Pearson Physics Level 20
Unit III Circular Motion, Work, and Energy: Chapter 5 Solutions

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Concept Check

1. The axis of rotation is through the centre of the Frisbee directed straight up and down.
2. A typical bicycle has six axes of rotation:
- the axis for each pedal of the bike (2);
- the axis through which the pedal arms rotate, which is attached to the large gears at the front of the bike (1);
- the axes for each wheel of the bike (2); and
- the axis through which the handle bars rotate when the bike is steered (1).

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Concept Check

The force that acts as the centripetal force is friction. Friction holds the pebble in the tread of the tire as it follows a circular path.

5.1 Check and Reflect

Knowledge

1. Students' answers will vary. Examples of objects that move with uniform circular motion include:
- The propeller of an aircraft turning with a constant speed.
- The wheel of a car turning with a constant speed.
- The motion of the planets around the Sun, or moons around a planet. (This motion can be considered uniform until section 5.3.)
- Any circular motion where the speed is uniform.

Examples of non-uniform circular motion could include:
- The propeller of an aircraft that is slowing down or speeding up.
- The wheel of a car that is slowing down or speeding up.
- Any circular motion where the speed is not constant.

2. The centripetal force acting in each case is:
(a) friction between the wheels and the road
(b) tension of the rope
(c) force of gravity between the Moon and Earth

Applications

3. The speed varies with the square root of the radius.
4. The speed is the magnitude of the velocity. If the circular motion is uniform, the speed will be constant. The velocity is continually changing as the object's direction changes.

Extensions

5. If the wheels were oval, the bike would experience acceleration and deceleration each turn. The motion of the bike would not be uniform.

6. When the pebble comes in contact with the ground, it is not moving relative to the ground. This is why it does not easily dislodge. To get it to dislodge, there are two strategies:

- Stop the wheel from turning while the pebble is in contact with the ground. This has the effect of making the pebble move relative to the ground, and friction might wrench it loose.
- Speed up so that the frictional force holding the pebble in the tread of the wheel is insufficient to provide the centripetal force necessary to keep it moving in a circular path. The pebble will fly off at a tangent.

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Example 5.1 Practice Problems

1. ***Analysis and Solution***

$$\begin{aligned} f &= \frac{300\text{ rev}}{1\text{ s}} \\ &= \frac{1\text{ min}}{60\text{ s}} \\ &= \frac{5.00\text{ rev}}{\text{s}} \\ &= 5.00\text{ Hz} \end{aligned}$$

The frequency of the propeller is 5.00 Hz.

2. ***Analysis and Solution***

$$f = 40\text{ Hz}$$

$$\begin{aligned} \text{rpm} &= f \times \frac{60\text{s}}{\text{min}} \\ &= (40\,\frac{\text{rev}}{\cancel{\text{s}}})\frac{60\,\cancel{\text{s}}}{\text{min}} \\ &= 2.4 \times 10^3\text{ rpm} \end{aligned}$$

The rotational frequency of the motor is 2.4×10^3 rpm .

3. ***Analysis and Solution***

$$\begin{aligned} f &= 6.0\times10^4\text{ rpm} \\ &= 6.0\times10^4\,\frac{\text{rev}}{\cancel{\text{min}}}\times\frac{1\,\cancel{\text{min}}}{60\text{s}} \\ &= 1.0\times10^3\,\frac{\text{rev}}{\text{s}} \\ &= 1.0\times10^3\text{ Hz} \end{aligned}$$

$$\begin{aligned} T &= \frac{1}{f} \\ &= \frac{1}{1.0 \times 10^3\text{ Hz}} \\ &= 1.0 \times 10^{-3}\text{ s} \end{aligned}$$

The centrifuge's frequency is 1.0×10^3 Hz, and its period is 1.0×10^{-3} s.

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Example 5.2 Practice Problems

1. ***Given***

$v = 261.0$ km/h

$r = 0.350$ m

Required

period of the racecar's wheels (T)

Analysis and Solution

The speed of the outer edge of the wheel is the same as the racecar. To determine the period, manipulate the equation for the velocity around a circle to solve for T. Convert the car's speed to appropriate SI units before using it in the equation.

$$
\begin{aligned}
v &= \frac{261.0\ \cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1\ \cancel{\text{h}}}{3600\ \text{s}} \times \frac{1000\,\text{m}}{1\ \cancel{\text{km}}} \\
&= 72.50\,\text{m/s}
\end{aligned}
$$

$$
\begin{aligned}
v &= \frac{2\pi r}{T} \\
T &= \frac{2\pi r}{v} \\
&= \frac{2\pi\left(0.350\ \cancel{\text{m}}\right)}{72.50\ \frac{\cancel{\text{m}}}{\text{s}}} \\
&= 0.0303\,\text{s}
\end{aligned}
$$

Paraphrase

The period of the racecar's wheels when it is travelling at 261.0 km/h is 0.0303 s.

2. ***Given***

$r = 16.1$ km

$f = 716$ Hz

Required

speed at the pulsar's equator (v)

Analysis and Solution

Any point on the equator is at a radius of 16.1 km from the center of the pulsar. The speed can be determined by converting the frequency to period and using equation 1.

$$\begin{aligned} T &= \frac{1}{f} \\ &= \frac{1}{716 \text{ Hz}} \\ &= 0.001397 \text{ s} \end{aligned}$$

$$\begin{aligned} v &= \frac{2\pi r}{T} \\ &= \frac{2\pi(1.61 \times 10^4)}{0.001397 \text{ s}} \\ &= 7.24 \times 10^7 \text{ m/s} \end{aligned}$$

Paraphrase

The speed at the pulsar's equator is 7.24×10^7 m/s.

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Example 5.3 Practice Problems

1. ***Given***

$D = 28.0$ cm

$T = 0.110$ s

Required

centripetal acceleration (a_c)

Analysis and Solution

First, determine the speed at the outer edge of the Frisbee. Then use the equation for centripetal acceleration.

$$\begin{aligned} v &= \frac{2\pi r}{T} \\ &= \frac{2\pi(0.140 \text{ m})}{0.110 \text{ s}} \\ &= 7.997 \text{ m/s} \end{aligned}$$

$$\begin{aligned} a_c &= \frac{v^2}{r} \\ &= \frac{\left(7.997 \dfrac{\text{m}}{\text{s}}\right)^2}{0.140 \text{ m}} \\ &= 4.57 \times 10^2 \text{ m/s}^2 \end{aligned}$$

Paraphrase

The centripetal acceleration at the edge of the Frisbee is 4.57×10^2 m/s^2.

2. ***Given***

$a_c = 125.0 \text{ m/s}^2$

$r = 3.00$ cm

Required

speed at the edge of the spinning top (v)

Analysis and Solution
Determine the speed by manipulating the equation for centripetal acceleration.

$$a_c = \frac{v^2}{r}$$

$$\begin{aligned} v &= \sqrt{a_c r} \\ &= \sqrt{\left(125.0\ \frac{\text{m}}{\text{s}^2}\right)(0.0300\ \text{m})} \\ &= 1.94\ \text{m/s} \end{aligned}$$

Paraphrase
The speed at the edge of the spinning top is 1.94 m/s.

3. ***Given***
$D = 14.0$ m
$a_c = 2527.0\ \text{m/s}^2$

Required
period (T)

Analysis and Solution
First, calculate the radius of the blade. Then determine the speed of the rotor blades so that you can find the period using the equation for centripetal acceleration.

$$D = 14.0\,\text{m}$$

$$\begin{aligned} r &= \frac{D}{2} \\ &= \frac{14.0\,\text{m}}{2} \\ &= 7.00\,\text{m} \end{aligned}$$

$$a_c = \frac{v^2}{r}$$

$$\begin{aligned} v &= \sqrt{a_c r} \\ &= \sqrt{\left(2527.0\frac{\text{m}}{\text{s}^2}\right)(7.00\,\text{m})} \\ &= 133.0\,\text{m/s} \end{aligned}$$

$$v = \frac{2\pi \text{r}}{T}$$

$$\begin{aligned} T &= \frac{2\pi r}{v} \\ &= \frac{2\pi(7.00\ \cancel{\text{m}})}{133.0\frac{\cancel{\text{m}}}{\text{s}}} \\ &= 0.331\,\text{s} \end{aligned}$$

Paraphrase
The period of the helicopter blade is 0.331 s.

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Example 5.4 Practice Problems

1. ***Given***

m = 7.50 kg
v = 365.9 m/s
r = 73.7 cm

Required

centripetal force on the blade (F_c)

Analysis and Solution

For this question, it is best to assume that the entire mass of the fan blade is at the centre of the blade. This is often referred to as the centre of mass. You can determine the centripetal force on the blade most accurately by using the radius from the centre of rotation to the centre of mass.

$$\begin{aligned} F_c &= \frac{mv^2}{r} \\ &= \frac{(7.50\text{ kg})\left(365.9\,\frac{\text{m}}{\text{s}}\right)^2}{0.737\text{ m}} \\ &= 1.36 \text{ x } 10^6 \text{ m/s} \end{aligned}$$

Paraphrase

The centripetal force on the blade is 1.36×10^6 N.

