^2 + \left(v_{\text{b}_{\text{f}S}}\right)^2} \\
&= \sqrt{(2.25\ \text{m/s})^2 + (-4.96\ \text{m/s})^2} \\
&= 5.4\ \text{m/s}
\end{aligned}
$$

Use the tangent function to find the direction of $\vec{v}_{\text{b}_\text{f}}$.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

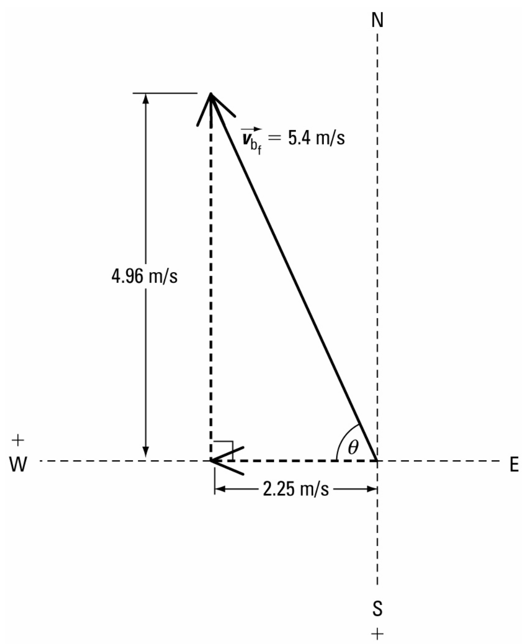
$$= \frac{4.96\ \cancel{\frac{m}{s}}}{2.25\ \cancel{\frac{m}{s}}}$$

$$= 2.203$$

$$\theta = \tan^{-1}(2.203)$$

$$= 65.6°$$

From the figure below, this angle is between $\vec{v}_{b_f}$ and the west direction.



$$\vec{v}_{b_f} = 5.4 \text{ m/s [65.6° N of W]}$$

Paraphrase
The velocity of the skateboard will be 5.4 m/s [65.6° N of W] immediately after the jump.

Student Book page 497

Example 9.15 Practice Problems

1. ***Given***

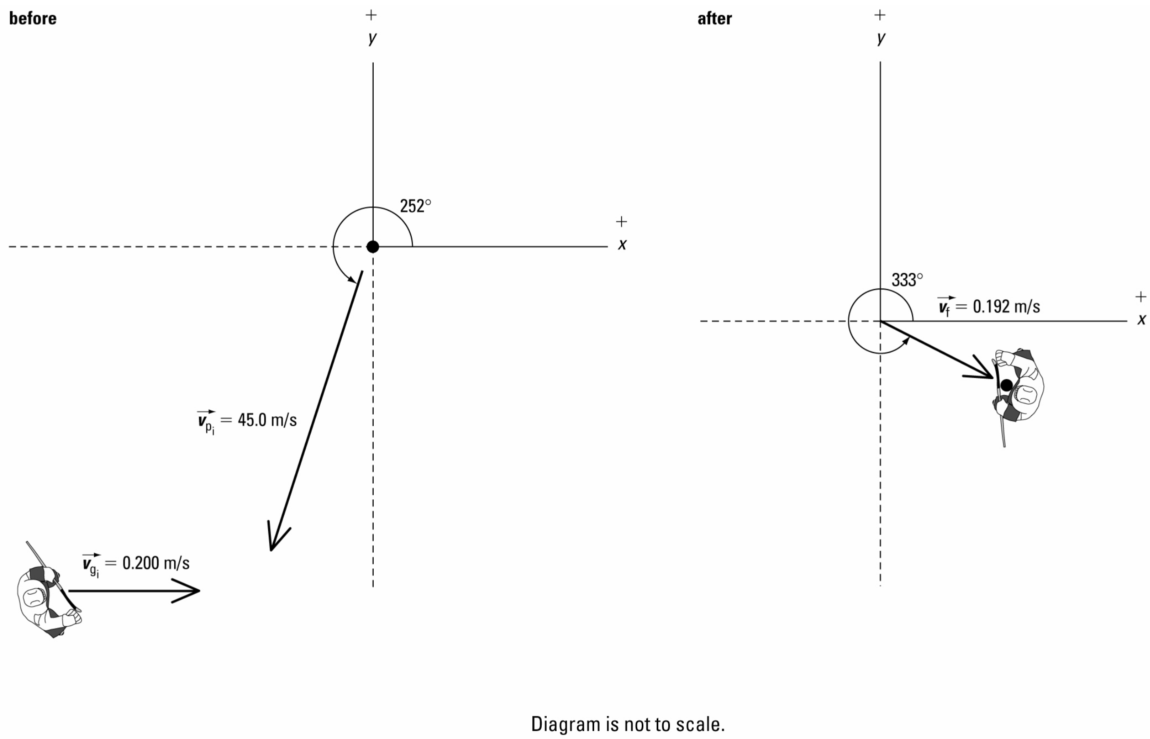
m_p = 0.168 kg

$\vec{v}_{p_i}$ = 45.0 m/s [252º]

$\vec{v}_f$ = 0.192 m/s [333º]

m_g = 82.0 kg

$\vec{v}_{g_i}$ = 0.200 m/s [0º]



Required
determine if the collision is elastic

Analysis and Solution
Calculate the total initial kinetic energy and the total final kinetic energy of the system.

$$
\begin{aligned}
E_{k_i} &= \frac{1}{2} m_p \left(v_{p_i}\right)^2 + \frac{1}{2} m_g \left(v_{g_i}\right)^2 \\
&= \frac{1}{2}(0.168 \text{ kg})(45.0 \text{ m/s})^2 + \frac{1}{2}(82.0 \text{ kg})(0.200 \text{ m/s})^2 \\
&= 171.7 \text{ kg•m}^2/\text{s}^2 \\
&= 171.7 \text{ J}
\end{aligned}
$$

$$
\begin{aligned}
E_{k_f} &= \frac{1}{2}(m_p + m_g)\left(v_f\right)^2 \\
&= \frac{1}{2}(0.168 \text{ kg} + 82.0 \text{ kg})(0.192 \text{ m/s})^2 \\
&= 1.515 \text{ kg•m}^2/\text{s}^2 \\
&= 1.515 \text{ J}
\end{aligned}
$$

Since $E_{k_i} \neq E_{k_f}$, the collision is inelastic.

Calculate the percent of kinetic energy retained.

$$\% E_k \text{ retained} = \frac{E_{k_f}}{E_{k_i}} \times 100\%$$
$$= \frac{1.515 \text{ J}}{171.7 \text{ J}} \times 100\%$$
$$= 0.882\%$$

Paraphrase

The collision is inelastic, and 0.882% of the kinetic energy is retained.

2. ***Given***

$m_A = 19.0$ kg

$v_{A_f} = 0.663$ m/s

$m_B = 19.0$ kg

$\vec{v}_{B_i} = 0$ m/s

$v_{B_f} = 1.31$ m/s

Required

initial speed of curling stone A if collision is elastic (v_{A_i})

Analysis and Solution

Choose both curling stones as an isolated system.

If the collision is elastic, the total kinetic energy of the system is conserved.

$$E_{k_i} = E_{k_f}$$
$$\frac{1}{2} m_A \left(v_{A_i}\right)^2 + \frac{1}{2} m_B \left(v_{B_i}\right)^2 = \frac{1}{2} m_A \left(v_{A_f}\right)^2 + \frac{1}{2} m_B \left(v_{B_f}\right)^2$$
$$\left(v_{A_i}\right)^2 + 0 = \left(v_{A_f}\right)^2 + \left(\frac{m_B}{m_A}\right)\left(v_{B_f}\right)^2$$
$$\left(v_{A_i}\right)^2 = \left(v_{A_f}\right)^2 + \left(\frac{m_B}{m_A}\right)\left(v_{B_f}\right)^2$$
$$= (0.663 \text{ m/s})^2 + \left(\frac{19.0 \text{ kg}}{19.0 \text{ kg}}\right)(1.31 \text{ m/s})^2$$
$$= 2.156 \text{ m}^2/\text{s}^2$$
$$v_{A_i} = 1.47 \text{ m/s}$$

Paraphrase

If the collision between both curling stones is elastic, the initial speed of the first curling stone would be 1.47 m/s.

Student Book page 499

9.4 Check and Reflect

Knowledge

1. In a one-dimensional collision, the initial and final velocities of all objects in the system lie along the same line, but are not necessarily in the same direction. This type of interaction occurs when two objects collide dead centre. An example is a billiard ball hitting an identical stationary ball dead centre so that after collision the second ball moves in the same direction as the incoming ball.