2. ***Analysis and Solution***

$$\begin{aligned} F_c &= \frac{mv^2}{r} \\ v &= \sqrt{\frac{F_c r}{m}} \\ &= \sqrt{\frac{(0.660\,\text{N})(0.230\,\text{m})}{0.00210\,\text{kg}}} \\ &= 8.50\,\text{m/s} \end{aligned}$$

The speed of the wheel is 8.50 m/s.

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Example 5.5 Practice Problems

1. ***Given***

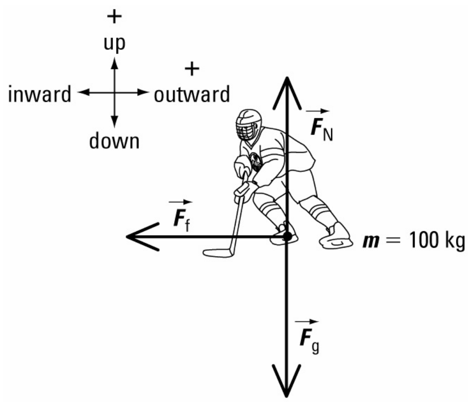
m = 100 kg
r = 7.17 m
μ = 0.80

Required

speed of the hockey player as he begins to slip (v)

Analysis and Solution

The force of friction provides the centripetal force. Start with this equality and solve for speed.



$$F_c = F_f$$

$$\frac{mv^2}{r} = \mu F_N \qquad\qquad F_N = -F_g$$

$$\frac{\cancel{m}v^2}{r} = \mu \cancel{m}g$$

$$v = \sqrt{\mu r g}$$

$$= \sqrt{(0.80)(7.17\,\text{m})\left(9.81\frac{\text{m}}{\text{s}^2}\right)}$$

$$= 7.5\,\text{m/s}$$

Paraphrase

The speed of the hockey player is 7.5 m/s.

2. ***Given***

r = 100 m

m = 1200 kg

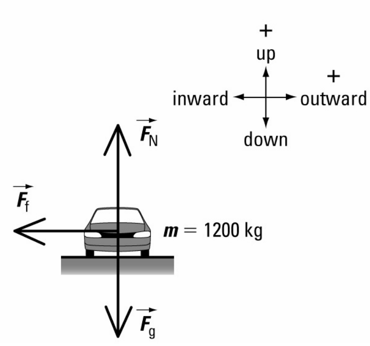
v = 95.0 km/h

Required

coefficient of friction (μ)

Analysis and Solution

Convert the speed of the car to appropriate SI units. The force of friction is the centripetal force, so you can write the equation $F_c = F_f$, and then solve for μ. Mass will cancel and is not needed.



$$v = \frac{95.0\ \cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1\ \cancel{\text{h}}}{3600\ \text{s}} \times \frac{1000\ \text{m}}{1\ \cancel{\text{km}}}$$

$$= 26.4\ \text{m/s}$$

$$\frac{\cancel{m} v^2}{r} = \cancel{m} \mu g$$

$$\frac{v^2}{r} = \mu g$$

$$\mu = \frac{v^2}{rg}$$

$$= \frac{\left(26.4\ \frac{\text{m}}{\text{s}}\right)^2}{(100\ \text{m})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}$$

$$= 0.710$$

Paraphrase

The coefficient of friction is 0.710.

3. ***Given***

m = 600.0 g

v = 3.0 m/s

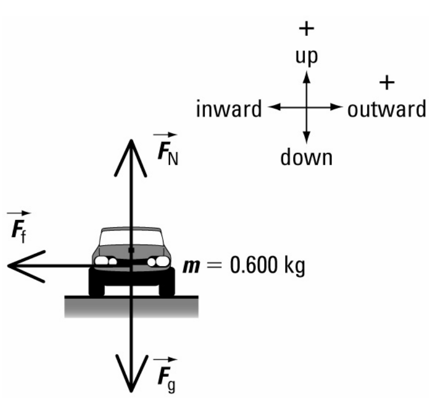
μ = 0.90

Required

radius (r)

Analysis and Solution

The force of friction is the centripetal force, so $F_c = F_f$. Solve for the radius.



$$
\begin{aligned}
F_c &= F_f \\
\frac{\cancel{m}v^2}{r} &= \mu \cancel{m}g \\
r &= \frac{v^2}{\mu g} \\
&= \frac{\left(3.0\,\frac{\text{m}}{\text{s}}\right)^2}{(0.90)\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)} \\
&= 1.0\,\text{m}
\end{aligned}
$$

Paraphrase

The radius of the car's turn is 1.0 m.

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Concept Check

The centripetal force exerted on the Moon is the force of gravity between Earth and the Moon. It will not change if the velocity of the Moon decreases, so the orbital radius must also decrease to maintain the same centripetal force. Similarly, if the Moon's velocity were to increase, its orbital radius would also increase because the centripetal force must remain the same.

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Example 5.6 Practice Problems

1. ***Given***

$r = 0.15$ m

$g = 9.81$ m/s^2

Required

speed (v)

Analysis and Solution

The minimum speed the car can have without falling off is determined by $F_c = F_g$. By equating the two forces, you can solve for the minimum speed.

$$
\begin{aligned}
F_c &= F_g \\
\frac{\cancel{m}v^2}{r} &= \cancel{m}g \\
v^2 &= rg \\
v &= \sqrt{rg} \\
&= \sqrt{(0.15\text{ m})\left(9.81\,\frac{\text{m}}{\text{s}^2}\right)} \\
&= 1.2\,\text{m/s}
\end{aligned}
$$

Paraphrase
The minimum speed required for the car to move around the loop without falling off is 1.2 m/s.

2. ***Given***
$v = 20.0$ m/s
Required
radius of the loop (r)
Analysis and Solution
Determine the radius by equating $F_c = F_g$. The mass is not necessary as it will cancel out.

$$F_c = F_g$$

$$\frac{\cancel{m}v^2}{r} = \cancel{m}g$$

$$r = \frac{v^2}{g}$$

$$= \frac{\left(20.0\ \frac{\text{m}}{\text{s}}\right)^2}{9.81\ \frac{\text{m}}{\text{s}^2}}$$

$$= 40.8\ \text{m}$$

Paraphrase
The largest radius that the roller coaster loop can have is 40.8 m.

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Concept Check

1. The rope is most likely to break when the bucket is in the bottom of the swing because this is when the tension on the rope is the greatest. The tension exerted by the rope provides the centripetal force alone, whereas at the top of the swing the force of gravity provides a portion of the centripetal force.
2. The centripetal force does not depend on an object's position. It is always the same as long as the speed does not change.

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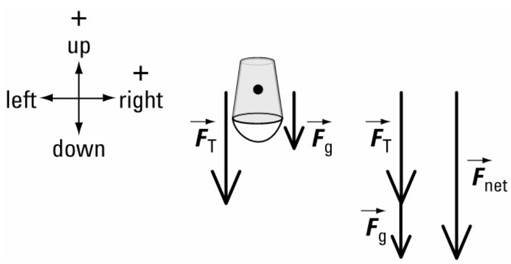
Example 5.7 Practice Problems

Note: For Practice Problems 1 and 2, the bucket is moving with uniform circular motion. The magnitude of the centripetal force is constant in all positions.

1. ***Given***
$m = 1.5$ kg
$r = 0.75$ m
$v = 3.00$ m/s
$g = 9.81$ m/s^2 [down]

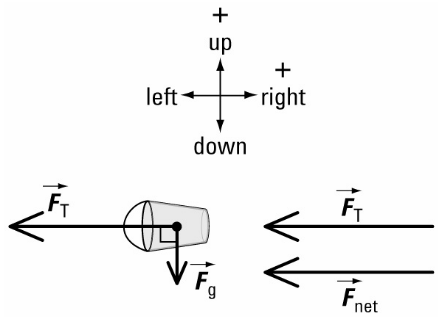
Required
tension on the rope at the top of its swing (T)
Analysis and Solution
The centripetal force is a net force that, in this case, is the sum of the tension and force of gravity. Use the same reference coordinates as in the example. Take up to be positive and down to be negative. At the top of the bucket's swing, the tension and force of gravity are both down. When you substitute them into the equation for centripetal force, they will be negative.



$$
\begin{aligned}
\vec{F}_c &= \vec{F}_T + \vec{F}_g \\
\vec{F}_T &= \vec{F}_c - \vec{F}_g \\
&= \left(-\frac{mv^2}{r}\right) - (m\vec{g}) \\
&= \left(-\frac{(1.5\text{ kg})\left(3.00\ \frac{\text{m}}{\text{s}}\right)^2}{0.75\text{ m}}\right) - \left((1.5\text{ kg})\left(-9.81\ \frac{\text{m}}{\text{s}^2}\right)\right) \\
&= (-18\,\text{N}) - (-14.715\,\text{N}) \\
&= -18\,\text{N} + 14.715\,\text{N} \\
&= -3.3\,\text{N}
\end{aligned}
$$

Paraphrase
The tension on the rope at position A is 3.3 N [down].