In a two-dimensional collision, the initial and final velocities of all objects in the system lie in a plane. So some or all of the velocity vectors have two components. This type of interaction occurs when two objects collide off centre. An example is a billiard ball hitting an identical stationary ball with a glancing blow so that both balls move off in directions different from the incoming ball.

2. The law of conservation of momentum for a two-dimensional collision states that in each direction, the initial momentum of the system is equal to the final momentum of the system.

Example: A 19.0-kg curling stone moving at 1.20 m/s [0º] collides with an identical stone moving at 0.850 m/s [270º]. Just after impact, the first stone moves off at 0.922 m/s [320º]. What will be the velocity of the second stone immediately after collision?

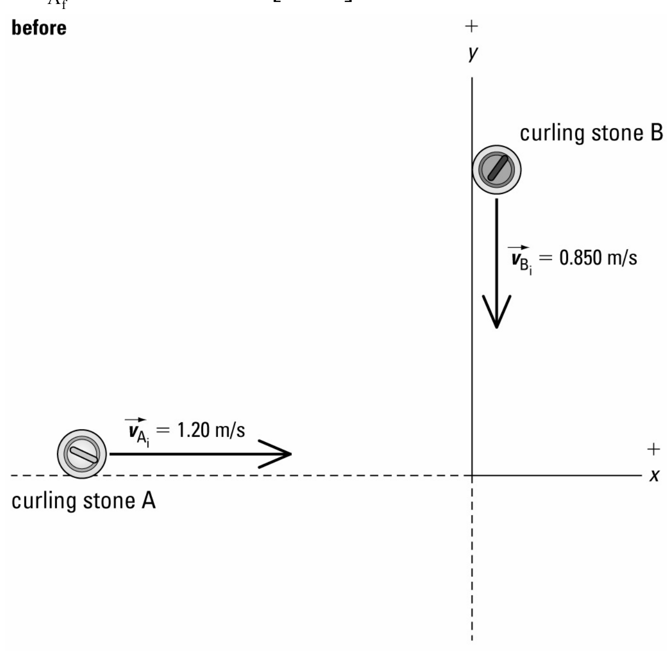
Solution:

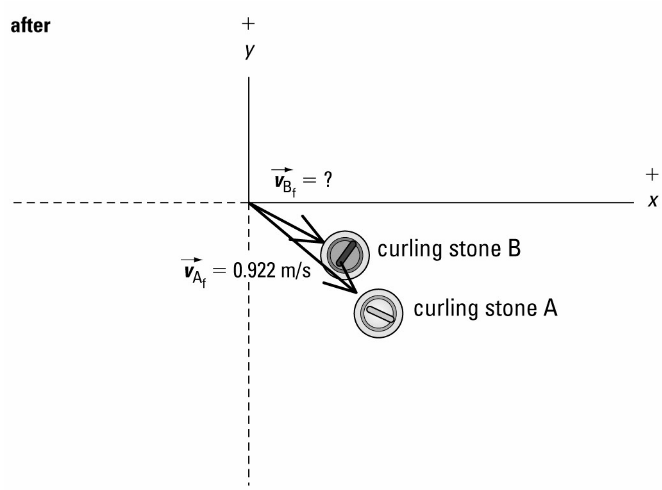
Given

$m_A = 19.0$ kg $\quad\quad m_B = 19.0$ kg

$\vec{v}_{A_i} = 1.20$ m/s [0º] $\quad\quad \vec{v}_{B_i} = 0.850$ m/s [270º]

$\vec{v}_{A_f} = 0.922$ m/s [320º]



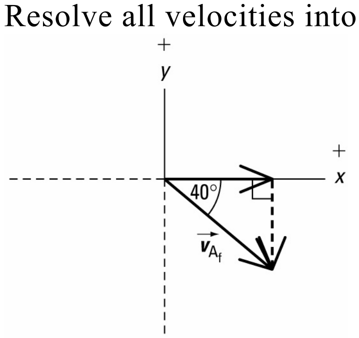


Required

final velocity of curling stone B ($\vec{v}_{B_f}$)

Analysis and Solution

Choose both curling stones as an isolated system.
Resolve all velocities into *x* and *y* components.



Vector	*x* component	*y* component
$\vec{v}_{A_i}$	1.20 m/s	0
$\vec{v}_{B_i}$	0	–(0.850 m/s)
$\vec{v}_{A_f}$	(0.922 m/s)(cos 40º)	–(0.922 m/s)(sin 40º)

Apply the law of conservation of momentum to the system in the *x* and *y* directions.

x direction

$$p_{\text{sys}_{ix}} = p_{\text{sys}_{fx}}$$

$$p_{A_{ix}} + p_{B_{ix}} = p_{A_{fx}} + p_{B_{fx}}$$

$$m_A v_{A_{ix}} + 0 = m_A v_{A_{fx}} + m_B v_{B_{fx}}$$

$$m_B v_{B_{fx}} = m_A (v_{A_{ix}} - v_{A_{fx}})$$

$$v_{B_{fx}} = \left(\frac{m_A}{m_B}\right)(v_{A_{ix}} - v_{A_{fx}})$$

$$= \left(\frac{19.0\ \cancel{kg}}{19.0\ \cancel{kg}}\right)\{1.20\text{ m/s} - (0.922\text{ m/s})(\cos 40°)\}$$

$$= 0.4937\text{ m/s}$$

y direction

$$p_{sys_{iy}} = p_{sys_{fy}}$$

$$p_{A_{iy}} + p_{B_{iy}} = p_{A_{fy}} + p_{B_{fy}}$$

$$0 + m_B v_{B_{iy}} = m_A v_{A_{fy}} + m_B v_{B_{fy}}$$

$$v_{B_{iy}} = \left(\frac{m_A}{m_B}\right) v_{A_{fy}} + v_{B_{fy}}$$

$$v_{B_{fy}} = v_{B_{iy}} - \left(\frac{m_A}{m_B}\right) v_{A_{fy}}$$

$$= -(0.850\text{ m/s}) - \left(\frac{19.0\ \cancel{kg}}{19.0\ \cancel{kg}}\right)\{-(0.922\text{ m/s})(\sin 40°)\}$$

$$= -0.2573\text{ m/s}$$

Use the Pythagorean theorem to find the magnitude of $\vec{v}_{B_f}$.

$$v_{B_f} = \sqrt{(v_{B_{fx}})^2 + (v_{B_{fy}})^2}$$

$$= \sqrt{(0.4937\text{ m/s})^2 + (-0.2573\text{ m/s})^2}$$

$$= 0.557\text{ m/s}$$

Use the tangent function to find the direction of $\vec{v}_{B_f}$.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{0.2573\ \cancel{\frac{m}{s}}}{0.4937\ \cancel{\frac{m}{s}}}$$

$$= 0.5213$$

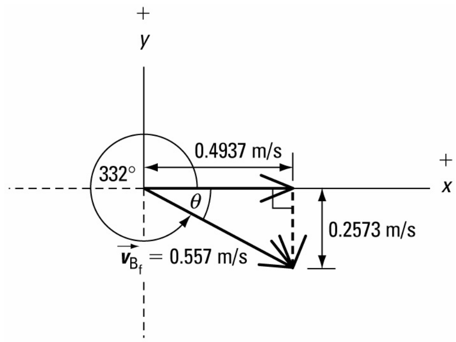
$$\theta = \tan^{-1}(0.5213)$$

$$= 27.5°$$

From the following figure, this angle is between $\vec{v}_{B_f}$ and the positive *x*-axis.

So, the direction of $\vec{v}_{B_f}$ measured *counterclockwise* from the positive *x*-axis is

360° – 27.5° = 332°.



$\vec{v}_{B_f}$ = 0.557 m/s [332°]

Paraphrase

The velocity of curling stone B will be 0.557 m/s [332°] immediately after collision.

3. The centre of mass of an object is a point in which all of the mass of the object can be assumed to be concentrated. The centre of mass may be located either on or outside the object, depending on its shape.

4. Scientists accepted the existence of the neutrino for so long without direct evidence because they believed that the conservation of momentum and the conservation of energy were universal laws that were always valid and had no exceptions. In order for these laws to hold, the neutrino had to exist. Later when direct evidence demonstrated the existence of the neutrino, these conservation laws were confirmed once again as being universal.

Applications

5. ***Given***

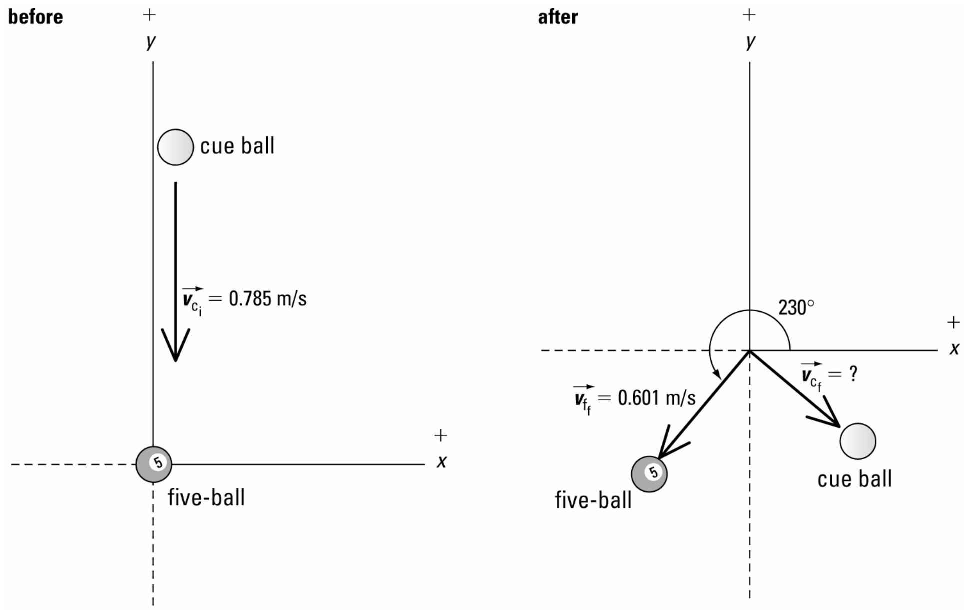
m_c = 160 g

$\vec{v}_{c_i}$ = 0.785 m/s [270º]

m_f = 160 g

$\vec{v}_{f_i}$ = 0 m/s

$\vec{v}_{f_f}$ = 0.601 m/s [230º]



Required

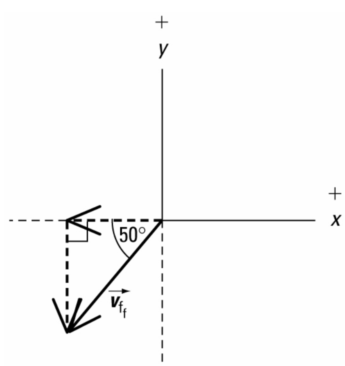
final velocity of cue ball ($\vec{v}_{c_f}$)

Analysis and Solution

Choose the cue ball and the five-ball as an isolated system.
The five-ball has an initial velocity of zero. So its initial momentum is zero.

$$\vec{p}_{f_i} = 0$$

Resolve all velocities into x and y components.



Vector	x component	y component
$\vec{v}_{c_i}$	0	–(0.785 m/s)
$\vec{v}_{f_f}$	–(0.601 m/s)(cos 50º)	–(0.601 m/s)(sin 50º)

Apply the law of conservation of momentum to the system in the x and y directions.

x direction

$$p_{\text{sys}_{\text{i}x}} = p_{\text{sys}_{\text{f}x}}$$

$$p_{\text{c}_{\text{i}x}} + p_{\text{f}_{\text{i}x}} = p_{\text{c}_{\text{f}x}} + p_{\text{f}_{\text{f}x}}$$

$$0 + 0 = m_{\text{c}} v_{\text{c}_{\text{f}x}} + m_{\text{f}} v_{\text{f}_{\text{f}x}}$$

$$m_{\text{c}} v_{\text{c}_{\text{f}x}} = -m_{\text{f}} v_{\text{f}_{\text{f}x}}$$

$$v_{\text{c}_{\text{f}x}} = -\left(\frac{m_{\text{f}}}{m_{\text{c}}}\right) v_{\text{f}_{\text{f}x}}$$

$$= -\left(\frac{160\ \cancel{\text{g}}}{160\ \cancel{\text{g}}}\right)\{-(0.601\text{ m/s})(\cos 50^{\circ})\}$$

$$= 0.3863\text{ m/s}$$

y direction

$$p_{\text{sys}_{\text{i}y}} = p_{\text{sys}_{\text{f}y}}$$

$$p_{\text{c}_{\text{i}y}} + p_{\text{f}_{\text{i}y}} = p_{\text{c}_{\text{f}y}} + p_{\text{f}_{\text{f}y}}$$

$$m_{\text{c}} v_{\text{c}_{\text{i}y}} + 0 = m_{\text{c}} v_{\text{c}_{\text{f}y}} + m_{\text{f}} v_{\text{f}_{\text{f}y}}$$

$$v_{\text{c}_{\text{i}y}} = v_{\text{c}_{\text{f}y}} + \left(\frac{m_{\text{f}}}{m_{\text{c}}}\right) v_{\text{f}_{\text{f}y}}$$

$$v_{\text{c}_{\text{f}y}} = v_{\text{c}_{\text{i}y}} - \left(\frac{m_{\text{f}}}{m_{\text{c}}}\right) v_{\text{f}_{\text{f}y}}$$

$$= -(0.785\text{ m/s}) - \left(\frac{160\ \cancel{\text{g}}}{160\ \cancel{\text{g}}}\right)\{-(0.601\text{ m/s})(\sin 50^{\circ})\}$$

$$= -0.3246\text{ m/s}$$

Use the Pythagorean theorem to find the magnitude of $\vec{v}_{\text{c}_{\text{f}}}$.

$$v_{\text{c}_{\text{f}}} = \sqrt{\left(v_{\text{c}_{\text{f}x}}\right)^2 + \left(v_{\text{c}_{\text{f}y}}\right)^2}$$

$$= \sqrt{(0.3863\text{ m/s})^2 + (-0.3246\text{ m/s})^2}$$

$$= 0.505\text{ m/s}$$

Use the tangent function to find the direction of $\vec{v}_{\text{c}_{\text{f}}}$.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{0.3246\ \cancel{\frac{\text{m}}{\text{s}}}}{0.3863\ \cancel{\frac{\text{m}}{\text{s}}}}$$

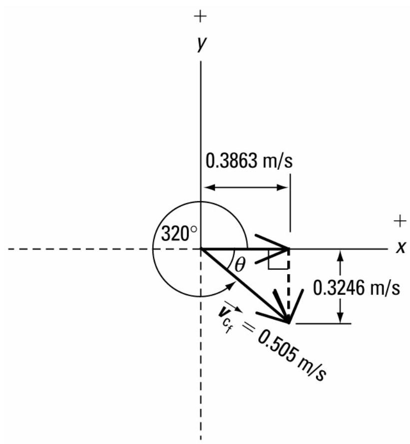
$= 0.8403$

$\theta = \tan^{-1}(0.8403)$

$= 40.0°$

From the figure below, this angle is between $\vec{v}_{c_f}$ and the positive *x*-axis. So the direction of $\vec{v}_{c_f}$ measured *counterclockwise* from the positive *x*-axis is

360° – 40.0° = 320°.



$\vec{v}_{c_f} = 0.505$ m/s [320°]

Paraphrase

The velocity of the cue ball will be 0.505 m/s [320°] immediately after impact.