2. ***Given***
$m = 1.5$ kg
$r = 0.75$ m
$v = 3.00$ m/s
$g = 9.81$ m/s^2 [down]
Required
tension on the rope when the rope is parallel to the ground (F_T)
Analysis and Solution
In this position, the tension is perpendicular to the force of gravity, and they are independent vectors. The force of gravity does not affect the tension. The tension is the centripetal force.



$$\begin{aligned}\vec{F}_T &= \vec{F}_c \\ &= \frac{mv^2}{r} \\ &= \frac{(1.5\text{ kg})\left(3.00\ \frac{\text{m}}{\text{s}}\right)^2}{0.75\,\text{m}} \\ &= 18\,\text{N [left]}\end{aligned}$$

Paraphrase

The tension on the rope in position B, is 18 N [left].

3. ***Given***

m = 0.98 kg

r = 0.40 m

F_T = 79.0 N [down]

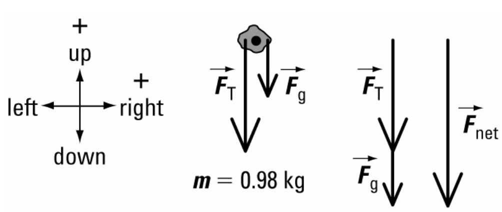
g = 9.81 m/s^2 [down]

Required

speed of the rock (v)

Analysis and Solution

To determine the speed of the rock, you need to know the centripetal force. At the top of the swing, the centripetal force is the sum of the tension and the force of gravity, which are both downward. Draw a diagram to show the reference coordinates, where up is positive and down is negative. Once you determine the centripetal force, find the speed of the rock by manipulating the equation and solving for v.



$$\begin{aligned}\vec{F}_c &= \vec{F}_T + \vec{F}_g \\ &= (-79.0\text{N}) + \left((0.98\,\text{kg})\left(-9.81\frac{\text{m}}{\text{s}^2}\right)\right) \\ &= -79.0\,\text{N} + -9.6138\,\text{N} \\ &= -88.6138\,\text{N}\end{aligned}$$

It is not necessary to keep the negative sign because you are solving for speed, which is a scalar quantity.

$$F_c = \frac{mv^2}{r}$$
$$v = \sqrt{\frac{F_c r}{m}}$$
$$= \sqrt{\frac{(88.6138\,\text{N})(0.40\,\text{m})}{0.98\,\text{kg}}}$$
$$= 6.0\,\text{m/s}$$

Paraphrase

The speed of the rock at the top of its circular path is 6.0 m/s.

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Example 5.8 Practice Problems

1. ***Analysis and Solution***

$$a_c = 4\pi^2 r f^2$$
$$f = \sqrt{\frac{a_c}{4\pi^2 r}}$$
$$= \sqrt{\frac{9.81\,\frac{\text{m}}{\text{s}^2}}{4\pi^2(30.0\text{ m})}}$$
$$= 0.0910\text{ Hz}$$
$$= 9.10 \times 10^{-2}\text{ Hz}$$

For the space station to simulate Earth's gravity, it would have to spin at a frequency of 9.10×10^{-2} Hz.

2. ***Given***

m = 454.0 g
r = 1.50 m
f = 150.0 rpm

Required

centripetal force (F_c)

Analysis and Solution

Convert frequency into SI units and substitute it into the equation.

$$f = \frac{150.0\,\text{rpm}}{60}$$
$$= 2.5\,\text{Hz}$$
$$F_c = 4\pi^2 rmf^2$$
$$= 4\pi^2(1.50\text{ m})(0.454\text{ kg})(2.5\text{ Hz})^2$$
$$= 1.68 \times 10^2\text{ N}$$

Paraphrase
The centripetal force acting on the mass is 1.68×10^2 N.

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5.2 Check and Reflect

Knowledge

1. The centripetal force is acting south.

2. The Moon experiences centripetal acceleration as it is pulled toward Earth by the centripetal force, and gravity.

3. Students' answers will vary. Possible answers include:
- increasing the speed
- decreasing the radius
- increasing the frequency
- decreasing the period

4. **(a)** The force of gravity does not vary as the object moves.
(b) The tension varies throughout the object's motion. The tension is least at the top of the circle and greatest at the bottom.

5. The air pressure exerted on the control surfaces of the plane (the ailerons and rudder) acts as the centripetal force.

Applications

6. ***Analysis and Solution***

$$\begin{aligned} v &= \frac{2\pi r}{T} \\ T &= \frac{2\pi r}{v} \\ &= \frac{2\pi(0.5\text{ m})}{15.0\ \frac{\text{m}}{\text{s}}} \\ &= 0.2\text{ s} \end{aligned}$$

The period of rotation of the wheels is 0.2 s.

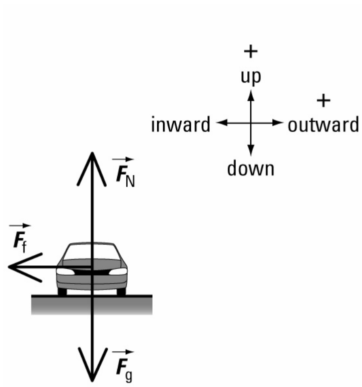
7. ***Analysis and Solution***

$$\begin{aligned} v &= \frac{2\pi r}{T} \\ &= \frac{2\pi(1.20\text{ m})}{0.0400\text{ s}} \\ &= 1.88\times10^2\text{ m/s} \end{aligned}$$

The velocity at the tip of the propeller blade is 1.88×10^2 m/s.

8. ***Given***
$m = 1500$ kg
$r = 100.0$ m
$\mu = 0.70$
Required
maximum speed with which the car can make the turn without skidding off the curve (v)

Analysis and Solution
The force of friction is the centripetal force. Use $F_f = F_c$, and solve for speed.



$$F_f = F_c$$

$$\mu F_N = \frac{mv^2}{r} \qquad\qquad F_N = F_g$$

$$\mu(\cancel{m}g) = \frac{\cancel{m}v^2}{r}$$

$$v = \sqrt{\mu r g}$$

$$= \sqrt{(0.70)(100.0\ \text{m})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}$$

$$= 2.6 \times 10^1\ \text{m/s}$$

Paraphrase
The maximum speed that the car must have to make the turn is 26 m/s.

9. ***Analysis and Solution***

$$v = \frac{100\ \cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1\ \cancel{\text{h}}}{3600\,\text{s}} \times \frac{1000\,\text{m}}{1\ \cancel{\text{km}}}$$

$$= 27.\overline{7}\ \text{m/s}$$

$$a_c = \frac{v^2}{r}$$

$$= \frac{\left(27.\overline{7}\ \frac{\text{m}}{\text{s}}\right)^2}{90.0\ \text{m}}$$

$$= 8.57\ \text{m/s}^2$$

The centripetal acceleration of the car is 8.57 m/s^2.

10. ***Given***
r = 8.9 m
f = 35 rpm
Required
centripetal acceleration (a_c)
magnitude of the centripetal acceleration in comparison to gravity (g = 9.81 m/s^2)

Analysis and Solution

First convert frequency to SI units and determine the magnitude of the centripetal acceleration of the arm. Then determine how many times bigger or smaller it is than gravity by using a simple ratio.

$$\begin{aligned} f &= 35\frac{\text{rev}}{\cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\text{ s}} \\ &= 0.583\,\text{Hz} \end{aligned}$$

$$\begin{aligned} a_c &= 4\pi^2 rf^2 \\ &= 4\pi^2(8.9\text{ m})(0.583\text{ Hz})^2 \\ &= 1.2 \times 10^2\text{ m/s}^2 \end{aligned}$$

The ratio of this acceleration to gravity is:

$$\frac{1.2 \times 10^2\ \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}}{9.81\ \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}} = 12$$

Paraphrase

The astronaut experiences an acceleration of 1.2×10^2 m/s^2, which is 12 times greater than the acceleration of gravity.

11. ***Given***

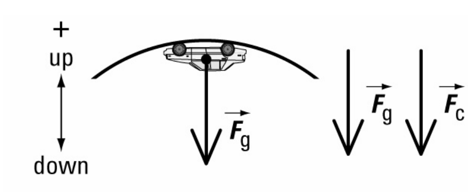
D = 30.0 cm

Required

minimum speed necessary to successfully move through the loop (v)

Analysis and Solution

At the top of the loop, the minimum speed occurs when the centripetal force is the force of gravity only, $F_c = F_g$. Use this equality to solve for the speed. Remember to convert the diameter to radius.



$$
\begin{aligned}
r &= \frac{D}{2} \\
&= \frac{0.300\ \text{m}}{2} \\
&= 0.150\ \text{m}
\end{aligned}
$$

$$
\begin{aligned}
F_{\text{c}} &= F_{\text{g}} \\
\frac{\cancel{m}v^2}{r} &= \cancel{m}g \\
v &= \sqrt{rg} \\
&= \sqrt{(0.150\ \text{m})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)} \\
&= 1.21\ \text{m/s}
\end{aligned}
$$

Paraphrase

The minimum speed that the toy racecar must have to go around the loop is 1.21 m/s.