6. ***Given***

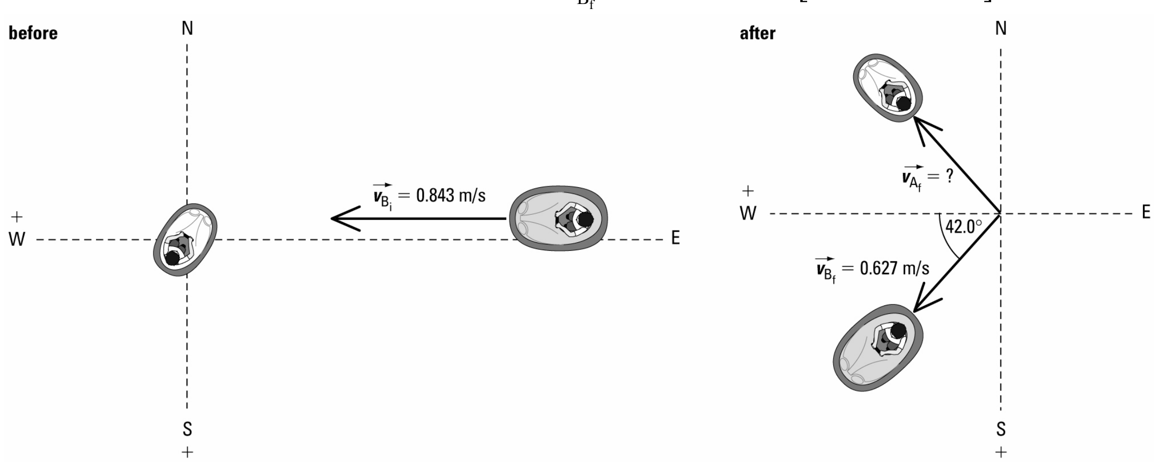
$m_A = 230$ kg

$\vec{v}_{A_i} = 0$ m/s

$m_B = 255$ kg

$\vec{v}_{B_i} = 0.843$ m/s [W]

$\vec{v}_{B_f} = 0.627$ m/s [42.0º S of W]



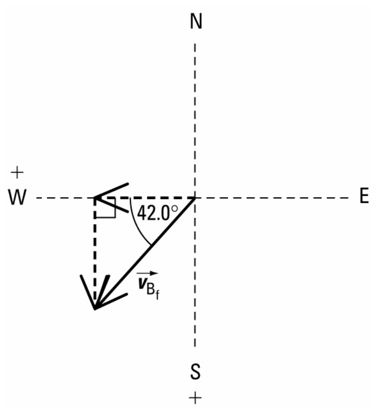
Required

final velocity of bumper car A ($\vec{v}_{A_f}$)

Analysis and Solution
Choose both bumper cars as an isolated system.
Bumper car A has an initial velocity of zero. So its initial momentum is zero.

$\vec{p}_{A_i} = 0$

Resolve all velocities into west and south components.



Vector	West component	South component
$\vec{v}_{B_i}$	0.843 m/s	0
$\vec{v}_{B_f}$	(0.627 m/s)(cos 42.0º)	(0.627 m/s)(sin 42.0º)

Apply the law of conservation of momentum to the system in the west and south directions.

W direction

$$\begin{aligned} p_{sys_{iW}} &= p_{sys_{fW}} \\ p_{A_{iW}} + p_{B_{iW}} &= p_{A_{fW}} + p_{B_{fW}} \\ 0 + m_B v_{B_{iW}} &= m_A v_{A_{fW}} + m_B v_{B_{fW}} \\ m_A v_{A_{fW}} &= m_B(v_{B_{iW}} - v_{B_{fW}}) \\ v_{A_{fW}} &= \left(\frac{m_B}{m_A}\right)(v_{B_{iW}} - v_{B_{fW}}) \\ &= \left(\frac{255\ \cancel{\text{kg}}}{230\ \cancel{\text{kg}}}\right)\{0.843\text{ m/s} - (0.627\text{ m/s})(\cos 42.0^\circ)\} \\ &= 0.4180\text{ m/s} \end{aligned}$$

S direction

$$\begin{aligned} p_{sys_{iS}} &= p_{sys_{fS}} \\ p_{A_{iS}} + p_{B_{iS}} &= p_{A_{fS}} + p_{B_{fS}} \\ 0 + 0 &= m_A v_{A_{fS}} + m_B v_{B_{fS}} \end{aligned}$$

$$m_A v_{A_{fS}} = -m_B v_{B_{fS}}$$

$$v_{A_{fS}} = -\left(\frac{m_B}{m_A}\right) v_{B_{fS}}$$

$$= -\left(\frac{255\ \cancel{kg}}{230\ \cancel{kg}}\right)\{(0.627\ m/s)(\sin 42.0°)\}$$

$$= -0.4651\ m/s$$

Use the Pythagorean theorem to find the magnitude of $\vec{v}_{A_f}$.

$$v_{A_f} = \sqrt{\left(v_{A_{fW}}\right)^2 + \left(v_{A_{fS}}\right)^2}$$

$$= \sqrt{(0.4180\ m/s)^2 + (-0.4651\ m/s)^2}$$

$$= 0.625\ m/s$$

Use the tangent function to find the direction of $\vec{v}_{A_f}$.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

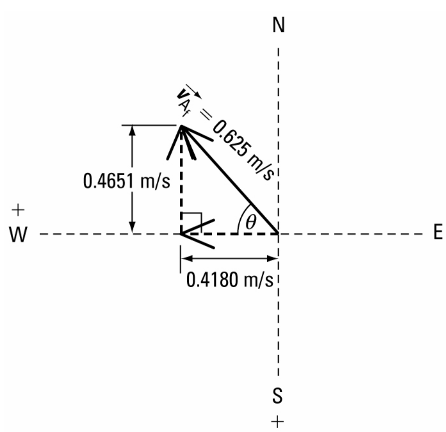
$$= \frac{0.4651\ \cancel{\frac{m}{s}}}{0.4180\ \cancel{\frac{m}{s}}}$$

$$= 1.113$$

$$\theta = \tan^{-1}(1.113)$$

$$= 48.1°$$

From the figure below, this angle is between $\vec{v}_{A_f}$ and the west direction.



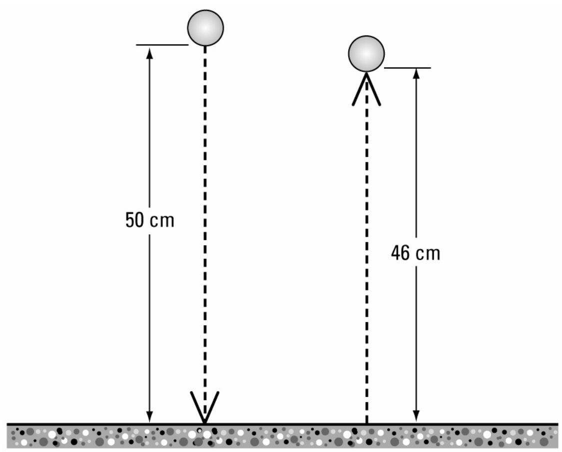
$$\vec{v}_{A_f} = 0.625\ m/s\ [48.1°\ N\ of\ W]$$

Paraphrase

The velocity of bumper car A will be 0.625 m/s [48.1° N of W] immediately after collision.

7. ***Analysis and Solution***

If the collision between the ball and the landing surface is elastic, the initial kinetic energy of the ball must be the same as its final kinetic energy. There must be no friction, deformation, sound, light, or heat present during the collision. Then *all* the kinetic energy would get converted to gravitational potential energy.



Since the rebound height is less than the initial height just before the ball was dropped, the collision is inelastic.

To determine the kinetic energy lost during the collision, calculate the loss in gravitational potential energy.

$$
\begin{aligned}
E_k \text{ lost} &= mg\Delta h \\
&= (0.25\text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(50\text{ cm} - 46\text{ cm})\left(\frac{1\text{ m}}{100\text{ cm}}\right) \\
&= 0.098\text{ kg}\bullet\text{m}^2/\text{s}^2 \\
&= 0.098\text{ J}
\end{aligned}
$$

There were 0.098 J of kinetic energy lost during the collision.

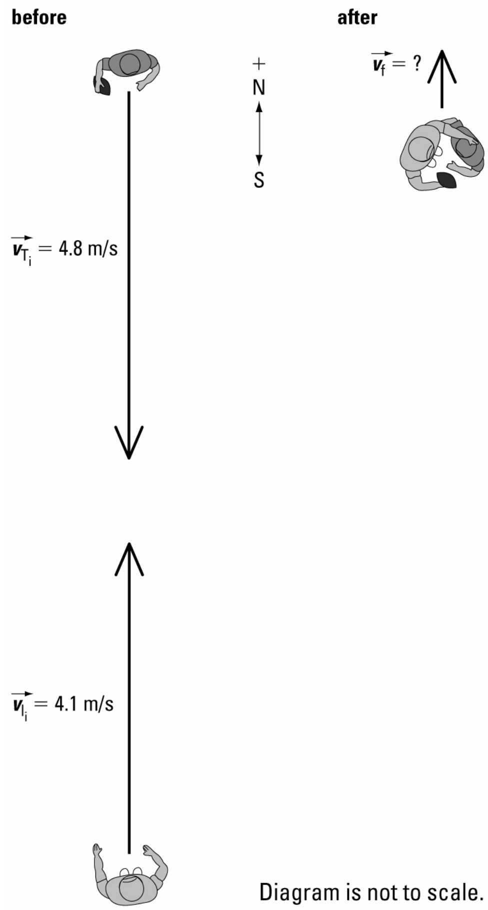
8. ***Given***

$m_T = 95$ kg

$m_l = 115$ kg

$\vec{v}_{T_i} = 4.8$ m/s [S]

$\vec{v}_{l_i} = 4.1$ m/s [N]



Required

if ball will cross goal line

Analysis and Solution

Choose the halfback, the ball, and the linebacker as an isolated system.
To determine if the ball will cross the goal line, calculate the final velocity of the system.
The halfback, the ball, and the linebacker move together as a unit after collision.
Apply the law of conservation of momentum to the system.

$$\vec{p}_{sys_i} = \vec{p}_{sys_f}$$

$$\vec{p}_{T_i} + \vec{p}_{l_i} = \vec{p}_{sys_f}$$

$$m_T \vec{v}_{T_i} + m_l \vec{v}_{l_i} = (m_T + m_l)\vec{v}_f$$

$$\vec{v}_f = \left(\frac{1}{m_T + m_l}\right)(m_T \vec{v}_{T_i} + m_l \vec{v}_{l_i})$$

$$= \left(\frac{1}{95 \text{ kg} + 115 \text{ kg}}\right)\{(95 \text{ kg})(-4.8 \text{ m/s}) + (115 \text{ kg})(+4.1 \text{ m/s})\}$$

$$= +0.074 \text{ m/s}$$

$$\vec{v}_f = 0.074 \text{ m/s [N]}$$

Paraphrase

The velocity of the halfback, the ball, and the linebacker is 0.074 m/s [N] immediately after collision. In order for the ball to cross the goal line, it would have to be moving south, not north. So the ball will not cross the line.

9. ***Given***

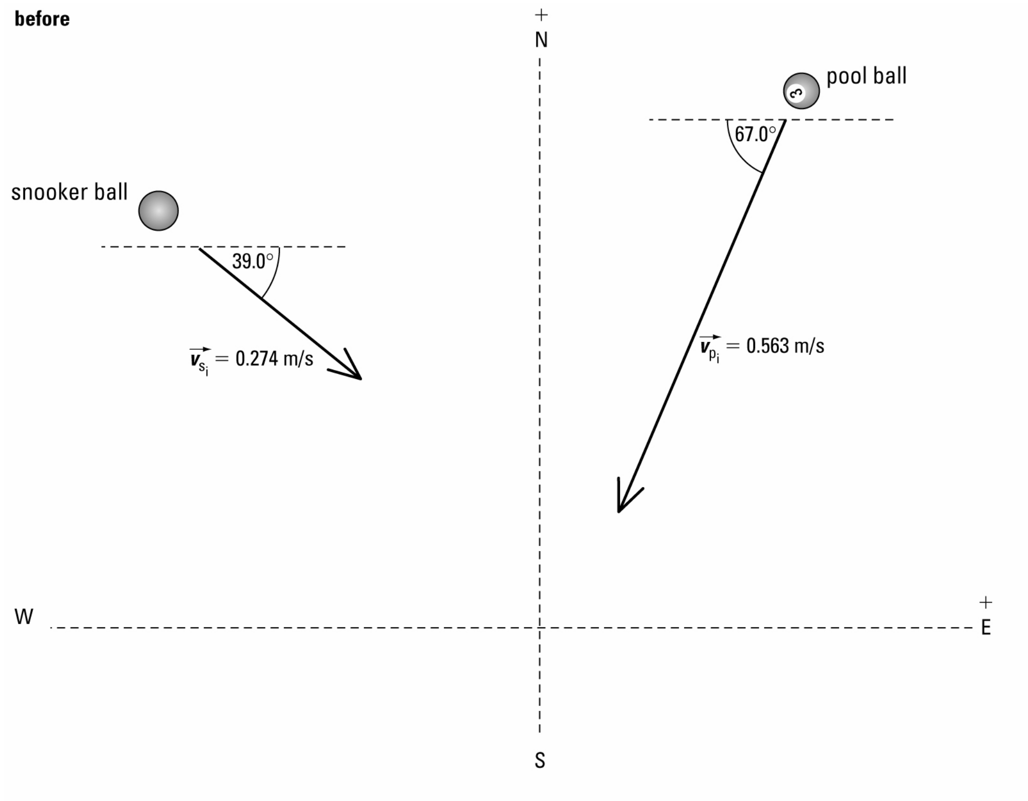
m_p = 0.160 kg

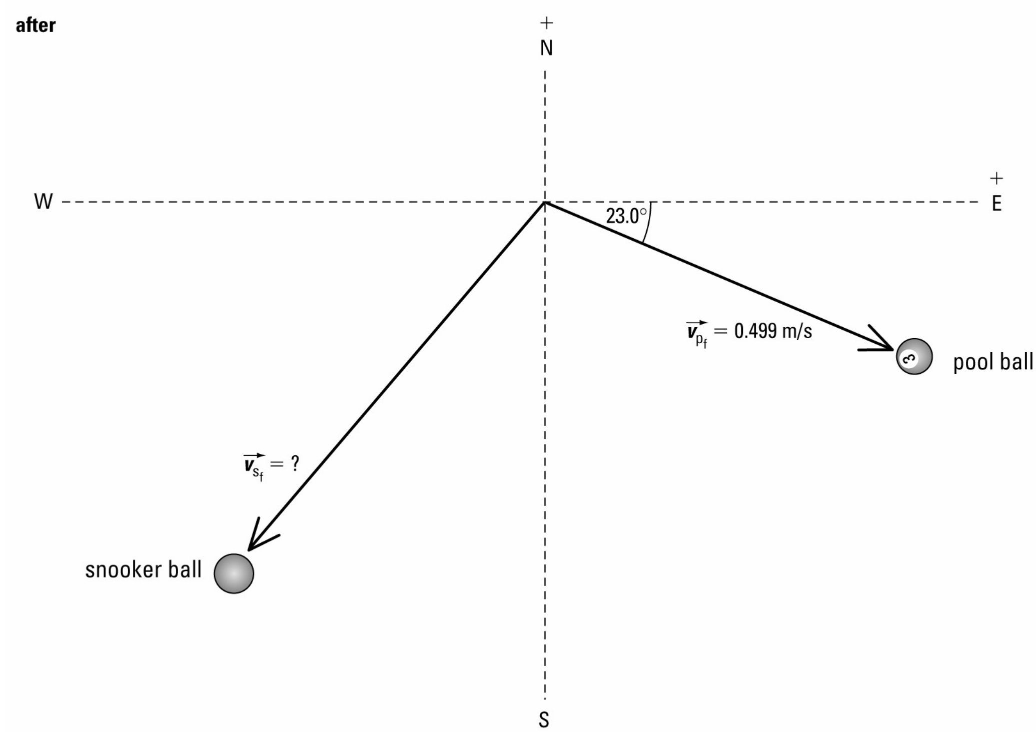
m_s = 0.180 kg

$\vec{v}_{p_i}$ = 0.563 m/s [67.0º S of W]

$\vec{v}_{s_i}$ = 0.274 m/s [39.0º S of E]

$\vec{v}_{p_f}$ = 0.499 m/s [23.0º S of E]



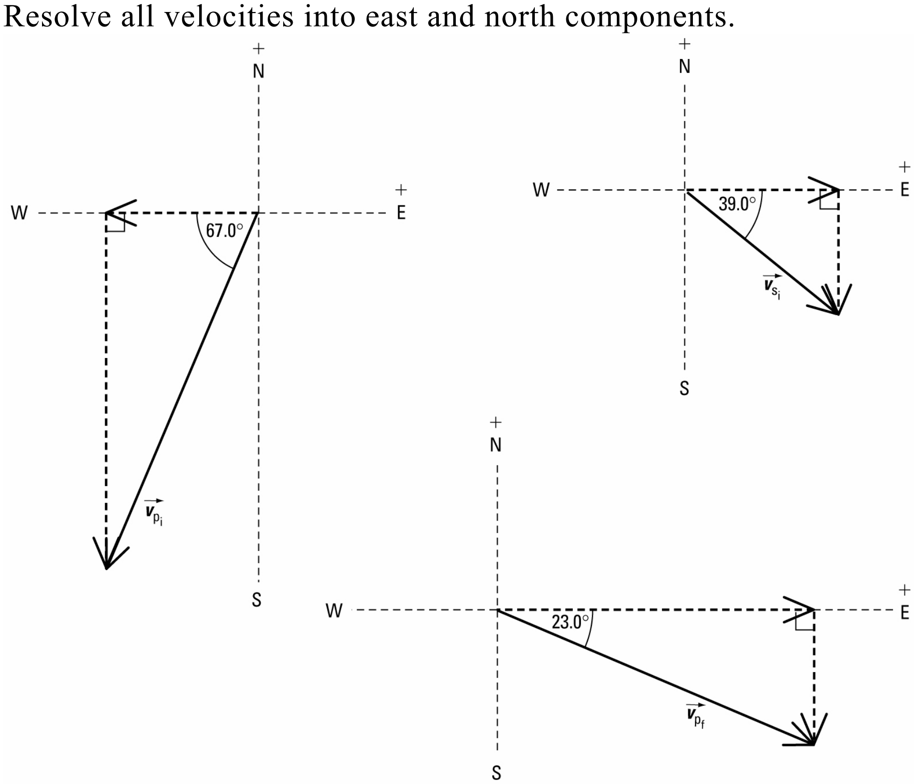


Required

final velocity of snooker ball ($\vec{v}_{s_f}$)

Analysis and Solution

Choose the pool ball and the snooker ball as an isolated system.
Resolve all velocities into east and north components.