12. ***Analysis and Solution***

$$
\begin{aligned}
T &= 24\ \text{h} \times 3600\ \text{s} \\
&= 86\,400\ \text{s}
\end{aligned}
$$

$$
\begin{aligned}
a_{\text{c}} &= \frac{4\pi^2 r}{T^2} \\
&= \frac{4\pi^2\left(6.37\times10^6\ \text{m}\right)}{\left(86\,400\ \text{s}\right)^2} \\
&= 0.0337\ \text{m/s}^2
\end{aligned}
$$

The centripetal acceleration of a person standing on the equator is 0.0337 m/s^2.

13. ***Given***

$r = 0.40$ m

$m = 0.010$ g

$F_{\text{f}} = 4.34 \times 10^{-4}$ N

Required

frequency of the tire's rotation that will fling the ant off the tire (*f*)

Analysis and Solution

The centripetal force is caused by the ant holding onto the tire (force of friction). The faster the tire rotates, the more force the ant must apply to stay on. Since the maximum force the ant can apply is 4.34×10^{-4} N, you need to determine the frequency that creates this force. Any higher frequency will cause the ant to fly off. Remember to convert the mass to the appropriate SI units.

$$F_c = F_f$$
$$= 4.34 \times 10^{-4}\ \text{N}$$
$$F_c = 4\pi^2 mrf^2$$
$$f = \sqrt{\frac{F_c}{4\pi^2 mr}}$$
$$= \sqrt{\frac{4.34 \times 10^{-4}\ \text{N}}{4\pi^2 (0.000010\ \text{kg})(0.40\ \text{m})}}$$
$$= 1.7\ \text{Hz}$$

Paraphrase

The ant will fly off the tire if the frequency of the wheel exceeds 1.7 Hz.

Extensions

14. ***Given***

frequency of pulley 1, f_1 = 200.0 rpm

Required

frequency of the larger pulley (f_2)

Analysis and Solution

Since both pulleys are connected by a belt along their outer rim, the speed of the two pulleys at their outer rim is the same. Write the equality $v_1 = v_2$ and solve using the equation for velocity, $v = \frac{2\pi r}{T}$.

$$f_1 = 200.0\ \text{rpm}$$
$$= \frac{200\ \text{rpm}}{60\ \text{s}}$$
$$= 3.33\ \text{Hz}$$
$$v = \frac{2\pi r}{T} \qquad f = \frac{1}{T}$$
$$= 2\pi rf$$
$$v_1 = v_2$$
$$\cancel{2\pi} r_1 f_1 = \cancel{2\pi} r_2 f_2$$
$$r_1 f_1 = r_2 f_2$$
$$f_2 = \frac{r_1 f_1}{r_2}$$
$$= \frac{(0.10\ \text{m})(3.33\ \text{Hz})}{0.25\ \text{m}}$$
$$= 1.3\ \text{Hz}$$

Paraphrase

The frequency of the larger pulley is 1.3 Hz or 78 rpm.

15. The car that has the larger turning radius will have the advantage because it had to move faster in the turn in order to stay beside the other car. As it comes out of the turn, it will be travelling faster than the other car.

Example 5.9 Practice Problems

1. ***Analysis and Solution***

$$T_{\mathrm{J}}^{2} = Kr_{\mathrm{J}}^{3}$$

where $K = 1\,\mathrm{y^2/AU^3}$

$$\begin{aligned} T_{\mathrm{J}} &= \sqrt{\left(1\,\frac{\mathrm{y}^2}{\mathrm{AU}^3}\right)(5.203\,\mathrm{AU})^3} \\ &= 11.87\,\mathrm{y} \end{aligned}$$

The orbital period of Jupiter is 11.87 Earth years.

2. ***Given***

$T_{\mathrm{p}} = 90\ 553$ d

Required

mean orbital radius (r_{p})

Analysis and Solution

Use Kepler's constant relating period and radius. To keep the same units, convert Pluto's period to years.

$$T_{\mathrm{p}} = \frac{90553\ \cancel{\mathrm{d}}}{365.25\,\frac{\cancel{\mathrm{d}}}{\mathrm{y}}} = 247.92\ \mathrm{y}$$

$$T_{\mathrm{P}}^{2} = Kr_{\mathrm{P}}^{3}$$

where $K = 1\,\mathrm{y^2/AU^3}$

$$\begin{aligned} r_{\mathrm{P}} &= \sqrt[3]{\frac{T_{\mathrm{P}}^{2}}{K}} \\ &= \sqrt[3]{\frac{(247.92\,\mathrm{y})^2}{1\,\frac{\mathrm{y}^2}{\mathrm{AU}^3}}} \\ &= 39.465\,\mathrm{AU} \end{aligned}$$

Paraphrase

The mean orbital radius of Pluto is 39.465 AU.

3. ***Analysis and Solution***

$$T_{\mathrm{d}}^{2} = Kr_{\mathrm{d}}^{3}$$

where $K = 1\,\mathrm{y^2/AU^3}$

$$\begin{aligned} T_{\mathrm{d}} &= \sqrt{Kr_{\mathrm{d}}^{3}} \\ &= \sqrt{\left(1\,\frac{\mathrm{y}^2}{\mathrm{AU}^3}\right)(45.0\,\mathrm{AU})^3} \\ &= 302\,\mathrm{y} \end{aligned}$$

The orbital period of the debris is 302 Earth years.

Student Book page 274

Concept Check

1. The orbit would be perfectly circular. Students could draw an "ellipse" with a semi-major axis equal to the semi-minor axis to see for themselves.

2. Earth's Moon and Charon orbit different foci. The Moon orbits Earth, and Charon orbits Pluto. Kepler's third law applies only to objects orbiting the same focus.

3. It is possible but not likely. The mass of the planet has nothing to do with its orbital radius and does not make a difference either way. However, there is no guarantee that Kepler's constant for our solar system could be used in this new solar system because the planet would have to have both the same orbital period and the same orbital radius. This may or may not be the case, depending on the mass of the new solar system's Sun.

4. Astronomers could pick one of the six planets at random and use its orbital period and radius to determine a new Kepler's constant that applies to that solar system. The new constant would then apply to all the planets in that system.

5. Planets orbit stars with a period that depends on their orbital radius as shown by Kepler's third law: $T_a^2 = Kr_a^3$, where "a" is any planet. Points moving on a rotating disk as shown in Figure 5.34 on page 265 in the Student Book all have the same period regardless of their orbital radius.

Student Book page 275

Example 5.10 Practice Problems

1. ***Analysis and Solution***

$$\frac{T_T^2}{r_T^3} = \frac{T_J^2}{r_J^3}$$

$$\begin{aligned} T_T &= \sqrt{\frac{\left(T_J^2\right)\left(r_T^3\right)}{r_J^3}} \\ &= \sqrt{\frac{(0.694\ \text{d})^2(147\,\text{d})^2}{\left(1.51472\times10^8\,\text{m}\right)^3}} \\ &= 15.9\ \text{d} \end{aligned}$$

Titan's orbital period is 15.9 days.

2. ***Given***

$T_{CH} = 147\,\text{d}$

Required

radius of Cassini-Huygens (r_{CH})

Analysis and Solution

To determine the average orbital radius of the Cassini-Huygens probe, you cannot use Kepler's constant as it only applies to planets orbiting the Sun. Instead use the moon Tethys and Kepler's third law.

$$\frac{T_{CH}^{\ 2}}{r_{CH}^{\ 3}} = \frac{T_T^{\ 2}}{r_T^{\ 3}}$$

$$r_{CH} = \sqrt[3]{\frac{r_T^{\ 3} T_{CH}^{\ 2}}{T_T^{\ 2}}}$$

$$= \sqrt[3]{\frac{(2.9466 \times 10^8\ \text{m})^3 (147\,\text{d})^2}{(1.887\,\text{d})^2}}$$

$$= 5.38 \times 10^9\ \text{m}$$

$$= 5.38 \times 10^6\ \text{km}$$

Paraphrase

The mean orbital radius of the Cassini-Huygens probe is 5.38×10^6 km.

3. ***Analysis and Solution***

The nearest moon is Callisto.

$$\frac{T_X^{\ 2}}{r_X^{\ 3}} = \frac{T_C^{\ 2}}{r_C^{\ 3}}$$

$$T_X = \sqrt{\frac{r_X^{\ 3} T_C^{\ 2}}{r_C^{\ 3}}}$$

$$= \sqrt{\frac{\left(9.38 \times 10^9\ \text{m}\right)^3 \left(16.689\,\text{d}\right)^2}{\left(1.883 \times 10^9\,\text{m}\right)^3}}$$

$$= 186\,\text{d}$$

The orbital period of moon X is 186 days.