Vector	East component	North component
$\vec{v}_{p_i}$	–(0.563 m/s)(cos 67.0º)	–(0.563 m/s)(sin 67.0º)
$\vec{v}_{s_i}$	(0.274 m/s)(cos 39.0º)	–(0.274 m/s)(sin 39.0º)
$\vec{v}_{p_f}$	(0.499 m/s)(cos 23.0º)	–(0.499 m/s)(sin 23.0º)

Apply the law of conservation of momentum to the system in the east and north directions.

E direction

$$p_{sys_{iE}} = p_{sys_{fE}}$$

$$p_{p_{iE}} + p_{s_{iE}} = p_{p_{fE}} + p_{s_{fE}}$$

$$m_p v_{p_{iE}} + m_s v_{s_{iE}} = m_p v_{p_{fE}} + m_s v_{s_{fE}}$$

$$\left(\frac{m_p}{m_s}\right) v_{p_{iE}} + v_{s_{iE}} = \left(\frac{m_p}{m_s}\right) v_{p_{fE}} + v_{s_{fE}}$$

$$v_{s_{fE}} = \left(\frac{m_p}{m_s}\right)(v_{p_{iE}} - v_{p_{fE}}) + v_{s_{iE}}$$

$$= \left(\frac{0.160\ \cancel{kg}}{0.180\ \cancel{kg}}\right)\{-(0.563\text{ m/s})(\cos 67.0^\circ) - (0.499\text{ m/s})(\cos 23.0^\circ)\}$$

$$+ (0.274\text{ m/s})(\cos 39.0^\circ)$$

$$= -0.3909\text{ m/s}$$

N direction

$$p_{sys_{iN}} = p_{sys_{fN}}$$

$$p_{p_{iN}} + p_{s_{iN}} = p_{p_{fN}} + p_{s_{fN}}$$

$$m_p v_{p_{iN}} + m_s v_{s_{iN}} = m_p v_{p_{fN}} + m_s v_{s_{fN}}$$

$$\left(\frac{m_p}{m_s}\right) v_{p_{iN}} + v_{s_{iN}} = \left(\frac{m_p}{m_s}\right) v_{p_{fN}} + v_{s_{fN}}$$

$$v_{s_{fN}} = \left(\frac{m_p}{m_s}\right)(v_{p_{iN}} - v_{p_{fN}}) + v_{s_{iN}}$$

$$= \left(\frac{0.160\ \cancel{kg}}{0.180\ \cancel{kg}}\right)\{-(0.563\text{ m/s})(\sin 67.0^\circ) - [-(0.499\text{ m/s})(\sin 23.0^\circ)]\}$$

$$+ \{-(0.274\text{ m/s})(\sin 39.0^\circ)\}$$

$$= -0.4598\text{ m/s}$$

Use the Pythagorean theorem to find the magnitude of $\vec{v}_{s_f}$.

$$v_{s_f} = \sqrt{\left(v_{s_{iE}}\right)^2 + \left(v_{s_{iN}}\right)^2}$$

$$= \sqrt{(-0.3909 \text{ m/s})^2 + (-0.4598 \text{ m/s})^2}$$

$$= 0.603 \text{ m/s}$$

Use the tangent function to find the direction of $\vec{v}_{s_f}$.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

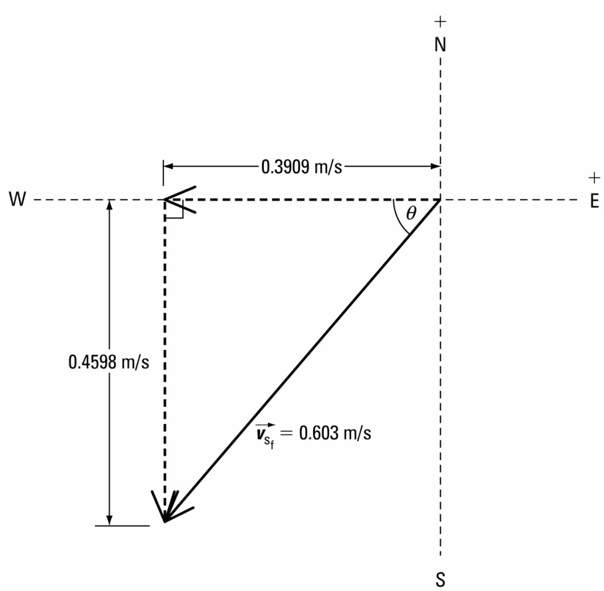
$$= \frac{0.4598 \frac{\cancel{m}}{\cancel{s}}}{0.3909 \frac{\cancel{m}}{\cancel{s}}}$$

$$= 1.176$$

$$\theta = \tan^{-1}(1.176)$$

$$= 49.6°$$

From the figure below, this angle is between $\vec{v}_{s_f}$ and the west direction.



$\vec{v}_{s_f}$ = 0.603 m/s [49.6° S of W]

Paraphrase

The velocity of the snooker ball will be 0.603 m/s [49.6° S of W] immediately after collision.

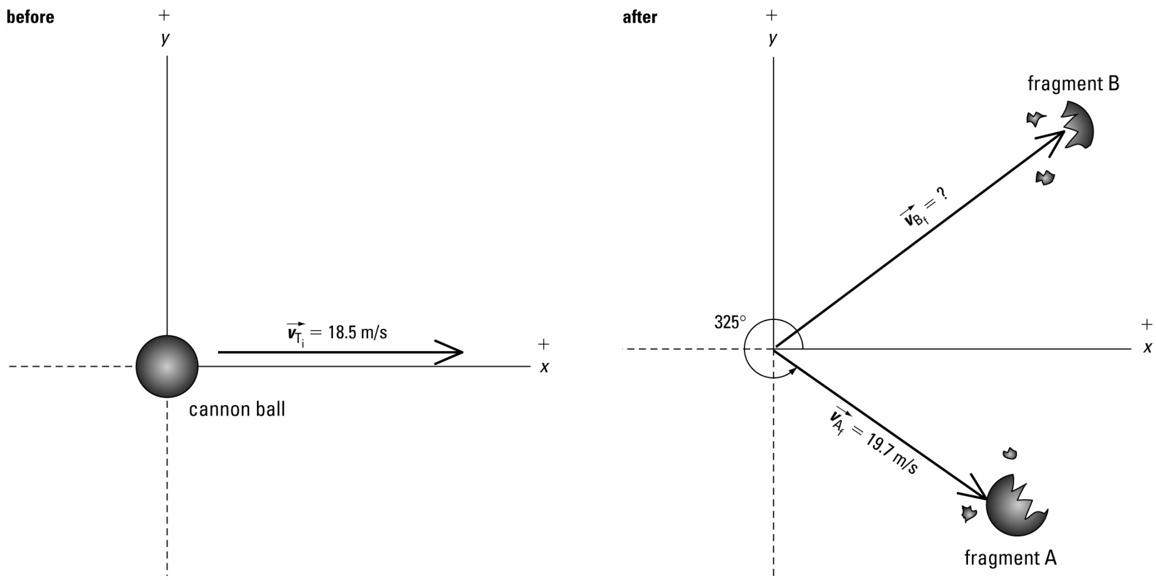
10. ***Given***

m_T = 4.00 kg

$\vec{v}_{T_i}$ = 18.5 m/s [0º]

m_A = 2.37 kg

$\vec{v}_{A_f}$ = 19.7 m/s [325º]



Required

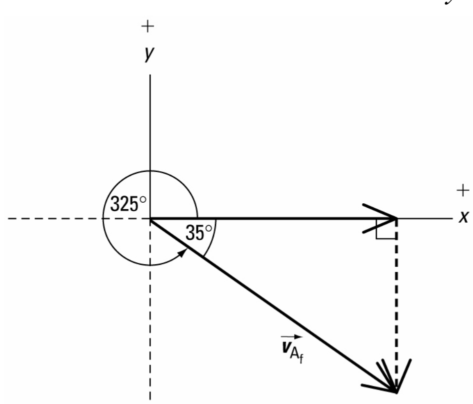
final velocity of fragment B ($\vec{v}_{B_f}$)

Analysis and Solution

Choose the cannon ball and the two fragments as an isolated system.
Since no mass is lost, find the mass of fragment B.

$$\begin{aligned} m_B &= m_T - m_A \\ &= 4.00 \text{ kg} - 2.37 \text{ kg} \\ &= 1.63 \text{ kg} \end{aligned}$$

Resolve all velocities into x and y components.