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Example 5.11 Practice Problems

1. ***Analysis and Solution***

$$F_c = F_g$$

$$\frac{\cancel{m_N} v^2}{\cancel{r}} = \frac{G \cancel{m_N} m_{Sun}}{r^{\cancel{2}}}$$

$$v^2 = \frac{Gm_{Sun}}{\text{r}}$$

$$v = \sqrt{\frac{Gm_{Sun}}{r}}$$

$$= \sqrt{\frac{\left(6.67 \times 10^{-11}\ \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}\right)\left(1.99 \times 10^{30}\ \text{kg}\right)}{4.50 \times 10^{12}\ \text{m}}}$$

$$= 5.43 \times 10^3\ \text{m/s}$$

Neptune's orbital speed is 5.43×10^3 m/s.

2. *Analysis and Solution*

$$F_c = F_g$$

$$\frac{\cancel{m_M} v^2}{\cancel{r}} = \frac{G\cancel{m_M} m_U}{r^{\cancel{2}}}$$

$$v^2 = \frac{Gm_U}{\mathrm{r}}$$

$$r = \frac{Gm_U}{v^2}$$

$$= \frac{\left(6.67 \times 10^{-11} \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}\right)\left(8.68 \times 10^{25} \text{ kg}\right)}{\left(6.68 \times 10^3 \frac{\text{m}}{\text{s}}\right)^2}$$

$$= 1.30 \times 10^8 \text{ m}$$

The radius of Miranda's orbit is 1.30×10^8 m.

Student Book page 279

Example 5.12 Practice Problems

1. *Given*

height of the space station above Earth, altitude = 359.2 km

Required

orbital speed of the space station (v)

Analysis and Solution

The space station is an artificial satellite that is relatively near to Earth. Add Earth's radius to its altitude so you can determine the space station's average speed.

$$\begin{aligned} \text{altitude } &= 359.2 \text{ km} \\ &= 3.59 \times 10^5 \text{ m} \end{aligned}$$

Distance from the centre of Earth:

$$\begin{aligned} \text{radius of Earth} + \text{altitude} &= 6.37 \times 10^6 \text{ m} + 3.59 \times 10^5 \text{ m} \\ &= 6.7292 \times 10^6 \text{ m} \end{aligned}$$

$$F_c = F_g$$

$$\frac{\cancel{m_{ISS}} v^2}{\cancel{r}} = \frac{G\cancel{m_{ISS}} m_{Earth}}{r^{\cancel{2}}}$$

$$v^2 = \frac{Gm_{Earth}}{\mathrm{r}}$$

$$v = \sqrt{\frac{Gm_{Earth}}{r}}$$

$$= \sqrt{\frac{\left(6.67 \times 10^{-11}\ \frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}\right)(5.98 \times 10^{24}\ \mathrm{kg})}{6.7292 \times 10^6\ \mathrm{m}}}$$

$$= 7.70 \times 10^3\ \mathrm{m/s}$$

Paraphrase

The orbital speed of the International Space Station is 7.70×10^3 m/s.

2. ***Given***

height of Chandra above Earth,
altitude = 114 593 km

Required

orbital period of Chandra (r)

Analysis and Solution

Chandra is an artificial satellite that is relatively near to Earth. Add Earth's radius to its altitude so you can determine Chandra's orbital radius.

$$\begin{aligned} \text{altitude} &= 114\ 593\ \mathrm{km} \\ &= 1.145\ 93 \times 10^8\ \mathrm{m} \end{aligned}$$

Distance from the centre of Earth:

$$\begin{aligned} \text{radius of Earth} + \text{altitude} &= 6.37 \times 10^6\ \mathrm{m} + 1.145\ 93 \times 10^8\ \mathrm{m} \\ &= 1.209\ 63 \times 10^8\ \mathrm{m} \end{aligned}$$

$$F_c = F_g$$

$$\frac{\cancel{m}_{\text{Chandra}} v^2}{\cancel{r}} = \frac{G\cancel{m}_{\text{Chandra}} m_{\text{Earth}}}{r^{\cancel{2}}}$$

$$v^2 = \frac{Gm_{\text{Earth}}}{\text{r}}$$

$$v = \sqrt{\frac{Gm_{\text{Earth}}}{r}}$$

$$= \sqrt{\frac{\left(6.67 \times 10^{-11} \frac{\text{N•m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\,\text{kg})}{1.209\ 63 \times 10^8\ \text{m}}}$$

$$= 1.816 \times 10^3\ \text{m/s}$$

$$v = \frac{2\pi r}{T}$$

$$T = \frac{2\pi r}{v}$$

$$= \frac{2\pi(1.209\ 63 \times 10^8\ \cancel{\text{m}})}{1.816 \times 10^3\ \frac{\cancel{\text{m}}}{\text{s}}}$$

$$= 4.19 \times 10^5\ \text{s}$$

Paraphrase

The orbital period of Chandra is 4.19×10^5 s.

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Concept Check

1. It occurred to Newton that the force of gravity may not be confined to Earth but could also exist out in space. He reasoned that the gravitational attraction between Earth and the Moon was responsible for the centripetal force, and therefore the Moon's orbit.

2. Newton knew that gravity existed only between objects that have mass. He also knew it was the force of gravity that was responsible for the centripetal force on planets. If he were told our solar system is orbiting the centre of our galaxy, then he would assume that there is a large mass there to create the gravitational, and hence centripetal force.

3. By equating the force of gravity with the centripetal force, Newton was able to substitute his equation for the force of gravity to determine the mass of an object being orbited. He used it to determine the mass of Earth using the Moon's orbital radius and period.

Student Book page 286

5.3 Check and Reflect

Knowledge

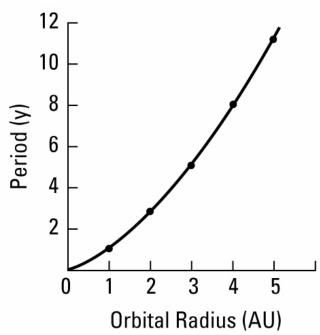
1. An astronomical unit is the length of the semi-major axis of Earth's orbit. This is the same as the mean orbital radius of Earth.

2. The orbital radius of a planet is not constant because planetary orbits are ellipses. An ellipse is an elongated circle with a radius that is not constant.
3. The larger the eccentricity, the more elongated (or squashed) the circle. Therefore, an eccentricity of 0.9 is very elongated. (A comet might have an eccentricity like this.)
4. A planet's orbital velocity is greatest at its perihelion, where it is closest in its orbit to the Sun. Its slowest speed occurs when it is farthest from the Sun at its aphelion.
5. For Kepler's third law to be valid, the two satellites that are equated must orbit the same focus.
6. Kepler's constant for moons orbiting the same planet is not the same as the constant for planets orbiting the Sun. The constant is different because the moons are orbiting a different focus than the planets are.
7. The Sun does experience orbital perturbation. It wobbles in its orbit because the combined mass of the planets causes it to revolve around their common centre of gravity.

Applications

8. ***Analysis and Solution***

The relationship between a planet's orbital period and its radius is given by Kepler's third law: $T \propto \sqrt{r^3}$ or $T = Kr^{3/2}$. Graphing this relation yields:



The graph of a planet's period as a function of its orbital radius will look like the graph above.

9. ***Analysis and Solution***

$$T_V{}^2 = Kr_V{}^3 \qquad \text{where } K = 1\,\text{y}^2/\text{AU}^3$$

$$\begin{aligned} r_V &= \sqrt[3]{\frac{T_V{}^2}{K}} \\ &= \sqrt[3]{\frac{(0.615\,\text{y})^2}{\frac{1\,\text{y}^2}{\text{AU}^3}}} \\ &= 0.723\,\text{AU} \end{aligned}$$

The orbital radius of Venus is 0.723 AU.

10. ***Analysis and Solution***

Determine the speed of Sedna from its orbital radius using equation 13. First convert the radius from astronomical units to metres.

$$\begin{aligned} r &= 479.5\ \cancel{\text{AU}} \times \frac{1.49 \times 10^{11}\ \text{m}}{1\ \cancel{\text{AU}}} \\ &= 7.145 \times 10^{13}\ \text{m} \end{aligned}$$

The force of gravity between the Sun and Sedna is the centripetal force:

$$F_c = F_g$$

$$\frac{\cancel{m_{\text{Sedna}}} v^2}{\cancel{r}} = \frac{G\cancel{m_{\text{Sedna}}} m_{\text{Sun}}}{r^{\cancel{2}}}$$

$$v^2 = \frac{Gm_{\text{Sun}}}{\text{r}}$$

$$v = \sqrt{\frac{Gm_{\text{Sun}}}{r}}$$

$$= \sqrt{\frac{\left(6.67 \times 10^{-11}\ \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}\right)(1.99 \times 10^{30}\ \text{kg})}{7.145 \times 10^{13}\ \text{m}}}$$

$$= 1.36 \times 10^3\ \text{m/s}$$

The average orbital speed of Sedna is 1.36×10^3 m/s.

11. ***Analysis and Solution***

$$\frac{{T_\text{E}}^2}{{r_\text{E}}^3} = \frac{T_\text{I}^2}{r_\text{I}^3}$$

$$T_\text{E} = \sqrt{\frac{{r_\text{E}}^3 T_\text{I}^2}{r_\text{I}^3}}$$

$$= \sqrt{\frac{\left(6.710 \times 10^8\ \cancel{\text{m}}\right)^3 (1.769\,\text{d})^2}{\left(4.220 \times 10^8\ \cancel{\text{m}}\right)^3}}$$

$$= 3.547\ \text{d}$$

Europa's mean orbital period is 3.547 d.