Vector	x component	y component
$\vec{v}_{T_i}$	18.5 m/s	0
$\vec{v}_{A_f}$	(19.7 m/s)(cos 35º)	–(19.7 m/s)(sin 35º)

Apply the law of conservation of momentum to the system in the x and y directions.

x direction

$$\begin{aligned}
p_{\text{sys}_{\text{i}x}} &= p_{\text{sys}_{\text{f}x}} \\
p_{\text{sys}_{\text{i}x}} &= p_{\text{A}_{\text{f}x}} + p_{\text{B}_{\text{f}x}} \\
m_\text{T} v_{\text{T}_{\text{i}x}} &= m_\text{A} v_{\text{A}_{\text{f}x}} + m_\text{B} v_{\text{B}_{\text{f}x}} \\
\left(\frac{m_\text{T}}{m_\text{B}}\right) v_{\text{T}_{\text{i}x}} &= \left(\frac{m_\text{A}}{m_\text{B}}\right) v_{\text{A}_{\text{f}x}} + v_{\text{B}_{\text{f}x}} \\
v_{\text{B}_{\text{f}x}} &= \left(\frac{m_\text{T}}{m_\text{B}}\right) v_{\text{T}_{\text{i}x}} - \left(\frac{m_\text{A}}{m_\text{B}}\right) v_{\text{A}_{\text{f}x}} \\
&= \left(\frac{4.00\ \cancel{\text{kg}}}{1.63\ \cancel{\text{kg}}}\right)(18.5\text{ m/s}) - \left(\frac{2.37\ \cancel{\text{kg}}}{1.63\ \cancel{\text{kg}}}\right)\{(19.7\text{ m/s})(\cos 35^\circ)\} \\
&= 21.94\text{ m/s}
\end{aligned}$$

y direction

$$\begin{aligned}
p_{\text{sys}_{\text{i}y}} &= p_{\text{sys}_{\text{f}y}} \\
p_{\text{sys}_{\text{i}y}} &= p_{\text{A}_{\text{f}y}} + p_{\text{B}_{\text{f}y}} \\
0 &= m_\text{A} v_{\text{A}_{\text{f}y}} + m_\text{B} v_{\text{B}_{\text{f}y}} \\
m_\text{B} v_{\text{B}_{\text{f}y}} &= -m_\text{A} v_{\text{A}_{\text{f}y}} \\
v_{\text{B}_{\text{f}y}} &= -\left(\frac{m_\text{A}}{m_\text{B}}\right) v_{\text{A}_{\text{f}y}} \\
&= -\left(\frac{2.37\ \cancel{\text{kg}}}{1.63\ \cancel{\text{kg}}}\right)\{-(19.7\text{ m/s})(\sin 35^\circ)\} \\
&= 16.43\text{ m/s}
\end{aligned}$$

Use the Pythagorean theorem to find the magnitude of $\vec{v}_{\text{B}_\text{f}}$.

$$\begin{aligned}
v_{\text{B}_\text{f}} &= \sqrt{\left(v_{\text{B}_{\text{f}x}}\right)^2 + \left(v_{\text{B}_{\text{f}y}}\right)^2} \\
&= \sqrt{(21.94\text{ m/s})^2 + (16.43\text{ m/s})^2} \\
&= 27.4\text{ m/s}
\end{aligned}$$

Use the tangent function to find the direction of $\vec{v}_{\text{B}_\text{f}}$.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

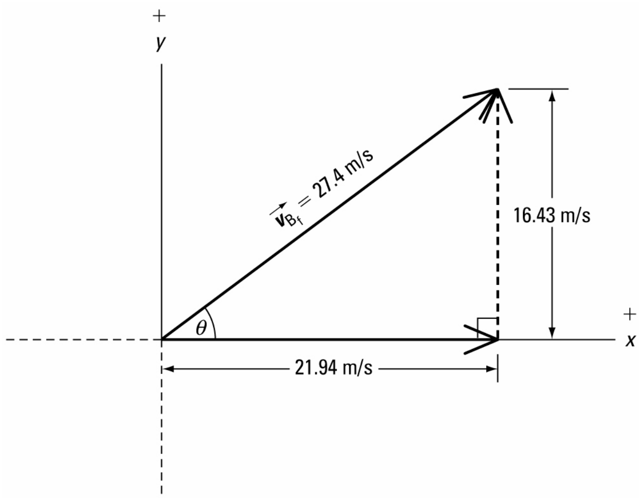
$$= \frac{16.43\ \frac{\cancel{m}}{\cancel{s}}}{21.94\ \frac{\cancel{m}}{\cancel{s}}}$$

$$= 0.7490$$

$$\theta = \tan^{-1}(0.7490)$$

$$= 37°$$

From the figure below, this angle is between $\vec{v}_{B_f}$ and the positive x-axis.



$$\vec{v}_{B_f} = 27.4 \text{ m/s } [37°]$$

Paraphrase

The velocity of fragment B will be 27.4 m/s [37°] immediately after the explosion.

11. ***Given***

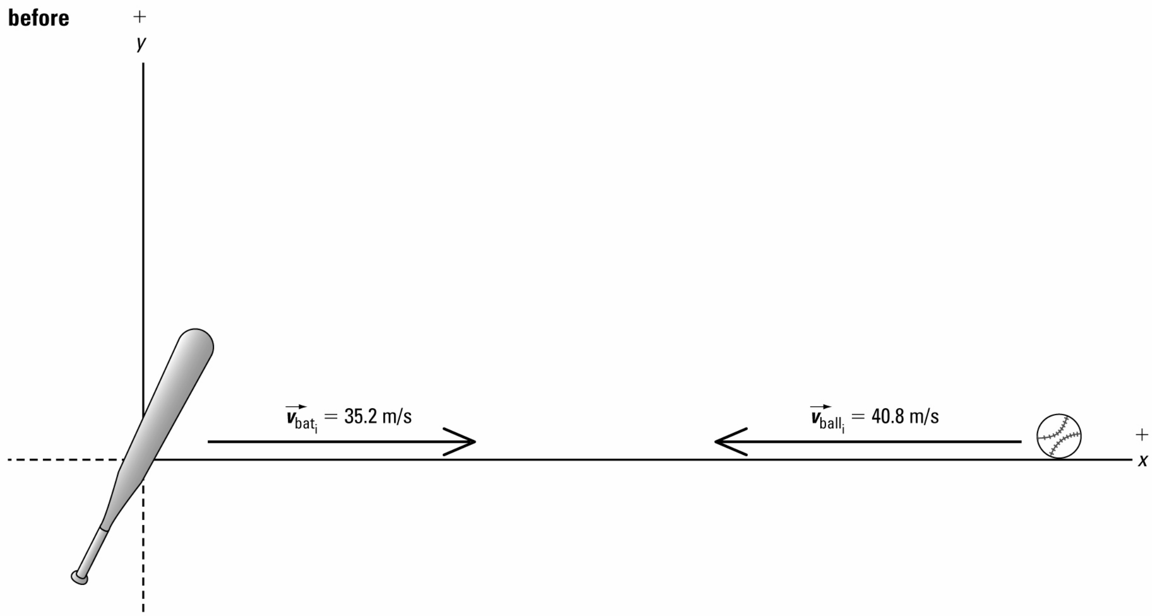
$m_{bat} = 0.952$ kg

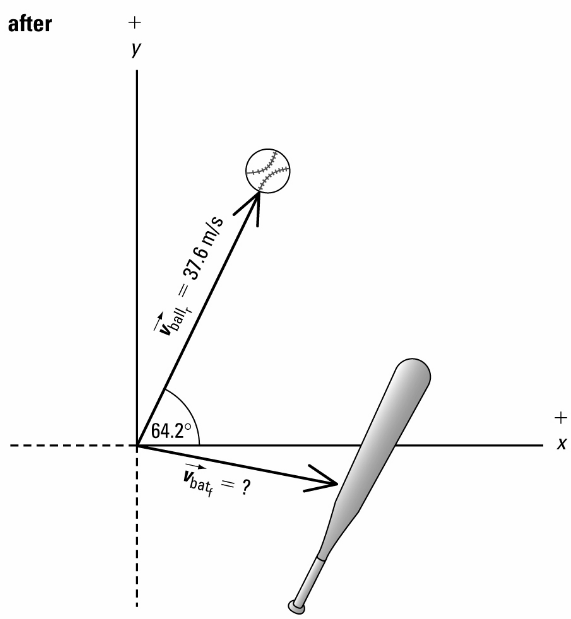
$\vec{v}_{bat_i} = 35.2$ m/s [0º]