12. ***Given***

$r_\text{M} = 1.8552 \times 10^8\,\text{m}$

$T_\text{M} = 0.942\,\text{d}$

Required

mean orbital speed (v)

Analysis and Solution

First convert the orbital period to SI units before substituting them into the equation.

$$T_\text{M} = 0.942\ \cancel{\text{d}} \times \frac{86\ 400\,\text{s}}{\cancel{\text{d}}}$$

$$= 8.139 \times 10^4\ \text{s}$$

$$v = \frac{2\pi r}{T}$$

$$= \frac{2\pi\left(1.8552 \times 10^8\ \text{m}\right)}{8.139 \times 10^4\ \text{s}}$$

$$= 1.43 \times 10^4\ \text{m/s}$$

Paraphrase

The mean orbital speed of Mimas is 1.43×10^4 m/s.

13. ***Analysis and Solution***

$$T = 224.7\ \cancel{\text{d}} \times \frac{86\ 400\ \text{s}}{\cancel{\text{d}}}$$

$$= 1.9414 \times 10^{7}\ \text{s}$$

$$F_{\text{c}} = F_{\text{g}}$$

$$\frac{4\pi^2 \cancel{m_{\text{Venus}}} r}{T^2_{\text{Venus}}} = \frac{G\cancel{m_{\text{Venus}}} m_{\text{Sun}}}{r^2}$$

$$m_{\text{Sun}} = \frac{4\pi^2 r^3}{T^2_{\text{Venus}} G}$$

$$= \frac{4\pi^2 \left(1.08 \times 10^{11}\ \text{m}\right)^3}{\left(1.9414 \times 10^{7}\ \text{s}\right)^2 \left(6.67 \times 10^{-11}\ \frac{\text{N•m}}{\text{s}^2}\right)}$$

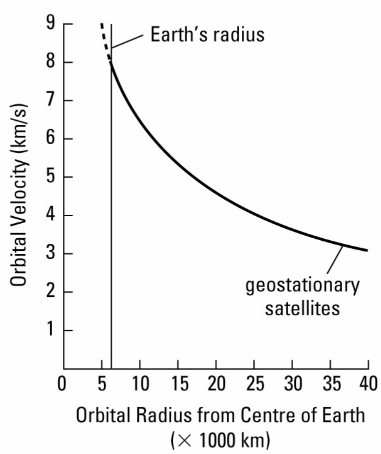
$$= 1.98 \times 10^{30}\ \text{kg}$$

The mass of the Sun is 1.98×10^{30} kg.

Extensions

14. Students' answers will vary. They might suggest that satellites can reserve some fuel so that when they wear out or are no longer useful, they can fire their rockets to put them on a re-entry path into Earth's atmosphere where they will burn up. Conversely, the satellites could push themselves to a higher orbit where they would be out of the way.

15. Graph of Velocity vs. Radius for an Artificial Satellite Orbiting Earth



Student Book pages 288–289

Chapter 5 Review

Knowledge

1. The direction of centripetal acceleration is toward the centre of the circle.

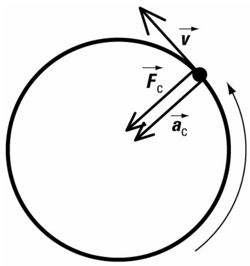
2. (a) The centripetal force is produced by water against the boat's rudder.

(b) The centripetal force is produced by air against the wings, ailerons, and rudder.

(c) The centripetal force is produced by Earth's gravity.

3. The tug your hand feels as you swing an object in a circle on the end of a rope is the reaction force of the rope on your hand. The force your hand exerts on the rope is the centripetal force.

4.



5. A spinning tire stretches because the tire continually tries to move in a straight line. The centripetal force exerted by the hub pulls the tire inward, making it move in a circle.

6. The cable is likely to break at the bottom of its circular path. In this position, the tension in the cable is the greatest, as it alone is providing the centripetal force. At the top of the circular path, the centripetal force is provided by the tension in the cable and the force of gravity.

7. A motorcycle making a turn experiences the same centripetal force a car does. This is the force of friction exerted on the tires by the road.

8. A truck is more massive than a car so the centripetal force needed to make the truck turn the same radius as a car is greater. If the centripetal force provided by friction is not enough, the truck will skid. To prevent this, a truck's turning radius must be larger at high speeds so that the required centripetal force is lower, and it can make the turn safely.

9. A planet moves fastest in its orbit when it is nearest the Sun, at its perihelion.

10. The semi-major axis of an elliptical orbit represents the mean orbital radius of the planet.

11. (a) Equation 14 is $m_{\text{Earth}} = \dfrac{4\pi^2 r^3}{T_{\text{Moon}}^2 G}$; m_{Earth} represents the mass of Earth, and T_{Moon} represents the period of Earth's Moon. Equation 14 is presented with these variables to show how Newton found the mass of Earth.

(b) Equation 14 can be used in the most general case where *m* is the mass of the object being orbited, and *T* is the period of its satellite.

12. Kepler's second law states that a planet, in its orbit around the Sun, will sweep out equal areas in equal times. To do this, the planet must move faster when it is closer to the Sun than when it is farther away.

13. Kepler's constant (*K*) is $1\,\text{y}^2/\text{AU}^3$ and is based on the orbit of Earth around the Sun, so according to Kepler's third law, it is only applicable to other celestial bodies orbiting the same focus (the Sun). It is possible to determine Kepler's constant for any set of bodies orbiting the same focus, but the values will be different.

Applications

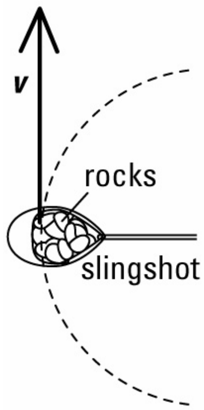
14. ***Analysis and Solution***

$a_c = 4\pi^2 r f^2$ if *f* is doubled: $f \rightarrow 2f$

$$\begin{aligned} a_c &= 4\pi^2 r (2f)^2 \\ &= 4\pi^2 r 4 f^2 \end{aligned}$$

If the frequency of a spinning object is doubled, the centripetal acceleration becomes four times bigger.

15. It must be released at the point where it is moving vertically upward. This occurs when the rope is in the horizontal position.



16. ***Analysis and Solution***

$$F_c = F_g$$

$$\frac{\cancel{m}v^2}{r} = \cancel{m}g$$

$$v = \sqrt{rg}$$

Mass does not play a role in whether people stay in their seats in a roller coaster.

17. ***Analysis and Solution***

$$\begin{aligned} v &= \frac{2\pi r}{T} \\ &= \frac{2\pi(25.0 \text{ m})}{8.0 \text{ s}} \\ &= 2.0 \times 10^1 \text{ m/s} \end{aligned}$$

The speed of the eagle is 2.0×10^1 m/s.

18. ***Analysis and Solution***

$$\begin{aligned} F_c &= \frac{mv^2}{r} \\ &= \frac{(80.0 \text{ kg})\left(7.0 \frac{\text{m}}{\text{s}}\right)^2}{5.0 \text{ m}} \\ &= 7.8 \times 10^2 \text{ N} \end{aligned}$$

The centripetal force exerted on the passenger is 7.8×10^2 N.

19. ***Given***

$r = 200.0$ m

Required

minimum speed the glider must fly (v)

Analysis and Solution

The relation $F_c = F_g$ describes the minimum speed.

$$F_c = F_g$$

$$\frac{\cancel{m}v^2}{r} = \cancel{m}g$$

$$\begin{aligned} v &= \sqrt{rg} \\ &= \sqrt{(200.0\text{ m})(9.81\ \frac{\text{m}}{\text{s}^2})} \\ &= 44.3\text{ m/s} \end{aligned}$$

Paraphrase

The minimum speed the glider must fly to make a perfectly circular vertical loop is 44.3 m/s.

20. ***Given***

m = 800.0 g

r = 60.0 cm

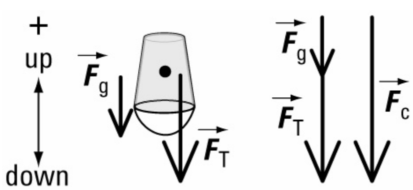
f = 2.0 Hz

Required

tension on the rope at the top of the circle ($\vec{F}_T$)

Analysis and Solution

At the top of the swing, the force of gravity and tension are both acting down. Take down to be negative and up to be positive. The centripetal force is the sum of these two forces given by $\vec{F}_c = \vec{F}_g + \vec{F}_T$. Manipulate this equation to solve for tension.



$$\vec{F}_c = \vec{F}_g + \vec{F}_T$$

$$\vec{F}_T = \vec{F}_c - \vec{F}_g$$

Solve for F_g and F_c separately, then substitute them into the equation. Since both forces are acting down, they will be negative.

$$\begin{aligned} \vec{F}_c &= -4\pi^2 rmf^2 \\ &= -4\pi^2(0.60\text{ m})(0.800\text{ kg})(2.0\,\text{Hz})^2 \\ &= -75.799\,\text{N} \end{aligned}$$

$$\begin{aligned} \vec{F}_g &= mg \\ &= (0.800\,\text{kg})\left(-9.81\frac{\text{m}}{\text{s}^2}\right) \\ &= -7.848\,\text{N} \end{aligned}$$

$$\begin{aligned} \vec{F}_T &= (-75.799\,\text{N}) - (-7.848\,\text{N}) \\ &= -68.0\,\text{N} \end{aligned}$$

Paraphrase

The tension on the rope is 68.0 N [down].

21. ***Given***

$r = 40.0$ m
$m = 1600.0$ kg
$\mu = 0.500$
$v = 30.0\,\text{km/h}$

Required

determine if the car skids off the road

Analysis and Solution

To determine if the car skids off the road, calculate the maximum speed at which the car can round the curve without skidding. Compare it with the speed the car is travelling (30.0 km/h).

To determine the maximum possible speed, use the equality $F_c = F_f$.

$$F_c = F_g$$

$$\frac{mv^2}{r} = \mu F_N \qquad \text{where } F_N = F_g$$

$$\frac{\cancel{m}v^2}{r} = \mu(\cancel{m}g)$$

$$\begin{aligned} v &= \sqrt{\mu r g} \\ &= \sqrt{(0.500)(40.0\,\text{m})\left(9.81\frac{\text{m}}{\text{s}^2}\right)} \\ &= 14.0\,\text{m/s} \\ &= \frac{14.0\,\cancel{\text{m}}}{\cancel{\text{s}}} \times \frac{3600\,\cancel{\text{s}}}{\text{h}} \times \frac{1\,\text{km}}{1000\,\cancel{\text{m}}} \\ &= 50.4\,\text{km/h} \end{aligned}$$

Paraphrase

The maximum possible speed that the car can go around the turn without skidding is 50.4 km/h. The car is travelling at 30.0 km/h, so it won't skid.

22. ***Given***

$D = 25.4$ cm
$f = 750$ rpm

Required

centripetal acceleration at the edge of the blade (F_c)

Analysis and Solution

Determine the centripetal acceleration from the frequency of rotation and the radius. Remember to convert both the frequency and radius to appropriate SI units first.

$D = 25.4\,\text{cm}$ or $0.254\,\text{m}$

$$\begin{aligned} r &= \frac{D}{2} \\ &= \frac{0.254\,\text{m}}{2} \\ &= 0.1270\,\text{m} \end{aligned}$$

$$f = \frac{750\,\text{rpm}}{60\,\text{s}}$$
$$= 12.5\,\text{Hz}$$
$$a_c = 4\pi^2 r f^2$$
$$= 4\pi^2(0.1270\,\text{m})(12.5\,\text{Hz})^2$$
$$= 7.83 \times 10^2\ \text{m/s}^2$$

Paraphrase

The centripetal acceleration at the edge of the blade is 7.83×10^2 m/s^2.

23. ***Analysis and Solution***

$$a_c = 4\pi^2 r f^2$$
$$f = \sqrt{\frac{a_c}{4\pi^2 r}}$$
$$= \sqrt{\frac{8.88\times10^4\ \frac{\text{m}}{\text{s}^2}}{4\pi^2\left(0.90\,\text{m}\right)}}$$
$$= 49.99\ \text{Hz}$$
$$= 49.99\ \text{Hz} \times \frac{60\ \text{s}}{\text{min}}$$
$$= 3.0\times10^3\ \text{rpm}$$

The frequency of the propeller is 3.0×10^3 rpm.

24. ***Given***

v = 107 000 km/h

Required

mathematical verification that the speed of Earth in its orbit around the Sun is 107 000 km/h

Analysis and Solution

Determine the speed of Earth from the Sun's mass and from Earth's mean orbital radius using equation 13. Convert the speed to km/h to compare it with the value given.

$$F_c = F_g$$
$$\frac{\cancel{m}_{\text{Earth}} v^2}{\cancel{r}} = \frac{G\cancel{m}_{\text{Earth}} m_{Sun}}{r^{\cancel{2}}}$$
$$v^2 = \frac{Gm_{\text{Sun}}}{\text{r}}$$
$$v = \sqrt{\frac{Gm_{\text{Sun}}}{r}}$$
$$= \sqrt{\frac{\left(6.67\times10^{-11}\ \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}\right)(1.99\times10^{30}\ \text{kg})}{1.49\times10^{11}\ \text{m}}}$$
$$= 2.98467\times10^4\ \text{m/s}$$

Now convert this speed to km/h:

$$\frac{2.98467\times10^{4}\ \cancel{\text{m}}}{\cancel{\text{s}}}\times\frac{3600\ \cancel{\text{s}}}{\text{h}}\times\frac{1\ \text{km}}{1000\ \cancel{\text{m}}}$$

$=107\ 448$ km/h

Paraphrase

The average orbital speed of Earth is 107 448 km/h.

25. ***Analysis and Solution***

$$a_{\text{c}}=\frac{v^{2}}{r}$$

$$\begin{aligned}v&=\sqrt{a_{\text{c}}r}\\&=\sqrt{\left(6.87\frac{\text{m}}{\text{s}^{2}}\right)(25.0\ \text{m})}\\&=13.1\ \text{m/s}\end{aligned}$$

The speed of the hubcap is 13.1 m/s.

26. (a) ***Given***

$T_{\text{e}}=3.14\times10^{-8}$ s

$r_{\text{e}}=0.300$ m

$m_{\text{e}}=9.11\times10^{-31}$ kg

Required

speed of the electron (v)

Analysis and Solution

Determine the speed of the electron from its orbital period and radius.

$$\begin{aligned}v&=\frac{2\pi r}{T}\\&=\frac{2\pi(0.300\ \text{m})}{3.14\times10^{-8}\ \text{s}}\\&=6.00\times10^{7}\ \text{m/s}\end{aligned}$$

Paraphrase

The speed of the electron is 6.00×10^{7} m/s.

(b) ***Given***

$T_{\text{e}}=3.14\times10^{-8}$ s

$r_{\text{e}}=0.300$ m

$m_{\text{e}}=9.11\times10^{-31}$ kg

Required

centripetal acceleration of the electron (a_{c})

Analysis and Solution

Determine the centripetal acceleration of the electron using equation 5.

$$a_c = \frac{v^2}{r}$$

$$= \frac{\left(6.00 \times 10^7 \ \frac{\text{m}}{\text{s}}\right)^2}{0.300 \text{ m}}$$

$$= 1.20 \times 10^{16} \text{ m/s}^2$$

Paraphrase

The centripetal acceleration of the electron is 1.20×10^{16} m/s^2.

27. ***Analysis and Solution***

$$T_H{}^2 = K r_H{}^3$$

$$r_H = \sqrt[3]{\frac{T_H{}^2}{K}}$$

$$= \sqrt[3]{\frac{(76.5 \text{ y})^2}{1 \frac{\text{y}^2}{\text{AU}^3}}}$$

$$= 18.0 \text{ AU}$$

The mean orbital radius of Halley's comet is 18.0 AU.

28. ***Given***

$T_p = 400.0 \text{ d}$

$r_p = 1.30 \times 10^{11} \text{ m}$

Required

mass of the star being orbited (m_{Star})

Analysis and Solution

Newton's version of Kepler's third law determines the mass of a body being orbited. Make sure to use the appropriate SI units.

$$T_p = 400.0 \ \cancel{\text{d}} \times \frac{86\,400 \text{ s}}{\cancel{\text{d}}}$$

$$= 3.456 \times 10^7 \text{ s}$$

$$F_c = F_g$$

$$\frac{4\pi^2 \cancel{m_p} r}{T_p{}^2} = \frac{G \cancel{m_p} m_{Star}}{r^2}$$

$$m_{Star} = \frac{4\pi^2 r^3}{T_p{}^2 G}$$

$$m_{Star} = \frac{4\pi^2 r_p{}^3}{G T_p{}^2}$$

$$= \frac{4\pi^2 \left(1.30 \times 10^{11} \text{ m}\right)^3}{\left(6.67 \times 10^{-11} \ \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}\right)\left(3.456 \times 10^7 \text{ s}\right)^2}$$

$$= 1.09 \times 10^{30} \text{ kg}$$

Paraphrase

The mass of the star is 1.09×10^{30} kg.

29. (a) ***Analysis and Solution***

$$\begin{aligned} T &= \frac{2\pi r}{v} \\ &= \frac{2\pi\left(2.27\times10^{20}\ \cancel{\text{m}}\right)}{1234\ \frac{\cancel{\text{m}}}{\text{s}}} \\ &= 1.16\times10^{18}\ \text{s} \\ &= 3.68\times10^{10}\ \text{y} \end{aligned}$$

The period of our solar system in its orbit around the black hole is 1.16×10^{18} s or 36.8 billion years.