$m_{ball} = 0.145$ kg

$\vec{v}_{ball_i} = 40.8$ m/s [180º]

$\vec{v}_{ball_f} = 37.6$ m/s [64.2º]



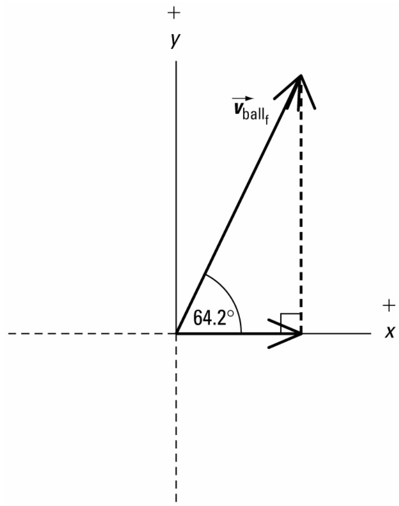


Required

final velocity of centre of mass of bat ($\vec{v}_{bat_f}$)

Analysis and Solution

Choose the bat and the ball as an isolated system.
Resolve all velocities into *x* and *y* components.



Vector	*x* component	*y* component
$\vec{v}_{bat_i}$	35.2 m/s	0
$\vec{v}_{ball_i}$	–(40.8 m/s)	0
$\vec{v}_{ball_f}$	(37.6 m/s)(cos 64.2º)	(37.6 m/s)(sin 64.2º)

Apply the law of conservation of momentum to the system in the *x* and *y* directions.

x direction

$$
\begin{aligned}
p_{sys_{ix}} &= p_{sys_{fx}} \\
p_{bat_{ix}} + p_{ball_{ix}} &= p_{bat_{fx}} + p_{ball_{fx}} \\
m_{bat} v_{bat_{ix}} + m_{ball} v_{ball_{ix}} &= m_{bat} v_{bat_{fx}} + m_{ball} v_{ball_{fx}} \\
v_{bat_{ix}} + \left(\frac{m_{ball}}{m_{bat}}\right) v_{ball_{ix}} &= v_{bat_{fx}} + \left(\frac{m_{ball}}{m_{bat}}\right) v_{ball_{fx}} \\
v_{bat_{fx}} &= \left(\frac{m_{ball}}{m_{bat}}\right)(v_{ball_{ix}} - v_{ball_{fx}}) + v_{bat_{ix}} \\
&= \left(\frac{0.145\ \cancel{kg}}{0.952\ \cancel{kg}}\right)\{-(40.8\text{ m/s}) - (37.6\text{ m/s})(\cos 64.2^\circ)\} + 35.2\text{ m/s} \\
&= 26.49\text{ m/s}
\end{aligned}
$$

y direction

$$
\begin{aligned}
p_{sys_{iy}} &= p_{sys_{fy}} \\
p_{bat_{iy}} + p_{ball_{iy}} &= p_{bat_{fy}} + p_{ball_{fy}} \\
0 + 0 &= m_{bat} v_{bat_{fy}} + m_{ball} v_{ball_{fy}} \\
m_{bat} v_{bat_{fy}} &= -m_{ball} v_{ball_{fy}}
\end{aligned}
$$

$$v_{bat_{fy}} = -\left(\frac{m_{ball}}{m_{bat}}\right) v_{ball_{fy}}$$

$$= -\left(\frac{0.145\ \cancel{kg}}{0.952\ \cancel{kg}}\right)\{(37.6\text{ m/s})(\sin 64.2°)\}$$

$$= -5.156\text{ m/s}$$

Use the Pythagorean theorem to find the magnitude of $\vec{v}_{bat_f}$.

$$v_{bat_f} = \sqrt{\left(v_{bat_{fx}}\right)^2 + \left(v_{bat_{fy}}\right)^2}$$

$$= \sqrt{(26.49\text{ m/s})^2 + (-5.156\text{ m/s})^2}$$

$$= 27.0\text{ m/s}$$

Use the tangent function to find the direction of $\vec{v}_{bat_f}$.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{5.156\ \cancel{\frac{m}{s}}}{26.49\ \cancel{\frac{m}{s}}}$$

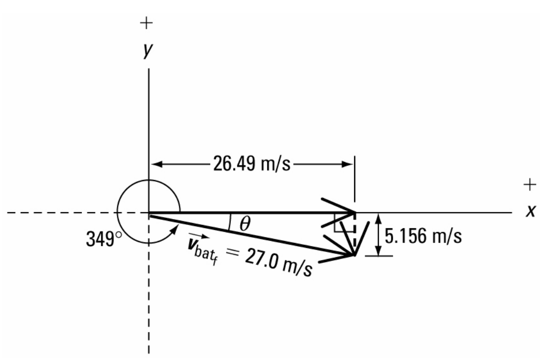
$$= 0.1946$$

$$\theta = \tan^{-1}(0.1946)$$

$$= 11.0°$$

From the figure below, this angle is between $\vec{v}_{bat_f}$ and the positive x-axis. So the direction of $\vec{v}_{bat_f}$ measured *counterclockwise* from the positive x-axis is

360° – 11.0° = 349°.



$$\vec{v}_{bat_f} = 27.0\text{ m/s [349°]}$$

Paraphrase

The velocity of the centre of mass of the baseball bat will be 27.0 m/s [349°] immediately after collision.

Extensions

12. This research question will appeal to kinesthetic learners and those interested in athletics. Running shoes that contain springs are currently banned by governing bodies in track and field such as International Amateur Athletic Federation (IAAF) and U.S.A. Track and Field (USATF). The springs in these shoes make the collision between a runner's foot and the running surface up to 96% elastic, compared to other materials such as foam. So very little kinetic energy from the runner's feet is converted to deformation, heat, or other types of energy. The springs also reduce impulsive forces by up to 20%, minimizing the incidence of injuries to joints and muscles. Since each spring must either compress or stretch, the time interval of interaction increases. Overall, a runner does not feel fatigued when running with these types of running shoes.

13. This question offers students a preview of some of the material they will learn about in Unit VIII. Enrico Fermi (1901–1954) had an early aptitude for mathematics and physics, which was encouraged by his father's colleagues. After gaining his doctorate in physics in 1922, he received a scholarship from the Italian government and spent some time doing research with Max Born in Göttingen, Germany.

In 1926, Fermi developed "Fermi statistics," statistical laws that apply to particles now called fermions (particles subject to Pauli's exclusion principle). From 1927 to 1938, he held the position of Professor of Theoretical Physics at the University of Rome. His early theoretical work consisted of electrodynamic problems and various spectroscopic phenomena. He eventually turned his attention to the atomic nucleus, and showed that almost every element bombarded with neutrons undergoes a nuclear transformation. This work eventually led to the discovery of nuclear fission and the ability to produce elements beyond uranium. In 1938, Fermi won the Nobel Prize in Physics for his work on producing radioactive elements using neutron bombardment, and for nuclear reactions brought about by slow neutrons.

Immediately after winning his Nobel Prize, he emigrated to the U.S. to escape Mussolini's fascist dictatorship. During World War II, Fermi was one of the team leaders of the physicists working on the Manhattan Project, a project aimed at developing an atomic bomb and nuclear energy. In 1942, he successfully created the first nuclear reactor in Chicago using a series of controlled nuclear chain reactions in an atomic pile. Toward the end of his life, Fermi turned his attention to high-energy physics and was investigating the mysterious origin of cosmic rays.