(b) ***Given***

$T_S = 1.16\times10^{18}$ s

$r_S = 1.30\times10^{11}$ m

Required

mass of the black hole (m_B)

Analysis and Solution

Since our Sun is orbiting the black hole at the centre of our galaxy, use Newton's version of Kepler's third law to determine the black hole's mass.

$$F_c = F_g$$

$$\frac{4\pi^2 \cancel{m_{\text{Sun}}} r}{T^2_{\text{Sun}}} = \frac{G\cancel{m_{\text{Sun}}} m_B}{r^2}$$

$$\begin{aligned} m_B &= \frac{4\pi^2 r^3}{T^2_{\text{Sun}} G} \\ &= \frac{4\pi^2\left(2.27\times10^{20}\ \text{m}\right)^3}{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(1.16\times10^{18}\ \text{s}\right)^2} \\ &= 5.15\times10^{36}\ \text{kg} \end{aligned}$$

Paraphrase

The mass of the black hole is 5.15×10^{36} kg.

(c) ***Analysis and Solution***

$$\begin{aligned} a_c &= \frac{v^2}{r} \\ &= \frac{\left(1234\ \frac{\text{m}}{\text{s}}\right)^2}{2.27\times10^{20}\ \text{m}} \\ &= 6.71\times10^{-15}\ \text{m/s}^2 \end{aligned}$$

The centripetal acceleration of our solar system caused by the black hole is 6.71×10^{-15} m/s^2.

30. (a) ***Given***

$r_E = 6.37 \times 10^6$ m

Required

speed of the cannon ball (v)

Analysis and Solution

Determine the speed by treating the cannonball as a satellite and equating $F_c = F_g$.

$$F_c = F_g$$

$$\frac{\cancel{m}v^2}{r} = \cancel{m}g \quad \text{where } r \text{ is the radius of Earth}$$

$$\frac{v^2}{r} = g$$

$$\begin{aligned} v &= \sqrt{rg} \\ &= \sqrt{(6.37 \times 10^6 \text{ m})\left(9.81 \frac{\text{m}}{\text{s}^2}\right)} \\ &= 7.91 \times 10^3 \text{ m/s} \end{aligned}$$

Paraphrase

The cannonball must travel at a speed of 7.91×10^3 m/s to orbit Earth at the surface.

(b) ***Given***

$v = 7.91 \times 10^3$ m/s

$r = 6.37 \times 10^6$ m

Required

mass of Earth (m_{Earth})

Analysis and Solution

Determine the mass of Earth from Newton’s version of Kepler’s third law.

$$F_c = F_g$$

$$\frac{\cancel{m_{\text{cannonball}}}v^2}{\cancel{r}} = \frac{G\cancel{m_{\text{cannonball}}}m_{Earth}}{r^{\cancel{2}}}$$

$$v^2 r = Gm_{\text{Earth}}$$

$$\begin{aligned} m_{\text{Earth}} &= \frac{v^2 r}{G} \\ &= \frac{(7.91 \times 10^3 \frac{\text{m}}{\text{s}})^2 (6.37 \times 10^6 \frac{\text{m}}{\text{s}})}{6.67 \times 10^{-11} \frac{\text{N•m}^2}{\text{kg}^2}} \\ &= 5.98 \times 10^{24} \text{ kg} \end{aligned}$$

Paraphrase

The mass of Earth is 5.98×10^{24} kg.

(c) ***Analysis and Solution***

The mass of the cannonball is not related to its orbital speed.

$$v = \frac{2\pi r_{\text{Earth}}}{T}$$

$$T = \frac{2\pi r_{\text{Earth}}}{v}$$

$$= \frac{2\pi(6.37 \times 10^6\ \cancel{\text{m}})}{7.91 \times 10^3\ \frac{\cancel{\text{m}}}{\text{s}}}$$

$$= 5.06 \times 10^3\ \text{s}$$

The time it takes the cannonball to orbit Earth once is independent of its mass, and is 5.06×10^3 s.

31. ***Analysis and Solution***

$$T = 27.3\,\text{d}$$

$$T = 2.359 \times 10^6\,\text{s}$$

$$a_c = \frac{4\pi^2 r_{\text{Earth}}}{T^2}$$

$$= \frac{4\pi^2 \left(3.844 \times 10^8\ \text{m}\right)}{\left(2.359 \times 10^6\ \text{s}\right)^2}$$

$$= 0.002\,73\,\text{m/s}^2$$

$$F_c = ma_c$$

$$= \left(7.36 \times 10^{22}\ \text{kg}\right)\left(0.002\,73 \frac{\text{m}}{\text{s}^2}\right)$$

$$= 2.01 \times 10^{20}\ \text{N}$$

The centripetal acceleration of the Moon in orbit around Earth is 0.002 73 m/s^2.
The Moon experiences a centripetal force of 2.01×10^{20} N.

32. ***Given***

$$r_G = 6.20 \times 10^7\ \text{m}$$

Required

Galatea's orbital period (T_G)

Analysis and Solution

Use the orbital period and radius of one of Neptune's moons, such as Nereid, to find the orbital period of Galatea using Kepler's third law.

$$T_N = 360.14\ \text{d}$$

$$r_N = 5.5134 \times 10^9\ \text{m}$$

$$\frac{T_G^{\ 2}}{r_G^{\ 3}} = \frac{T_N^{\ 2}}{r_N^{\ 3}}$$

$$T_G = \sqrt{\frac{T_N^{\ 2} r_G^{\ 3}}{r_N^{\ 3}}}$$

$$= \sqrt{\frac{(360.14\,\text{d})^2 \left(6.20\times10^7\ \text{m}\right)^3}{\left(5.513\times10^9\ \text{m}\right)^3}}$$

$$= 0.430\ \text{d}$$

$$= 0.430\ \cancel{\text{d}} \times \frac{86\ 400\ \text{s}}{\cancel{\text{d}}}$$

$$= 3.72\times10^4\ \text{s}$$

Paraphrase

The orbital period of Galatea is 0.430 d or 3.72×10^4 s.

33. ***Given***

$m_U = 8.67\times10^{25}$ kg

$r = 1.909\ 45 \times 10^8$ m

Required

speed of Ariel (v)

Analysis and Solution

$$F_c = F_g$$

$$\frac{\cancel{m_A} v^2}{\cancel{r}} = \frac{G \cancel{m_A} m_U}{r^{\cancel{2}}}$$

$$v^2 = \frac{Gm_U}{\text{r}}$$

$$v = \sqrt{\frac{Gm_U}{r}}$$

$$= \sqrt{\frac{\left(6.67\times10^{-11}\ \frac{\text{N}\bullet\text{m}^2}{\text{kg}^2}\right)(8.67\times10^{25}\ \text{kg})}{1.909\,45\times10^8\ \text{m}}}$$

$$= 5.50\times10^3\ \text{m/s}$$

Paraphrase

The speed of Ariel is 5.50×10^3 m/s.

34. (a) ***Given***

$T = 4.46\ \text{x}\ 10^4$ s

$r = 2.38\ \text{x}\ 10^{10}$ m

Required

mass of star (m_{Star})

Analysis and Solution

$$F_c = F_g$$

$$\frac{4\pi^2 \cancel{m_p} r}{T^2} = \frac{G\cancel{m_p} m_{Star}}{r^2}$$

$$m_{Star} = \frac{4\pi^2 r^3}{T^2 G}$$

$$= \frac{4\pi^2 \left(2.38\times10^{10}\ \text{m}\right)^3}{\left(4.46\times10^4\ \text{s}\right)^2 \left(6.67\times10^{-11}\ \frac{\text{N•m}^2}{\text{kg}^2}\right)}$$

$$= 4.01\times10^{33}\ \text{kg}$$

Paraphrase

The mass of the star being orbited is 4.01×10^{33} kg.

(b) ***Given***

$T_A = 4.46 \text{ x } 10^4$ s

$r_A = 2.38 \text{ x } 10^{10}$ m

Required

orbital radius of second planet (r_B)

Analysis and Solution

$$\frac{{T_A}^2}{{r_A}^3} = \frac{{T_B}^2}{{r_B}^3}$$

$$r_B = \sqrt[3]{\frac{{r_A}^3 {T_B}^2}{{T_A}^2}}$$

$$= \sqrt[3]{\frac{\left(2.38\times10^{10}\ \text{m}\right)^3 \left(6.19\times10^6\ \text{s}\right)^2}{\left(4.46\times10^4 \text{s}\right)^2}}$$

$$= 6.38\times10^{11}\ \text{m}$$

The orbital radius of the second planet is 6.38×10^{11} m.

Extensions

35. Students' answers will vary. Two misconceptions were:
(i) Centripetal force acts radially outward.
(ii) Centripetal force is a force unto itself.

36. A person standing at the equator is moving around in a circle that has a radius of 6.37×10^6 m, and a period of 24 h. As a result, the person experiences a centripetal force given by $F_c = \frac{m4\pi^2 r}{T^2}$, which is the force of gravity.

A person standing on one of the poles is standing on Earth's axis of rotation. The person's period is the same: 24 h, but the radius of the circular path is zero. Therefore, the centripetal force is zero, even though the person experiences the force of gravity.

